

Using non-convex approximations for efficient analysis of timed automata

B. Srivathsan¹

Joint work with F. Herbreteau¹, D. Kini² and I. Walukiewicz¹

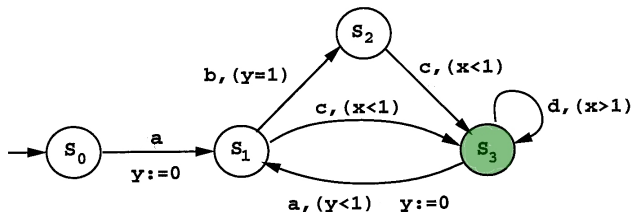
LaBRI, Université Bordeaux 1

Indian Institute of Technology Bombay, India

Verification Seminar

Université Libre de Bruxelles

Timed Automata [AD94]



Run: finite **sequence** of transitions,

$$(s_0, \overbrace{0}^x, \overbrace{0}^y) \xrightarrow{0.4, a} (s_1, 0.4, 0) \xrightarrow{0.5, c} (s_3, 0.9, 0.5)$$

- ▶ A run is **accepting** if it ends in a **green** state.

The problem we are interested in ...

Given a TA, does there **exist** an **accepting run**?

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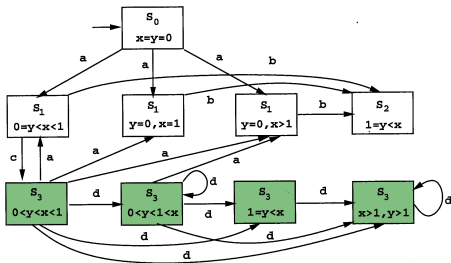
Given a TA, does there **exist** an **accepting run**?

Theorem [AD94, CY92]

This problem is **PSPACE-complete**

First solution to this problem

Key idea: Partition the space of valuations into a **finite** number of **regions**



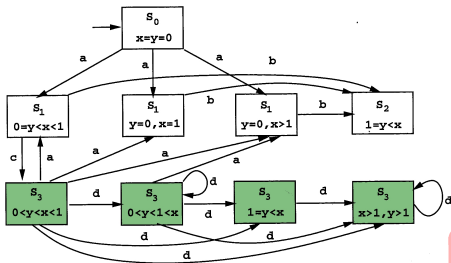
- ▶ **Region:** set of valuations satisfying the **same guards** w.r.t. time
- ▶ **Finiteness:** Parametrized by **maximal constant**

Sound and complete [AD94]

Region graph preserves state **reachability**

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► **Finiteness:** Parametrized by **maximal constant**

$\mathcal{O}(|X|! \cdot M^{|X|})$ many regions!

Sound and complete [AD94]

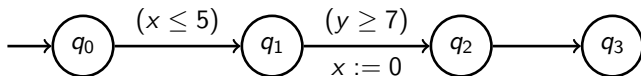
Region graph preserves state **reachability**

A more efficient solution...

Key idea: Maintain **all valuations** reachable along a path

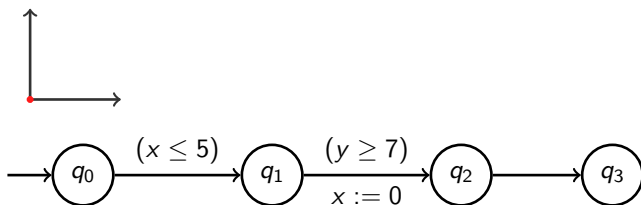
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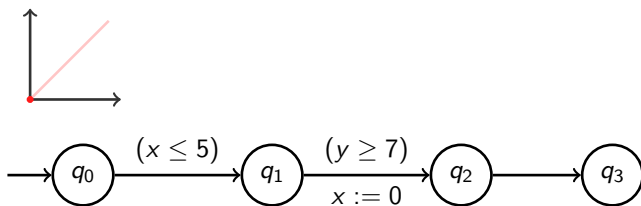
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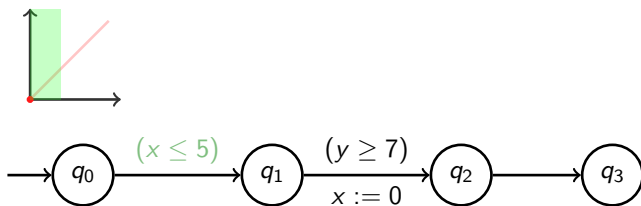
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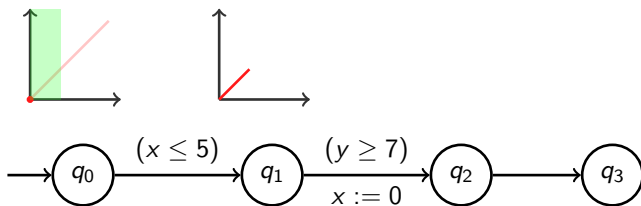
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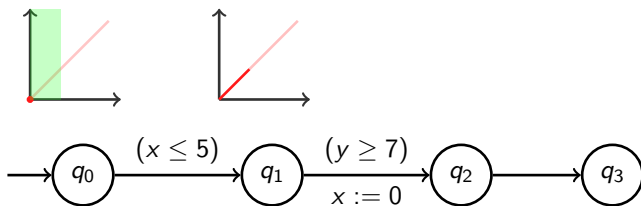
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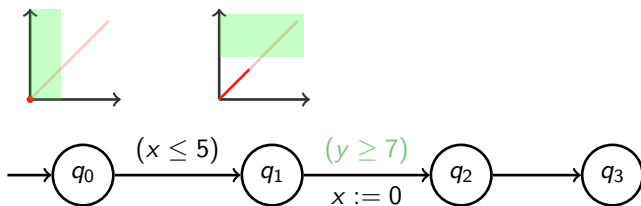
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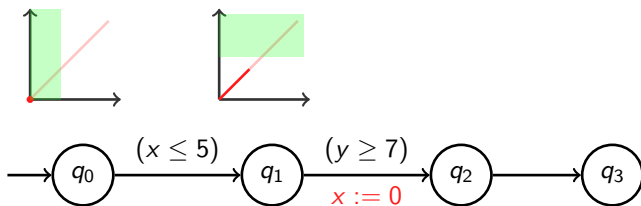
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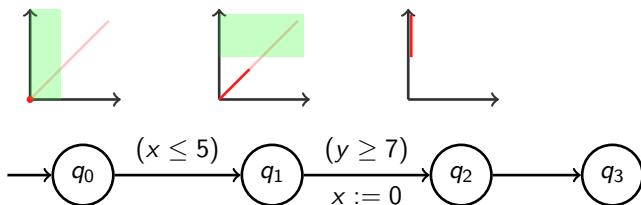
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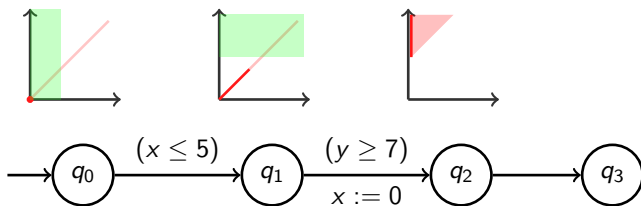
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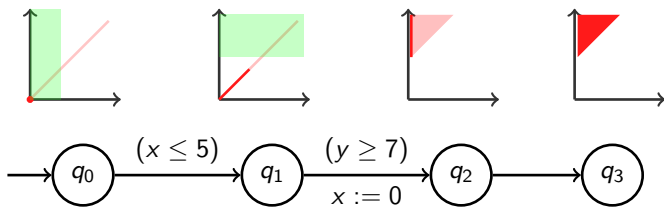
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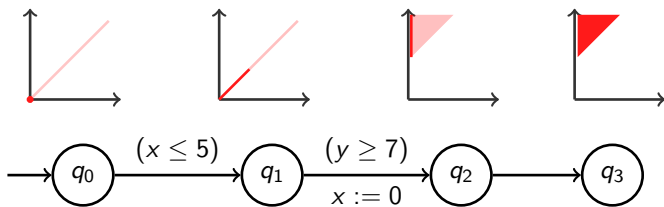
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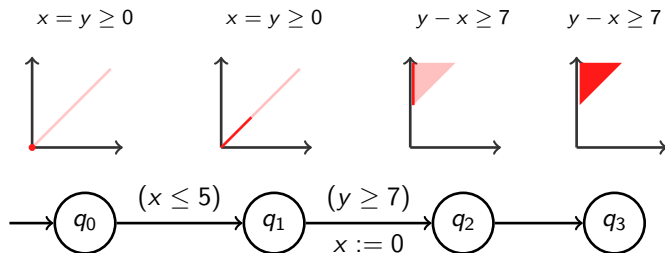
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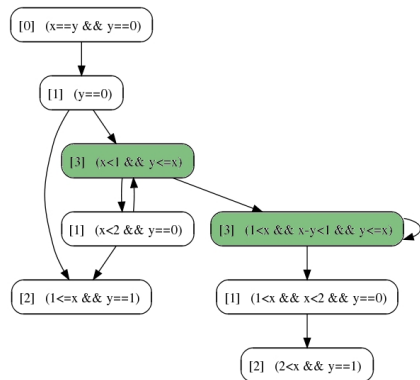
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Zones and zone graph

