Towards a logical model of some aspects of lexical semantics

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joint work with Bruno Mery and Christian Bassac
Lexical Semantics within Compositional Semantics

- A not-so-recent problem: polysemy
- Sense disambiguation and lexical semantics
- Linguistic and background knowledge
- The advent of the Generative Lexicon
- A gap within the formalism
Typical examples from Pustejovsky’s Generative Lexicon

- Qualia
  - A quick cigarette (telic)
  - A partisan article (agentive)

- Dot Objects
  - An interesting book (I)
  - A heavy book (ϕ)
  - A large city (T)
  - A cosmopolitan city (P)

- Co-predications
  - A heavy, yet interesting book
  - Paris is a large, cosmopolitan city
  - ? A fast, delicious salmon
  - ?? Washington is a small city and signed a trade agreement with Paris
Back to the roots: Montague semantics. Types.

Simply typed lambda terms \( types ::= e \mid t \mid types \rightarrow types \)

\( chair, sleep e \rightarrow t \)

\( likes \) transitive verb \( e \rightarrow (e \rightarrow t) \)
Back to the roots: Montague semantics. Syntax/semantics.

<table>
<thead>
<tr>
<th>(Syntactic type)*</th>
<th>=</th>
<th>Semantic type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^*$</td>
<td>=</td>
<td>$t$</td>
</tr>
<tr>
<td>$np^*$</td>
<td>=</td>
<td>$e$</td>
</tr>
<tr>
<td>$n^*$</td>
<td>=</td>
<td>$e \rightarrow t$</td>
</tr>
<tr>
<td>...</td>
<td>=</td>
<td>...</td>
</tr>
</tbody>
</table>
Back to the roots: Montague semantics. Logic within lambda-calculus 1/2.

Logical operations (and, or, some, all the,.....) need constants:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>∃</td>
<td>((e \rightarrow t) \rightarrow t)</td>
</tr>
<tr>
<td>∀</td>
<td>((e \rightarrow t) \rightarrow t)</td>
</tr>
<tr>
<td>∧</td>
<td>(t \rightarrow (t \rightarrow t))</td>
</tr>
<tr>
<td>∨</td>
<td>(t \rightarrow (t \rightarrow t))</td>
</tr>
<tr>
<td>⊃</td>
<td>(t \rightarrow (t \rightarrow t))</td>
</tr>
</tbody>
</table>
Back to the roots: Montague semantics. Logic within lambda-calculus 2/2.

Words in the lexicon need constants for their denotation:

<table>
<thead>
<tr>
<th>likes</th>
<th>( \lambda x \lambda y \text{(likes } y \text{)} x )</th>
<th>( x : e, y : e, \text{likes} : e \to (e \to t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;&lt; likes &gt;&gt;</td>
<td>is a two-place predicate</td>
<td></td>
</tr>
<tr>
<td>Garance</td>
<td>( \lambda P (P \text{ Garance}) )</td>
<td>( P : e \to t, \text{Pierre} : e )</td>
</tr>
<tr>
<td>&lt;&lt; Garance &gt;&gt;</td>
<td>is viewed as</td>
<td></td>
</tr>
<tr>
<td></td>
<td>the properties that &lt;&lt; Garance &gt;&gt; holds</td>
<td></td>
</tr>
</tbody>
</table>
Back to the roots: Montague semantics. Computing the semantics. 1/5

1. Replace in the lambda-term issued from the syntax the words by the corresponding term of the lexicon.

2. Reduce the resulting $\lambda$-term of type $t$ its normal form corresponds to a formula, the "meaning".
## Back to the roots: Montague semantics. Computing the semantics. 2/5

<table>
<thead>
<tr>
<th>word</th>
<th>semantic type $u^*$</th>
<th>semantics: $\lambda$-term of type $u^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>some</td>
<td>$(e \to t) \to ((e \to t) \to t)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda P e \to t \lambda Q e \to t \ (\exists (e \to t) \to t \ (\lambda x e (\land (t \to (t \to t)) (P x) (Q x))))$</td>
<td></td>
</tr>
<tr>
<td>statements</td>
<td>$e \to t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda x e (\text{statement}_{e \to t} \ x)$</td>
<td></td>
</tr>
<tr>
<td>speak about</td>
<td>$e \to (e \to t)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda y e \lambda x e ((\text{speak about}_{e \to (e \to t)} \ x) \ y)$</td>
<td></td>
</tr>
<tr>
<td>themselves</td>
<td>$(e \to (e \to t)) \to (e \to t)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\lambda P e \to (e \to t) \lambda x e ((P x) x)$</td>
<td></td>
</tr>
</tbody>
</table>
Back to the roots: Montague semantics. Computing the semantics. 3/5