- ▶ **Zone:** set of valuations defined by conjunctions of constraints:
 - ▶ $x \sim c$
 - ▶ $x - y \sim c$
 - ▶ e.g. $(x - y \geq 1) \wedge y < 2$
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Zones and zone graph



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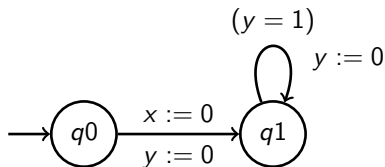
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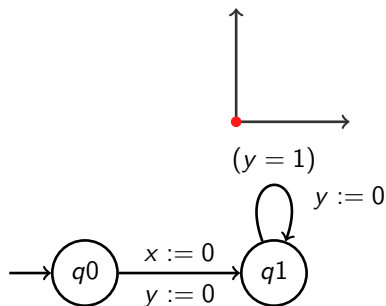
Sound and complete [DT98]

Zone graph preserves state **reachability**

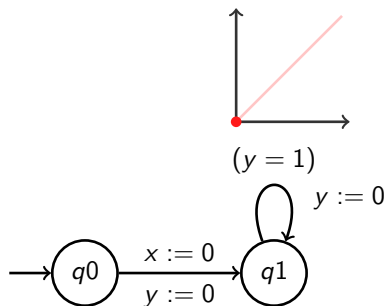
But the zone graph could be infinite ...



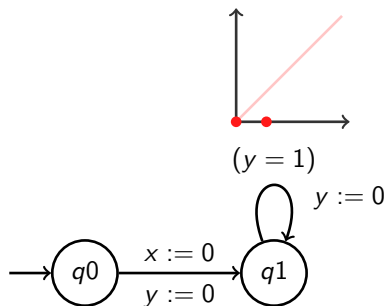
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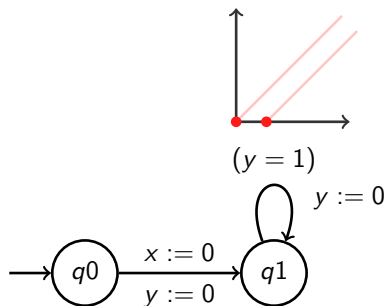
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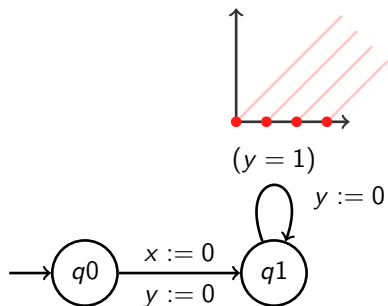
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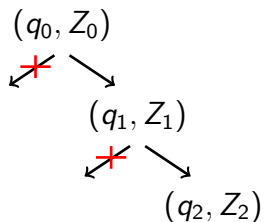


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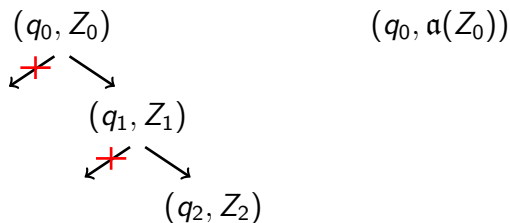
Use finite abstractions

Key idea: **Abstract** each zone in a **sound** manner



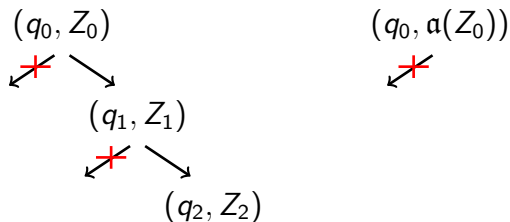
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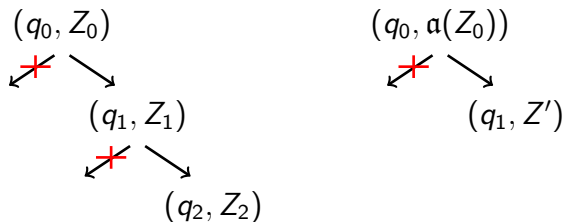
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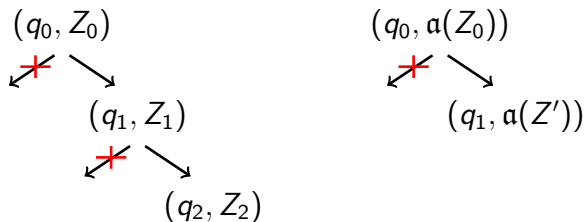
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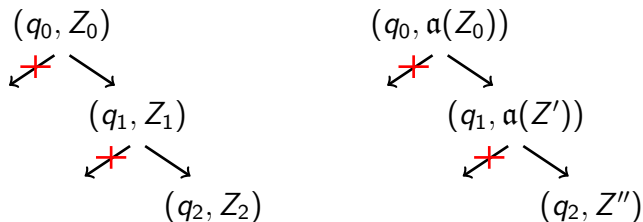
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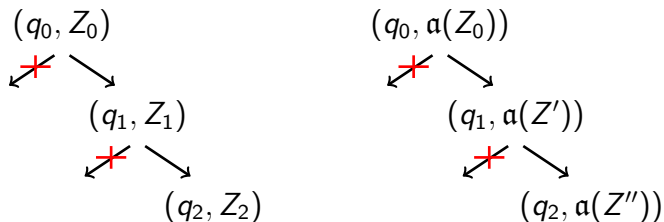
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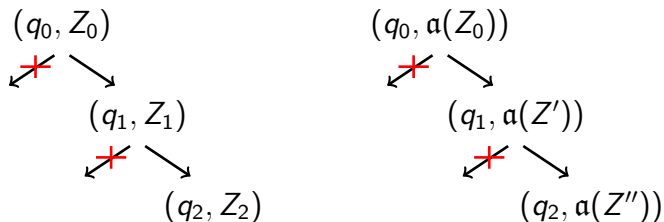
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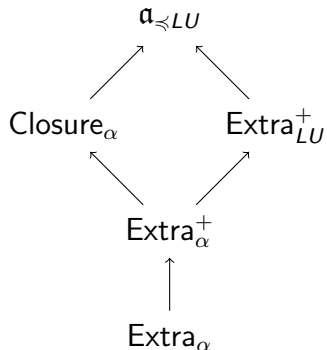
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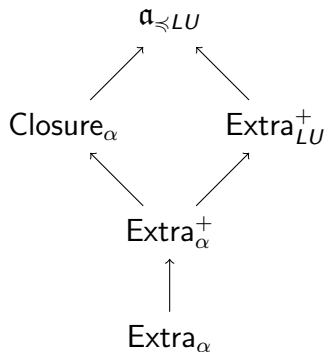


- ▶ Number of **abstracted zones** is **finite**
- ▶ **Coarser** abstraction \rightarrow fewer **abstracted zones**

Abstractions in literature [Bou04, BBLP06]



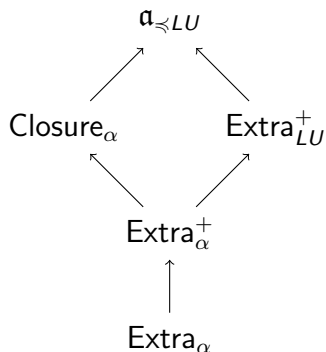
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Sound and complete

All the above abstractions preserve state **reachability**

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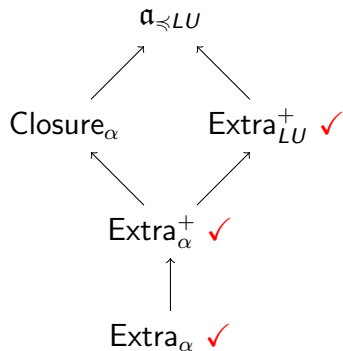


Sound and complete

All the above abstractions preserve state **reachability**

But for **implementation** abstracted zone should be a zone

Abstractions in literature [Bou04, BBLP06]

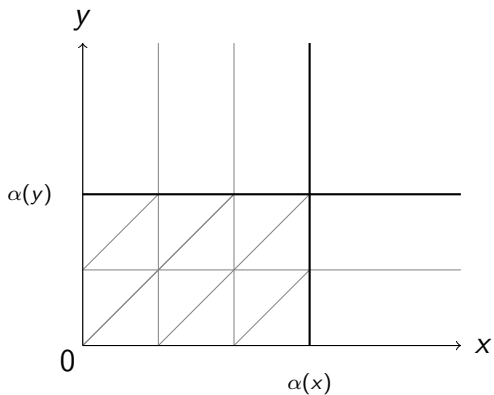


Only convex abstractions in implementations!

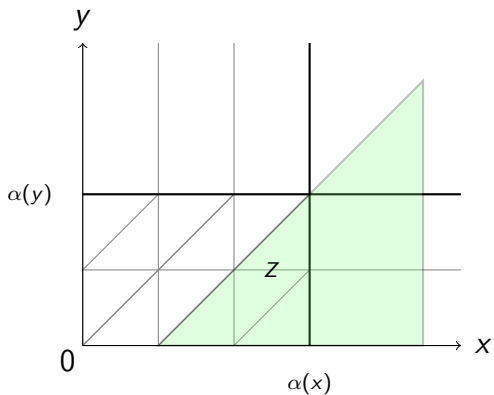
Here...

Efficient use of the **non-convex** Closure abstraction!

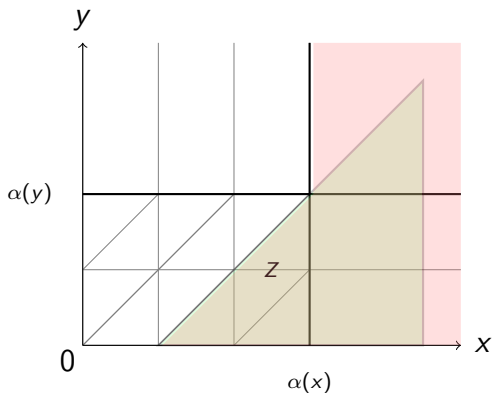
What is Closure $_{\alpha}$?



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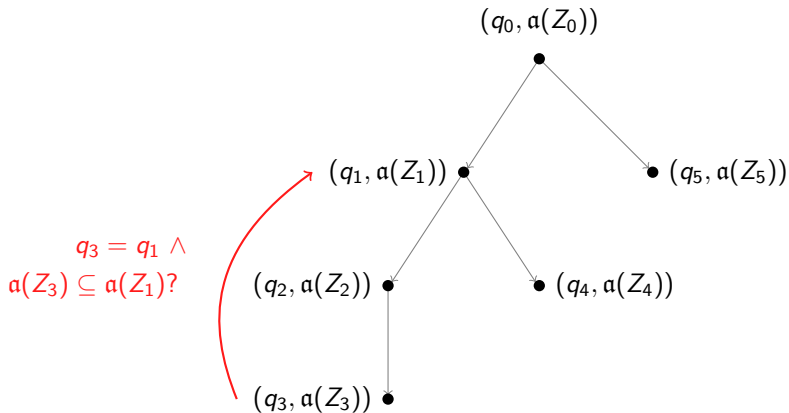


What is Closure_α ?



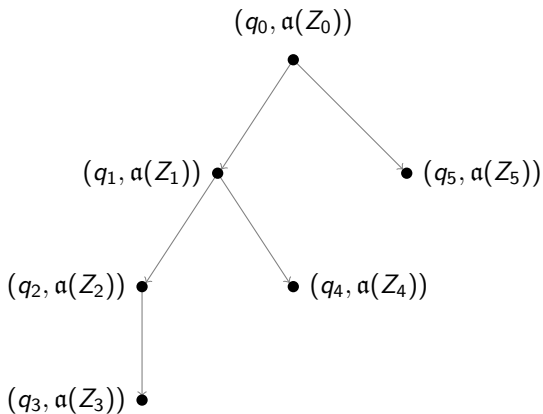
$\text{Closure}_\alpha(Z)$: set of regions that Z intersects

Using Closure_α for reachability



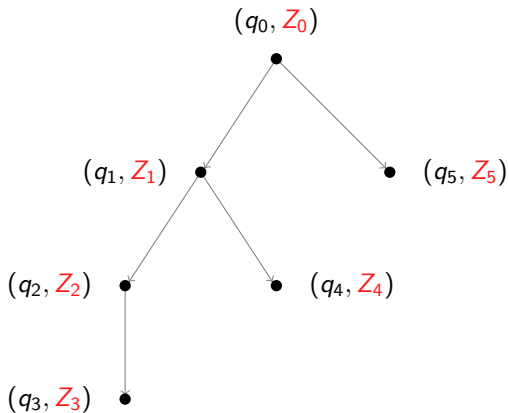
Standard algorithm: **covering tree**

Using Closure_α for reachability



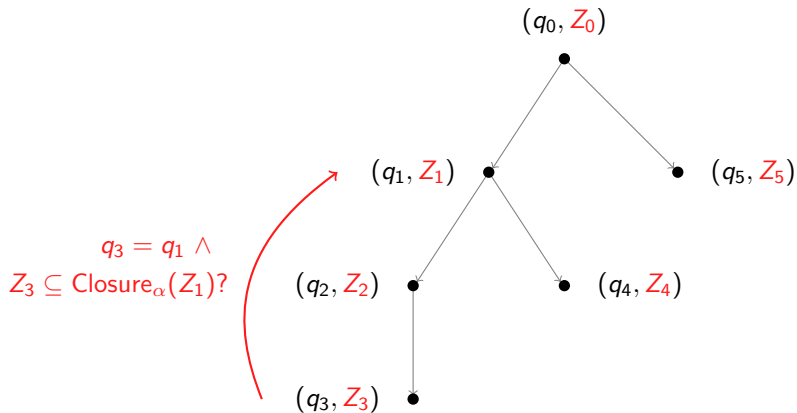
$\text{Closure}_\alpha(Z)$ **cannot be efficiently stored**

Using Closure_α for reachability



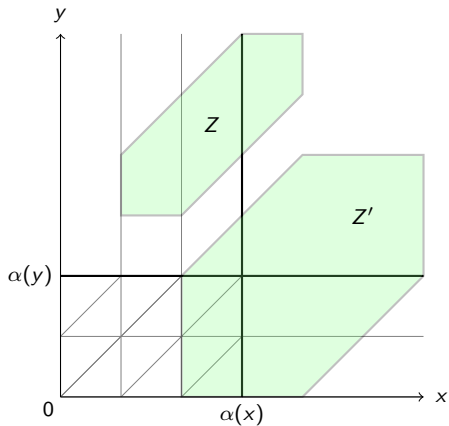
Do **not** store abstracted zones!

Using Closure_α for reachability

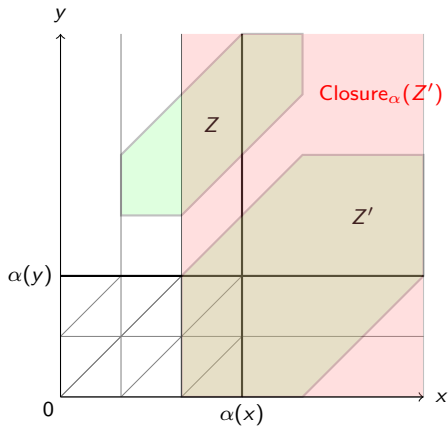


Use Closure for termination!

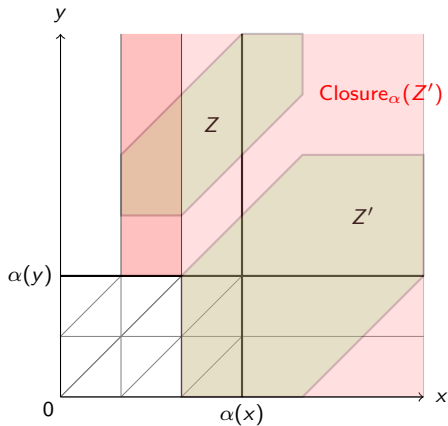
$Z \subseteq \text{Closure}_\alpha(Z')$?