The syntax (e.g. a Lambek categorial grammar) yields a $\lambda$-term representing this deduction simply is

$$((\text{some statements}) \ (\text{themselves speak\_about}))$$ of type $t$
Back to the roots: Montague semantics. Computing the semantics. 4/5

\[
\begin{align*}
\left( \lambda P_{e \rightarrow t} \lambda Q_{e \rightarrow t} (\exists_{(e \rightarrow t) \rightarrow t} (\lambda x_e (\wedge (P x)(Q x)))) \right) \\
\left( \lambda P_{e \rightarrow (e \rightarrow t)} \lambda x_e ((P x)x) \right) \\
\downarrow \beta \\
\left( \lambda Q_{e \rightarrow t} (\exists_{(e \rightarrow t) \rightarrow t} (\lambda x_e (\wedge_{(e \rightarrow t) \rightarrow t} (\text{statement}_{e \rightarrow t} x)(Q x)))) \right) \\
\left( \lambda x_e ((\text{speak\_about}_{e \rightarrow (e \rightarrow t)} x)x) \right) \\
\downarrow \beta \\
\left( \exists_{(e \rightarrow t) \rightarrow t} (\lambda x_e (\wedge (\text{statement}_{e \rightarrow t} x)((\text{speak\_about}_{e \rightarrow (e \rightarrow t)} x)x))) \right)
\end{align*}
\]
Back to the roots: Montague semantics. Computing the semantics. 5/5

This term represent the following formula of predicate calculus (in a more pleasant format):

$$\exists x : e (\text{statement}(x) \land \text{speak\_about}(x, x))$$

This is the semantics of the analyzed sentence.
More general types and terms. Many sorted logic. \( TY_n \)

Extension to \( TY_n \) without difficulty nor suprise: \( e \) can be divided in several kind of entities (a kind of a flat ontology).
More general types and terms. Second order types.

One can also add type variables and quantification over types.

- Constants $e$ and $t$, as well as any type variable $\alpha$ in $P$, are types.

- Whenever $T$ is a type and $\alpha$ a type variable which may but need not occur in $T$, $\Lambda \alpha. T$ is a type.

- Whenever $T_1$ and $T_2$ are types, $T_1 \to T_2$ is also a type.
More general types and terms. Second order terms.

- A variable of type $T$ i.e. $x : T$ or $x^T$ is a term. [For each type, a denumerable set of variables of this type.]

- $(f \tau)$ is a term of type $U$ whenever $\tau : T$ and $f : T \rightarrow U$.

- $\lambda x^T. \tau$ is a term of type $T \rightarrow U$ whenever $x : T$, and $\tau : U$.

- $\tau\{U\}$ is a term of type $T[U/\alpha]$ whenever $\tau : \Lambda \alpha. T$, and $U$ is a type.

- $\Lambda \alpha. \tau$ is a term of type $\Lambda \alpha.T$ whenever $\alpha$ is a type variable, and $\tau : T$ without any free occurrence of the type variable $\alpha$.
More general types and terms. Second order reduction.

The reduction is defined as follows:

- $(\Lambda \alpha \cdot \tau)\{U\}$ reduces to $\tau[U/\alpha]$ (remember that $\alpha$ and $U$ are types).

- $(\lambda x \cdot \tau)u$ reduces to $\tau[u/x]$ (usual reduction).
More general types and terms. A second order example.

How to coordinate over any type of entities predicates $P^{\alpha \rightarrow t}$ and $Q^{\beta \rightarrow t}$ over entities of respective kinds $\alpha$ and $\beta$

when we have a morphism from any type $\xi$ to $\alpha$ and one from $\xi$ to $\beta$?