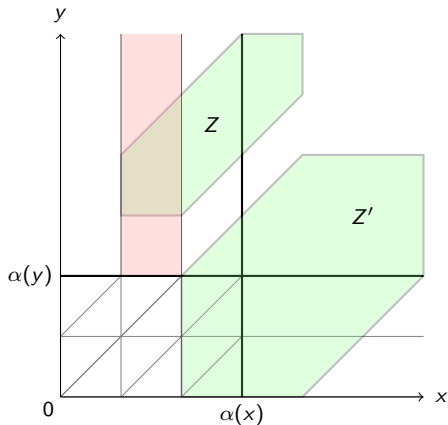
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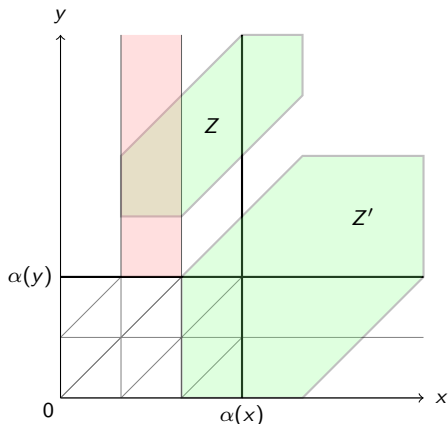


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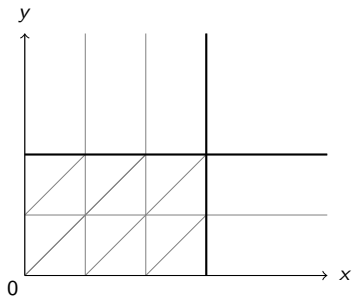
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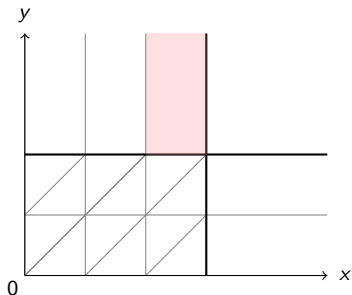
$Z \not\subseteq \text{Closure}_\alpha(Z') \Leftrightarrow \exists R. R \text{ intersects } Z, R \text{ does not intersect } Z'$

Coming next: An **efficient algorithm** for $Z \not\subseteq \text{Closure}_\alpha(Z')$

Step 1: Representing regions and zones



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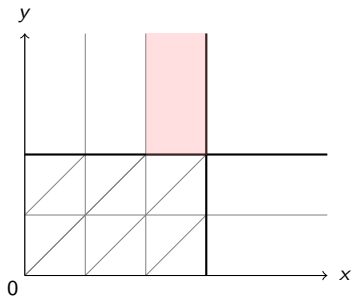
$$x < 3$$

$$y < \infty$$

$$x > 2$$

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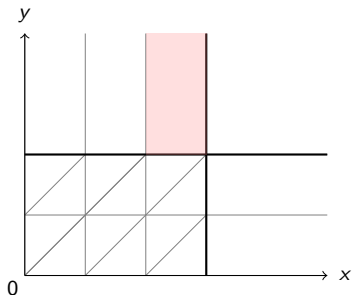
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•
0

•
x

•
y

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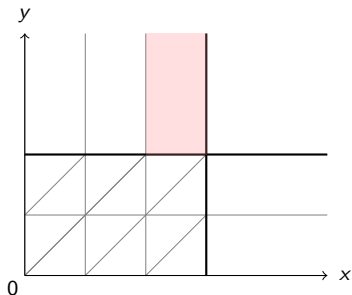
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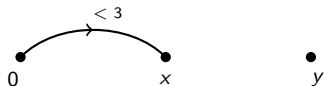


$$x - 0 < 3$$

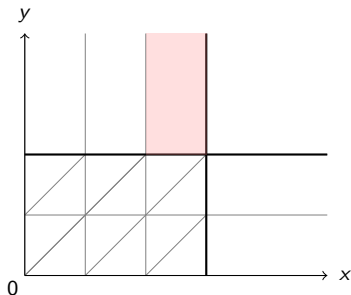
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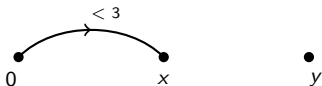


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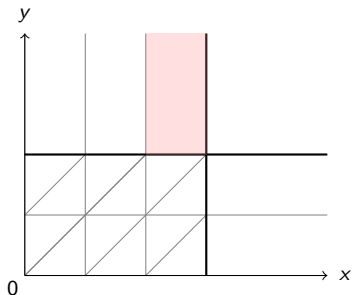
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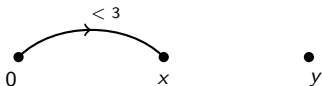
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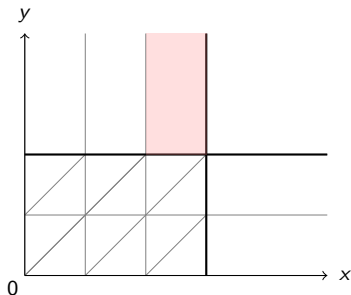
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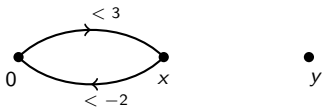
$$\begin{array}{ll} x - 0 < 3 & y < \infty \\ 0 - x < -2 & y > 2 \end{array}$$



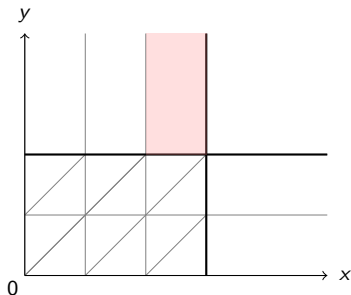
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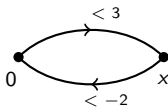
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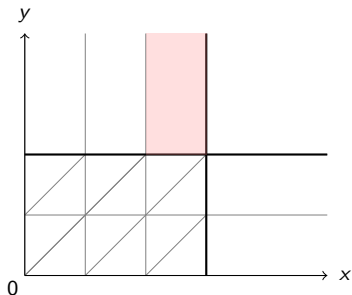


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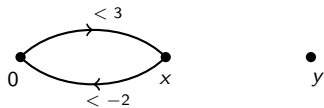


•
y

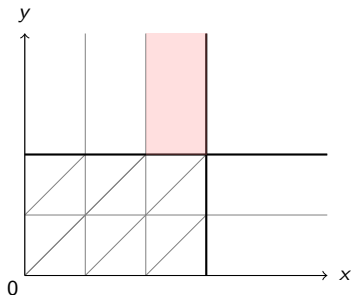
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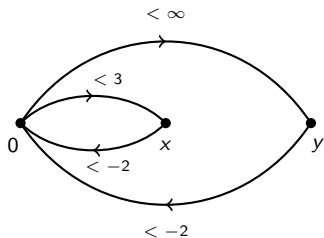
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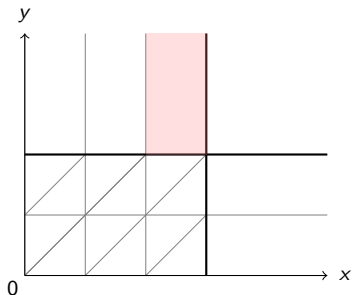
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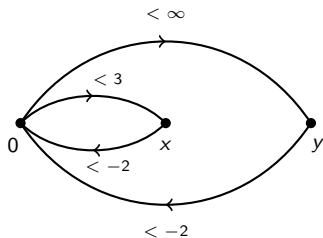
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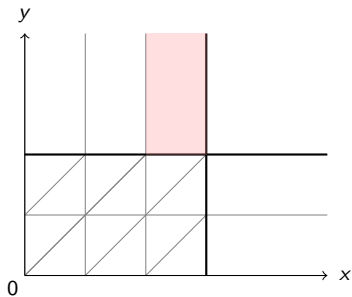
Step 1: Representing regions and zones



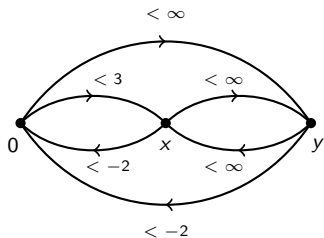
$$\begin{array}{ll} x - 0 < 3 & y - 0 < \infty \\ 0 - x < -2 & 0 - y < -2 \end{array}$$



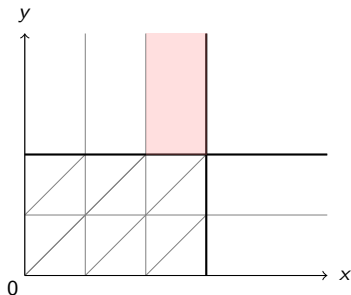
Step 1: Representing regions and zones



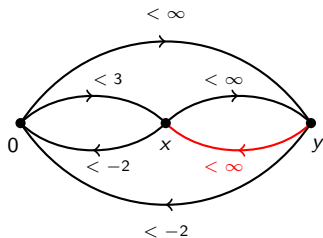
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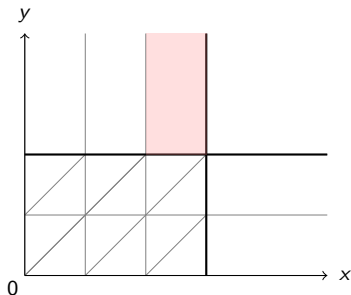


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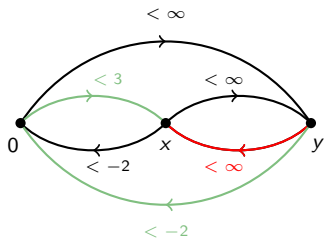