$\Lambda \xi \lambda x^{\xi} \lambda f^{\xi \rightarrow a} \lambda g^{\xi \rightarrow b}.(\text{and } (P( f \ x))(Q( g \ x)))$

One can even quantify over the predicates $P, Q$ and the types $\alpha, \beta$ to which they apply:

$\Lambda \alpha \Lambda \beta \lambda P^{\alpha \rightarrow t} \lambda Q^{\beta \rightarrow t} \Lambda \xi \lambda x^{\xi} \lambda f^{\xi \rightarrow a} \lambda g^{\xi \rightarrow b}.(\text{and } (P( f \ x))(Q( g \ x)))$
Principles of our lexicon

- Remain within reach of Montagovian compositional semantics
- Allow both predicate and argument to contribute lexical information to the compound
- Integrate within existing discourse models

We advocate a system based on *optional modifiers*. 
Overview of the Lexicon

How much information should a lexicon store?

- Basic compositional data: number, type, optional character of arguments
- Lexical data for adaptations: qualia, dot objects...
- Constraints on modifiers induced by lexical data
- Interpretation(s) of each term
The Types

- Montagovian composition:
  - Predicate include the typing and the order of its arguments.

- Generative Lexicon style concept hierarchy:
  - Types are different for every distinct lexical behavior
  - A kind of ontology details the specialization relations between types
  - The result is close to a language-independent hierarchy of concepts

Second-order typing, like Girard’s F system is needed for arbitrary modifiers:

\[
\Lambda \alpha \lambda x^A y^\alpha \ f^{\alpha \rightarrow R} \cdot ((\text{read}^{A \rightarrow R \rightarrow t} \ x) \ (f \ y))
\]
The Terms: main / standard term

• A standard λ-term attached to the main sense:
  – Used for compositional purposes
  – Comprising detailed typing information
  – Including slots for optional modifiers
  – $\Lambda \alpha \beta \lambda x^\alpha y^\beta f^{\alpha \rightarrow A} g^{\beta \rightarrow F}.((\text{eat}^{A \rightarrow F \rightarrow t}(f\ x))(g\ y))$
  – Paris$^T$
The Terms: Optional Morphisms

- Each a one-place predicate
- Used, or not, for adaptation purposes
- Each associated with a constraint: local, global, ∅

\[
\begin{align*}
&\text{(local)} \\
&\text{global} \\
&\varnothing
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix}
\text{Id}_{F \to F} & f_{\text{Living} \to F} \\
\emptyset & \text{grind}_{\text{global}}
\end{pmatrix} \\
\begin{pmatrix}
\text{Id}_{T \to T} & f_{T \to L} & f_{T \to P} & f_{T \to G} \\
\emptyset & \emptyset & \emptyset & \text{global}
\end{pmatrix}
\]
A Complete Lexical Entry

Every lexeme is associated to an $n$-uple such as:

$$\left( \text{Paris}^T, \frac{\lambda x^T \cdot x^T}{\emptyset}, \frac{\lambda x^T \cdot (f_{L\rightarrow L}^T x)}{\emptyset}, \frac{\lambda x^T \cdot (f_{P\rightarrow P}^T x)}{\emptyset}, \frac{\lambda x^T \cdot (f_{G\rightarrow G}^T x)}{\text{global}} \right)$$
Global vs local use of optional morphisms. GLOBAL

Type clash: \((\lambda x^V. (P^{V\rightarrow W} x)) \tau^U\)

\[(\lambda x^V. (P^{V\rightarrow W} x)) (f^{U\rightarrow V} \tau^U)\]

\(f\): optional term associated with either \(P\) or \(\tau\)

\(f\) applies once to the argument and not to the several occurrences of \(x\).

A conjunction yields \((\lambda x^V. (\land (P^{V\rightarrow W} x) (Q^{V\rightarrow W} x))) (f^{U\rightarrow V} \tau^U)\), the argument is uniformly transformed.

Second order is not needed, the type \(V\) of the argument is known and it is always the same for every occurrence of \(x\).
Global vs local use of optional morphisms. LOCAL

Type clash(es): $(\lambda x^? \cdot (\cdots (P^A \to^X x^?) \cdots (Q^B \to^Y x^?) \cdots) \tau^U [? = A = B \text{ e.g. } e \to t])$

$(\Lambda \xi. \lambda f^\xi \to^A. \lambda g^\xi \to^B. (\cdots (P^A \to^X (f^\xi x^\xi)) \cdots (Q^B \to^Y (g^\xi x^\xi)) \cdots)) \{U\} f^U \to^A g^U \to^B \tau^U$

$f, g$: optional terms associated with either $P$ or $\tau$.

This can be done for all the occurrences of $x$ and different $\alpha$ and different $f$ can be used each time.

Second order typing is required to anticipate the yet unknown type of the argument and to factor the different types for $f$ that will be use in the slots.