Need a **canonical** representation

Step 1: Representing regions and zones

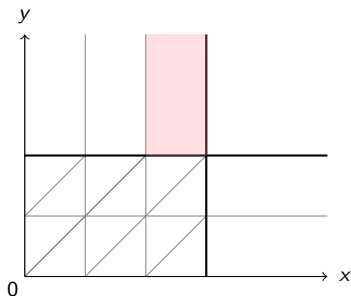


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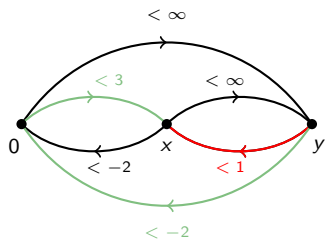


Shortest path should be given by the **direct edge**

Step 1: Representing regions and zones

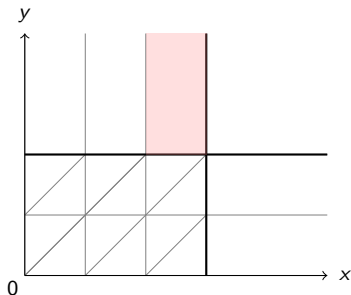


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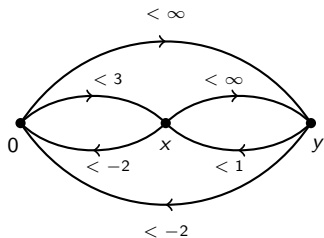


Shortest path should be given by the **direct edge**

Step 1: Representing regions and zones



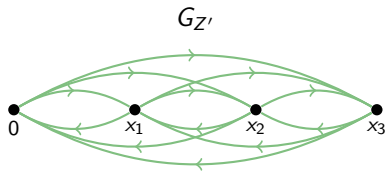
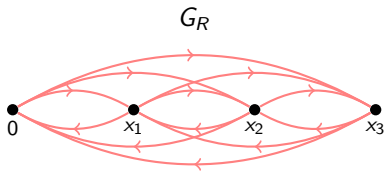
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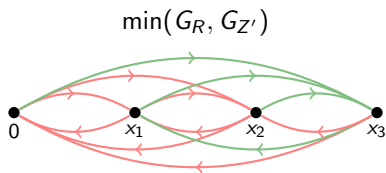
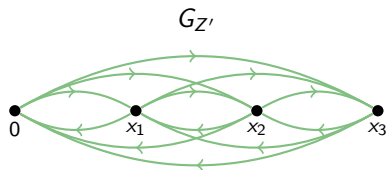
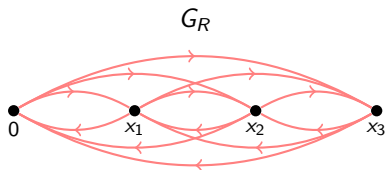
For every zone Z , **canonical distance graph** G_Z

Step 2: When is $R \cap Z'$ empty?

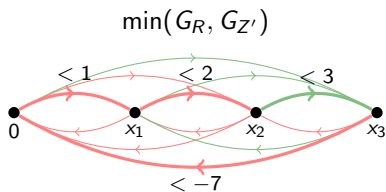
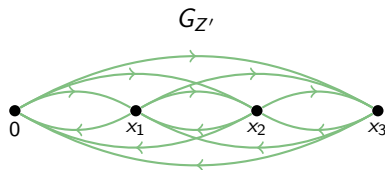
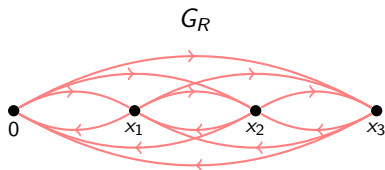
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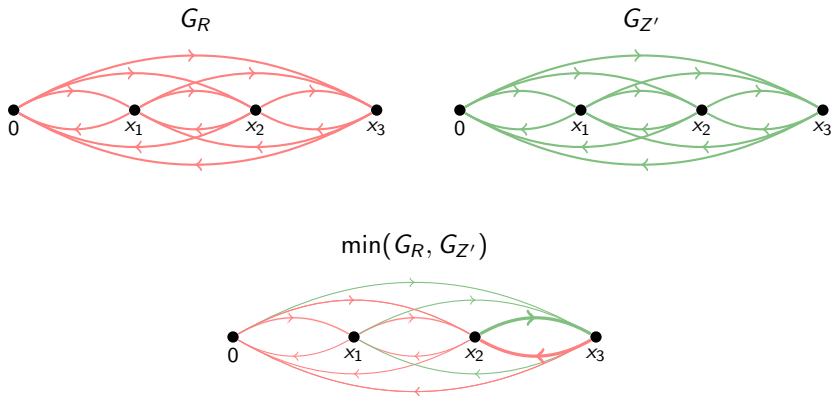
x_1	-	0	< 1
x_2	-	x_1	< 2
x_3	-	x_2	< 3
0	-	x_3	< -7

$0 < -1!$

Lemma

$R \cap Z'$ is **empty** \Leftrightarrow $\min(G_R, G_{Z'})$ has a **negative cycle**

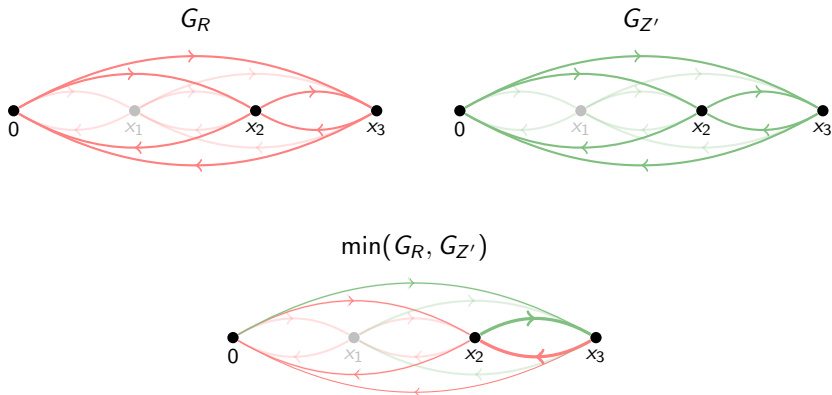
Step 2: When is $R \cap Z'$ empty?



Lemma [Bou04]

$R \cap Z'$ is **empty** \Leftrightarrow $\min(G_R, G_{Z'})$ has a **negative cycle** involving **2 clocks!**

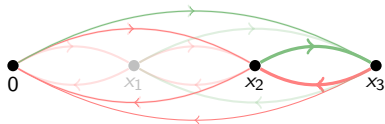
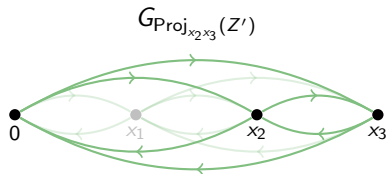
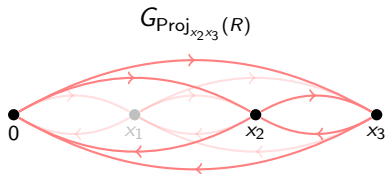
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Lemma

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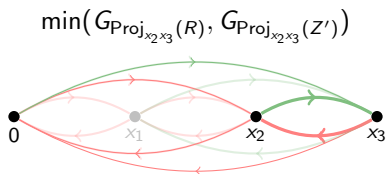
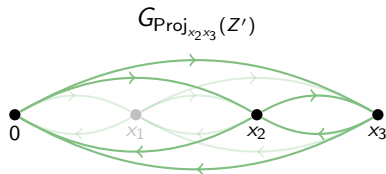
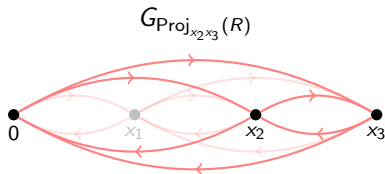
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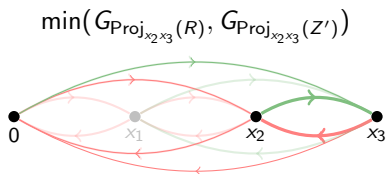
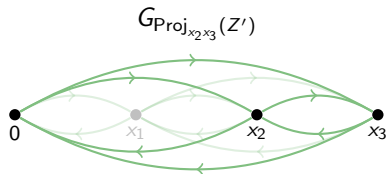
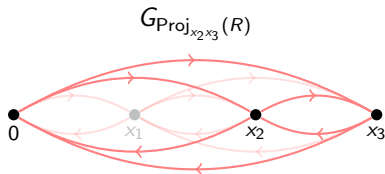
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Lemma

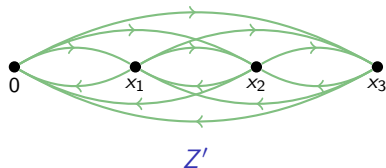
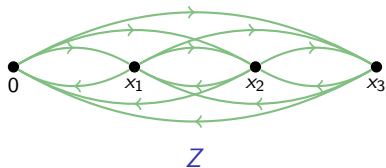
$R \cap Z'$ is **empty** $\Leftrightarrow \exists x, y. \text{Proj}_{xy}(R) \cap \text{Proj}_{xy}(Z')$ is **empty**