The types $\{U\}$ and the associated morphism $f$ are inferred from the original formula $(\lambda x^V. (P^V \to^W x)) \tau^U$. 
Standard behaviour

$\phi$: physical objects

small stone

$$\lambda x. \varphi \cdot (\text{small} \, \varphi \rightarrow \varphi \, x)$$

$$(\text{small} \, \tau) \varphi$$
Qualia exploitation

wondering, loving smile

\[
\begin{align*}
(\lambda x^P. (\text{and}^{t\to(t\to t)} (\text{wondering}^{P\to t} x) (\text{loving}^{P\to t} x))) \\
(\lambda x^P. (\text{and}^{t\to(t\to t)} (\text{wondering}^{P\to t} x) (\text{loving}^{P\to t} x))) (f_a^{S\to P} \tau^S) \\
(\text{and} (\text{loving} (f_a \tau)) (\text{loving} (f_a \tau)))
\end{align*}
\]
Facets (dot-objects): incorrect copredication

Incorrect co-predication. The global constraint blocks the copredication e.g. $f^g_{F_S \rightarrow F_d}$ cannot be *globally* used in

(??) *The tuna we had yesterday was lightning fast and delicious.*
Facets, correct co-predication. Town example 1/3

\[
T \text{ town} \quad L \text{ location} \quad P \text{ people} \\
\mathcal{f}_p^{T \rightarrow P} \quad \mathcal{f}_l^{T \rightarrow L} \quad k^T \text{ København} \\
\]

København is both a seaport and a cosmopolitan capital.
Conjunction of $\text{cospl}^{P \rightarrow t}$, $\text{cap}^{T \rightarrow t}$ and $\text{port}^{L \rightarrow t}$, applied to $tk^T$

If $T = P = L = e$, (Montague)

$$(\lambda x^e (\text{and}^{t \rightarrow (t \rightarrow t)} (\text{and}^{t \rightarrow (t \rightarrow t)} (\text{cospl} x) (\text{cap} x)) (\text{port} x))) k.$$ 

**AND** between three predicates over different kinds $P^\alpha \rightarrow t$, $Q^\beta \rightarrow t$, $R^\gamma \rightarrow t$

$$\Lambda \alpha \Lambda \beta \lambda P^\alpha \rightarrow t \lambda Q^\beta \rightarrow t \lambda R^\gamma \rightarrow t \Lambda \xi \lambda x^\xi \lambda f^\xi \rightarrow ^\alpha \lambda g^\xi \rightarrow ^\beta \lambda h^\xi \rightarrow ^\gamma. \text{ (and) (and) (P (f x)) (Q (g x)) (R (h x))}$$

The morphisms $f$, $g$ and $h$ convert $x$ to **different** types.
Facets, correct co-predication. Town example 3/3

AND applied to $P$ and $T$ and $L$ and to $\text{cospl}^{P\rightarrow t}$ and $\text{cap}^{T\rightarrow t}$ and $\text{port}^{L\rightarrow t}$ yields:

$$ \Lambda \xi \lambda x^{\xi} \lambda f^{\xi \rightarrow \alpha} \lambda g^{\xi \rightarrow \beta} \lambda h^{\xi \rightarrow \gamma}. (\text{and}(\text{and}(\text{cospl}^{P\rightarrow t} (f_p x)) (\text{cap}^{T\rightarrow t} (f_i x)))(\text{port}^{L\rightarrow t} (f_i x))) $$

We now wish to apply this to the type $T$ and to the transformations provided by the lexicon. No type clash with $\text{cap}^{T\rightarrow t}$, hence $\text{id}^{T\rightarrow T}$ works. For $L$ and $P$ we use the transformations $f_p$ and $f_i$.

$$(\text{and}^{t\rightarrow (t\rightarrow t)} (\text{and}^{t\rightarrow (t\rightarrow t)} (\text{cospl} (f_p k^T)^P)^t) (\text{cap} (\text{id} k^T)^T)^t) (\text{port} (f_i k^T)^L)^t)$$
Importing an existing lexicon

- Main type and argument structure: main $\lambda$-term
- \textit{Qualia}-roles: local modifiers
- Dot objects: local modifiers
- Some specific constructions are global modifiers (e.g. grinding).
- Inheritance structure: local modifier $\rightarrow$ parent
The calculus, summarized

- First-order $\lambda$-bindings: usual composition
- Open slots: generate all combinations of modifiers available
- As many interpretations as well-typed combinations