Step 3: Reduction to two clocks

Recall: $Z \not\subseteq \text{Closure}_\alpha(Z') \Leftrightarrow \exists R. R \text{ intersects } Z, R \text{ does not intersect } Z'$

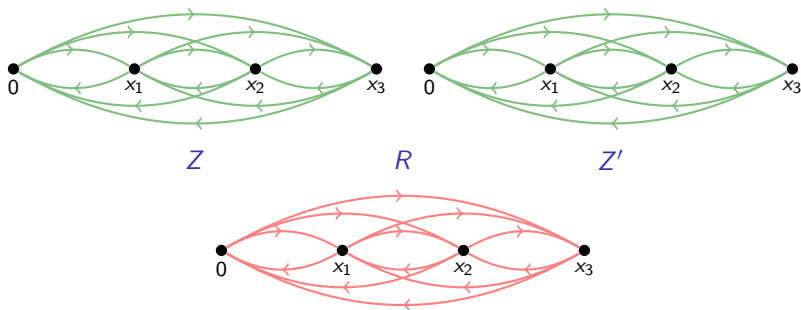
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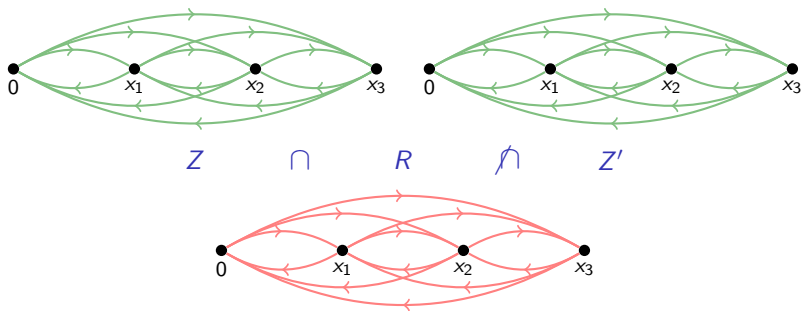
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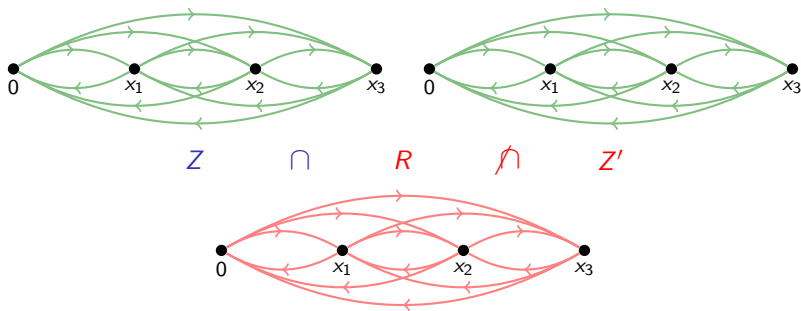
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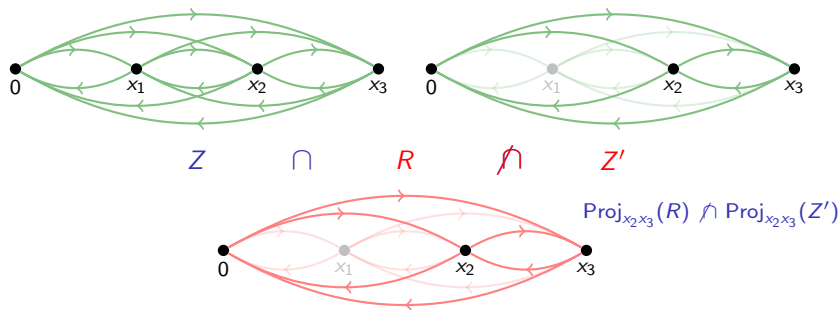
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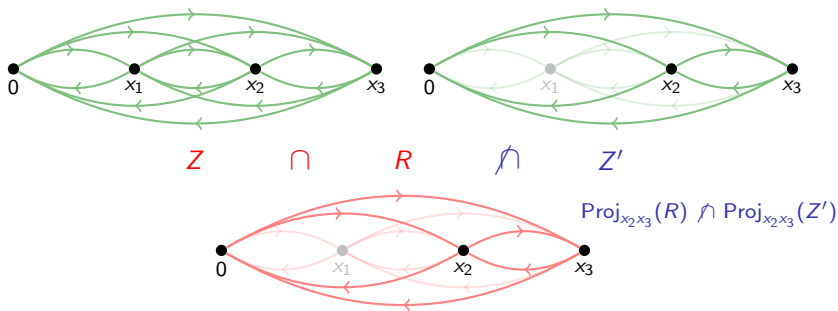
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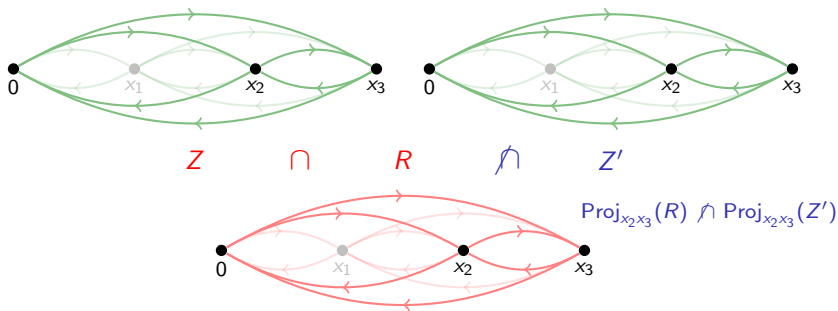
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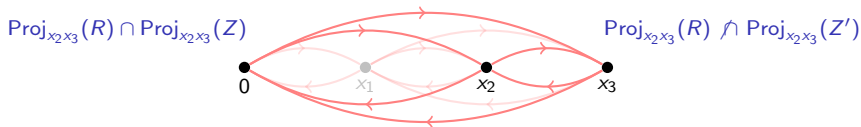
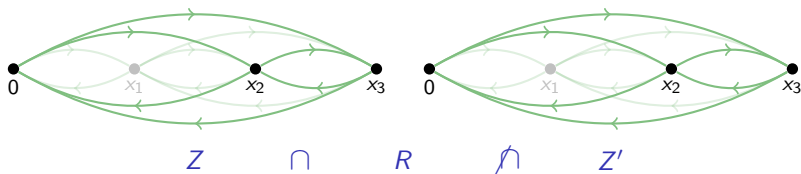
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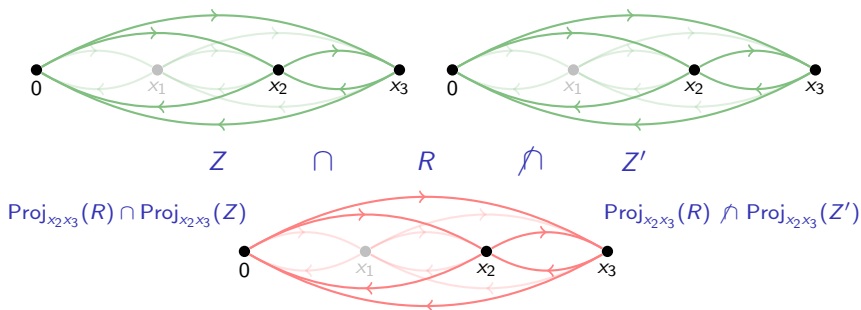
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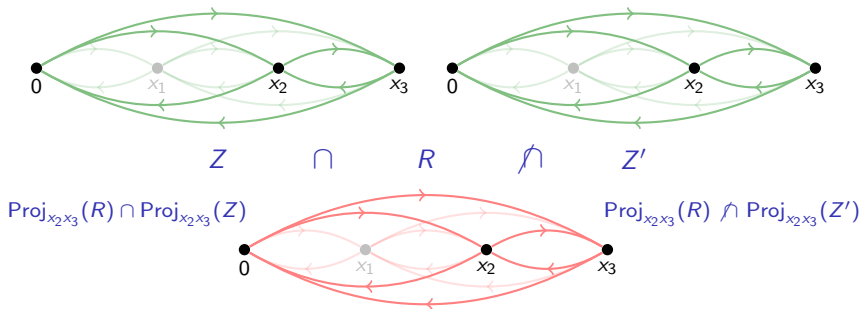


Theorem

$Z \not\subseteq \text{Closure}_\alpha(Z')$ if and only if there **exist 2 clocks** x, y s.t.

$$\mathbf{Proj}_{xy}(Z) \not\subseteq \text{Closure}_\alpha(\mathbf{Proj}_{xy}(Z'))$$

Step 3: Reduction to two clocks



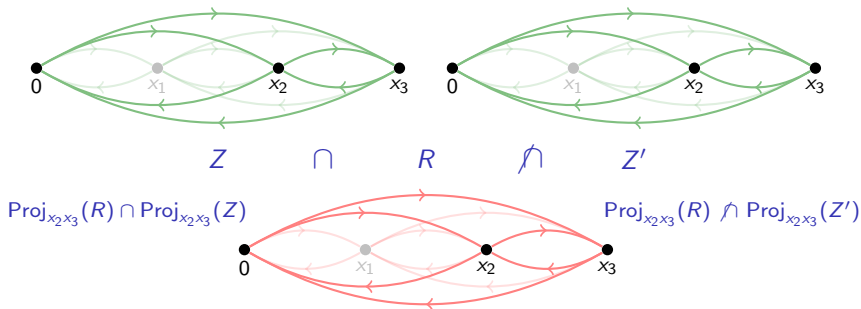
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Slightly **modified edge-edge comparison** is enough

Step 3: Reduction to two clocks



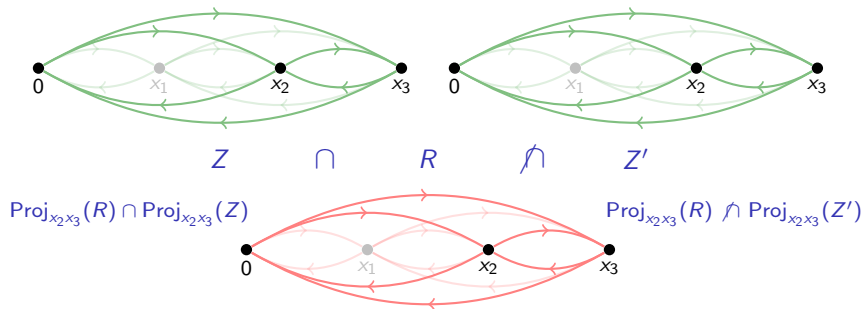
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Complexity: $\mathcal{O}(|X|^2)$, where X is the set of clocks

Step 3: Reduction to two clocks



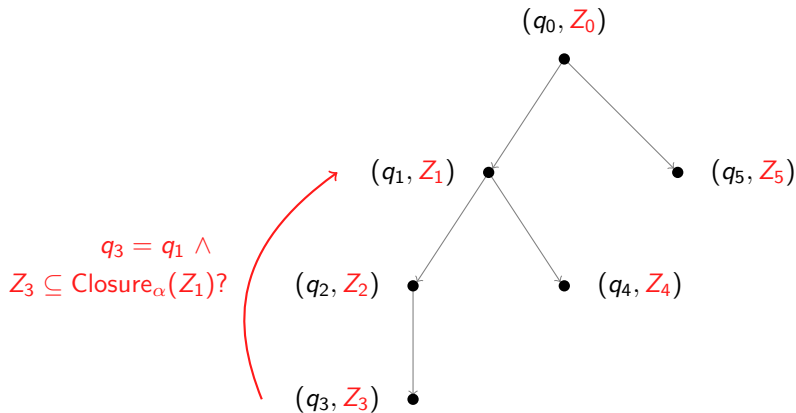
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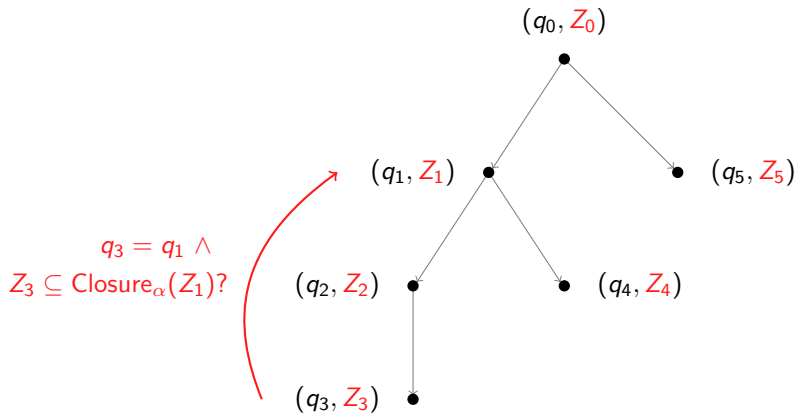
Same complexity as $Z \subseteq Z'$!

So what do we have now...



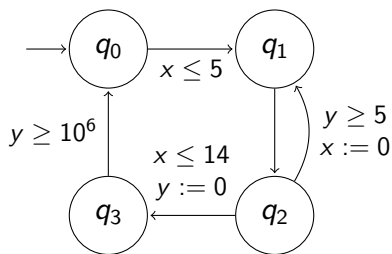
Efficient algorithm for $Z \subseteq \text{Closure}_\alpha(Z')$

So what do we have now...



Coming next: **prune** the **bound function** α !

Bound function α

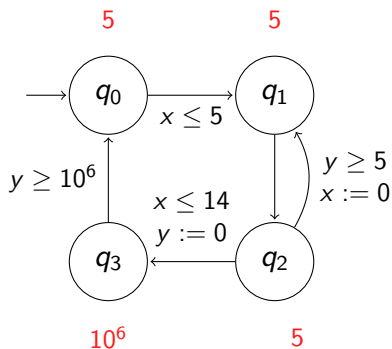


Naive: $\alpha(x) = 14$, $\alpha(y) = 10^6$

Size of graph $\sim 10^5$

Static analysis: bound function for every q

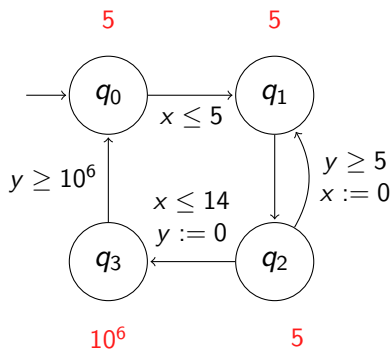
[BBFL03]



Naive: $\alpha(x) = 14$, $\alpha(y) = 10^6$

Static analysis: bound function for every q

[BBFL03]

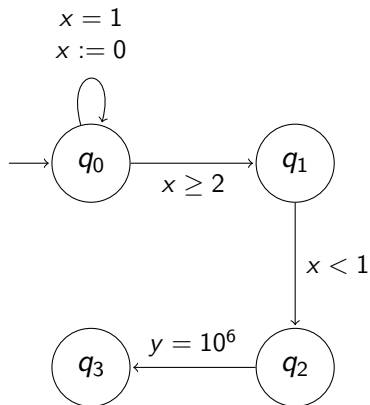


Naive: $\alpha(x) = 14$, $\alpha(y) = 10^6$

But this is not enough!

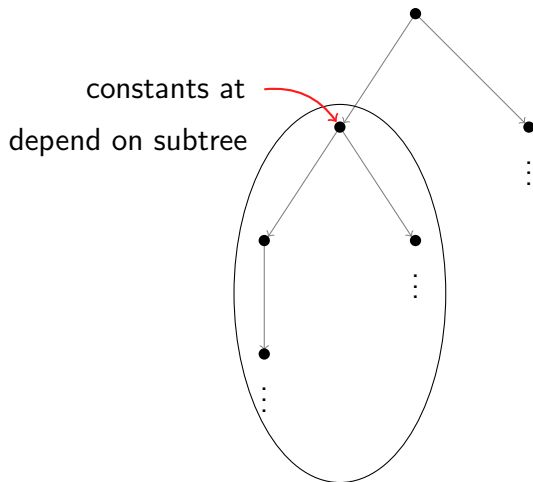
Need to look at semantics...