Paris is an populous city by the Seine river

\[(\Lambda x^\xi. \lambda x^\xi f^\xi \rightarrow ^P g^\xi \rightarrow ^L. (\text{and} (\text{populous}^P \rightarrow ^t (f \ x)) (\text{riverside}^L \rightarrow ^t (g \ x))))\]

\[\{T\} \ Paris^T \ \lambda x^T (f^T \rightarrow ^P x) \ \lambda x^T \ . (f^T \rightarrow ^L x)\]
Logical Formulae

- Many possible results
- Our choice: classical, higher-order predicate logic
- No modalities

\[ \text{and}(\text{populous}(f_P(\text{Paris}), \text{riverside}(f_L(\text{Paris})))) \]
Intermezzo: my favorite puzzle. Situation.

A shelf.

Three copies of *Madame Bovary*.

The collected novels of Flaubert in one volume (L’éducation sentimentale, Madame Bovary, Bouvard et Pécuchet)

One copy of *Jacques le fataliste*.

The volume also contains *Trois contes: Un coeur simple, La légende de Saint-Julien, Salammbô*
Intermezzo: my favorite puzzle. Questions.

- I carried down all the books to the cellar.
- Indeed, I read them all.
- How many books did you carry?
- How many books did you read?
Critics

- The classical solution with products: $\langle p_1(u), p_2(u) \rangle = u$

- (Asher’s solution with pullbacks) too tight relation type structure / morphisms (only and always canonical morphisms) and unavoidable relation to product

- (Ours) not enough relation types/morphisms (no relation at all), typing does not constrain morphisms,
Linear alternative

Direct representation with monoidal product $A \otimes B$ and replication!

- $A \otimes B$
  - without $\langle p_1(u), p_2(u) \rangle = u$
  - without canonical morphism
  - but the type of a transformation relates to the structure of the type.

- Types of morphims in a linear setting ($\vdash$ being $\rightarrow$) either:
  - irreversible: $A \rightarrow U$ since $A \not\rightarrow U \otimes A$
  - reusable: $A \rightarrow B = (!A) \rightarrow U$ since $(!A) \rightarrow U \otimes (!A)$

This leads to general questions....
Which logic for semantics? Linear Logic?

Two kind of logics:

• glue language?
  – usually base types $e, t$ constructor $\to$
  – not rich enough
  – composition better handled with linear types

• language of semantic representation
  – usually undefined, fragment of Higher Order Logic
  – too rich, but not enough fine grained enough. Linear logic?
A natural representation (too natural?)

In the usual system we use the following: if the lambda constants are connectives, quantifiers and relational or functional symbols, then every closed term of type $t$ is a formula, etc.

What about a closed term of type $e \rightarrow (e \otimes t)$ and other complex types.
Interpretation, models

- usually possible worlds
- too large, uncomputable, ...
- no well defined, unless free or categorical semantics
- can we use models of linear logic (of formulae or of composition)
Argument for and against linear logic. For.

For:

- Refined both for semantic representation and as a description of the computation leading to these representations.

- Encode usual formulae and even usual typed lambda calculus.
Argument for and against linear logic. Against.

Against

- As opposed to usual semantics, no good model of first order. Phase valued models unnatural, *ad hoc*

- Models of composition computation cannot handle proper axioms — coherence spaces (Scott domains), ludics (game semantics)

[Both are even needed for maths, hence for linguistics...]

A direction that I am exploring (me but also Melliès, Lamarche,) refinement *sheaves models* of intuitionistic logic (topoi, local notion — Grothendieck, Lawvere, Lambek)
Yet more general questions. 1

performance / competence

cognitive experiments versus formal computational complexity

Algorithmic complexity not adapted. Logical complexity:

- → nesting \((e \rightarrow t) \rightarrow t\)

- quantifier alternation

- order (individuals, predicates, predicates of predicates,...)
Yet more general questions. 2

How things are and works / How a specific language describes this

Ambiguity: does the lexicon (e.g. qualia structure) describe

• the world of the discourse universe (ontology)

• or a language dependent ontology:

  Ma voiture est crevée. even J’ai crevé. (une roue de ma voiture est crevée).
  * Ma voiture est bouchée. (le carburateur)    or    * Ma voiture est à plat. (la batterie)

Cross linguistic comparisons?
book and livre are already different wrt. the quantificational puzzle.