Static analysis: $\alpha(y) = 10^6$



More than 10^6 zones at q_0 **not necessary!**

Bound function for every (q, Z) in $ZG(\mathcal{A})$



Constant propagation

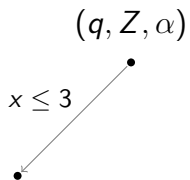
$$\alpha(x) = -\infty$$

$$(q, Z, \alpha)$$

•

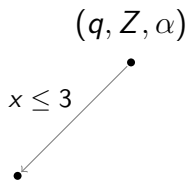
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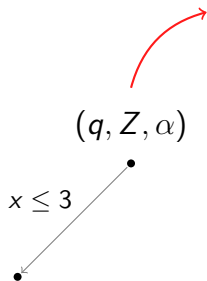
Constant propagation

$$\alpha(x) = 3$$



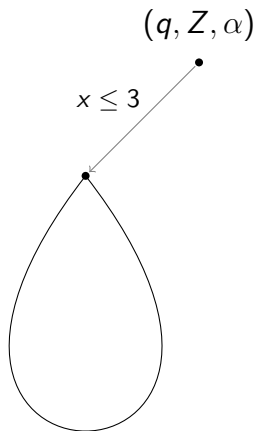
Constant propagation

$$\alpha(x) = 3$$



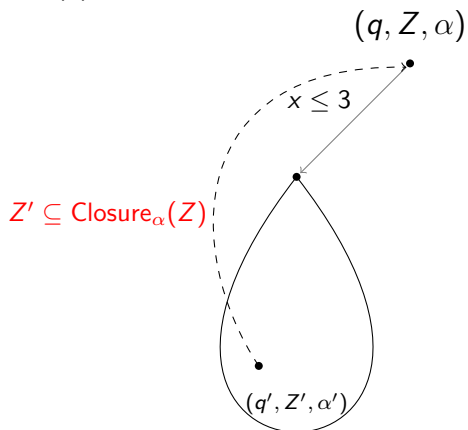
Constant propagation

$$\alpha(x) = 5$$



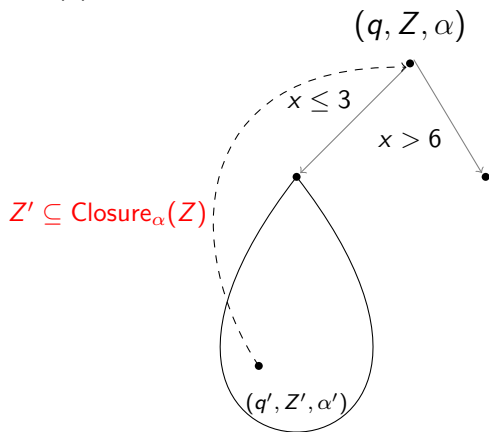
Constant propagation

$$\alpha(x) = 5$$



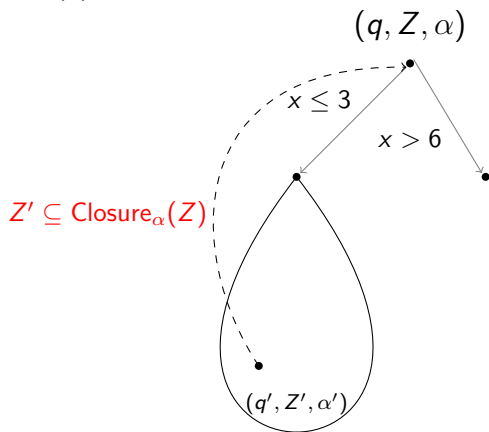
Constant propagation

$$\alpha(x) = 5$$



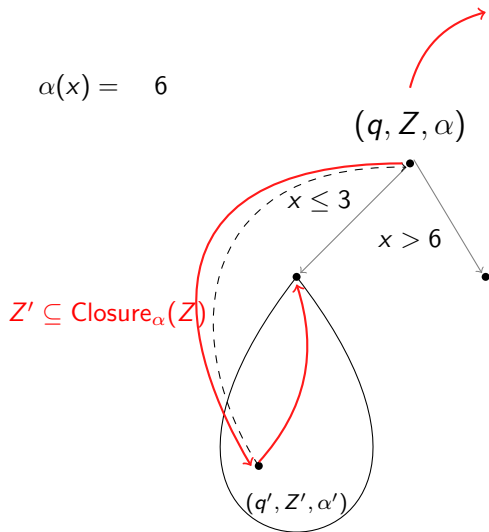
Constant propagation

$$\alpha(x) = 6$$



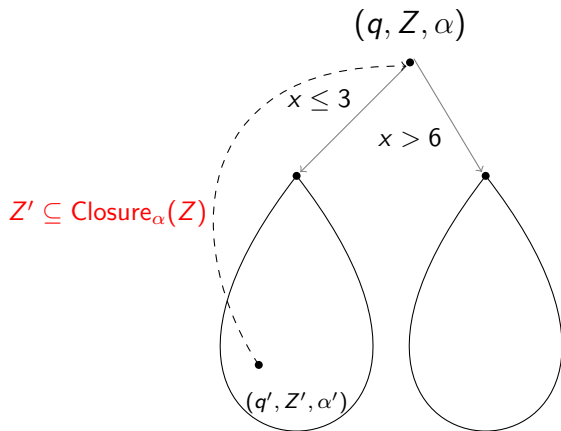
Constant propagation

$$\alpha(x) = 6$$



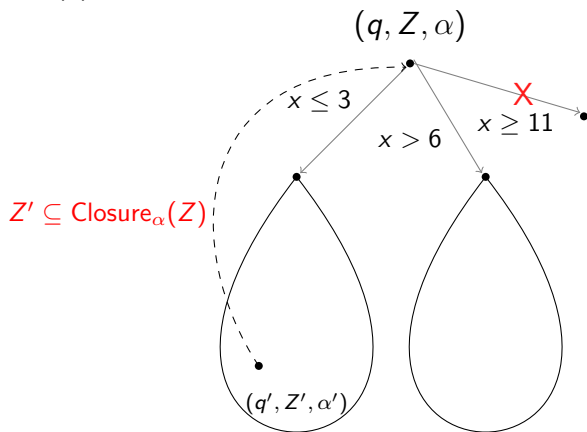
Constant propagation

$$\alpha(x) = 6$$



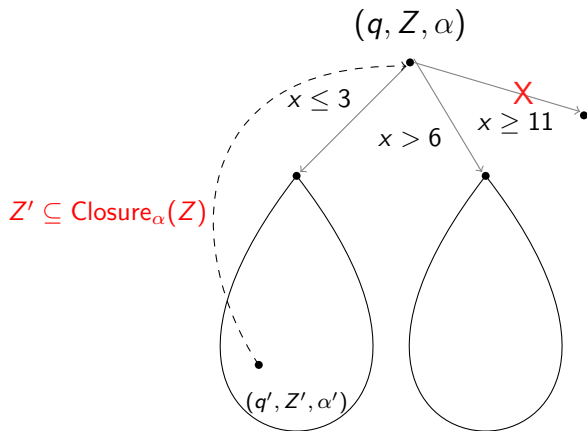
Constant propagation

$$\alpha(x) = 6$$



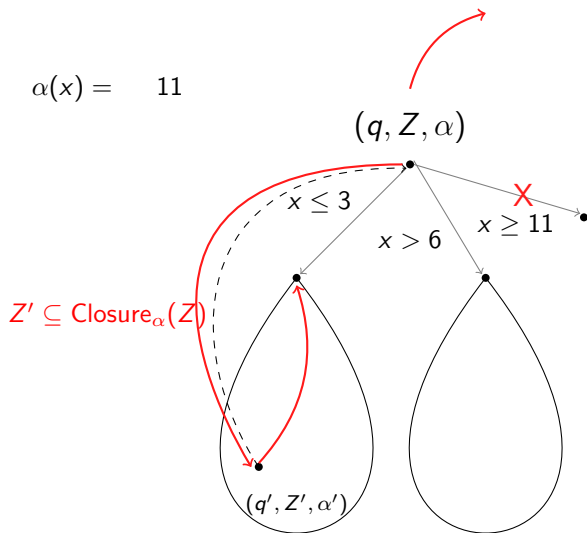
Constant propagation

$$\alpha(x) = 11$$



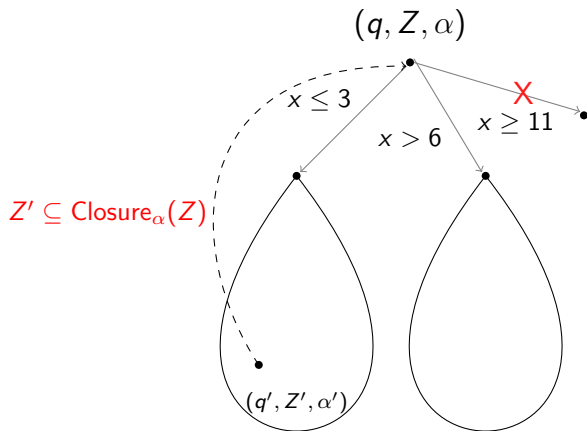
Constant propagation

$$\alpha(x) = 11$$



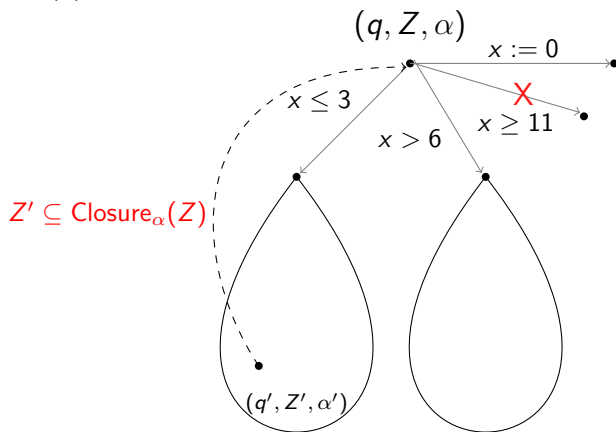
Constant propagation

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Constant propagation

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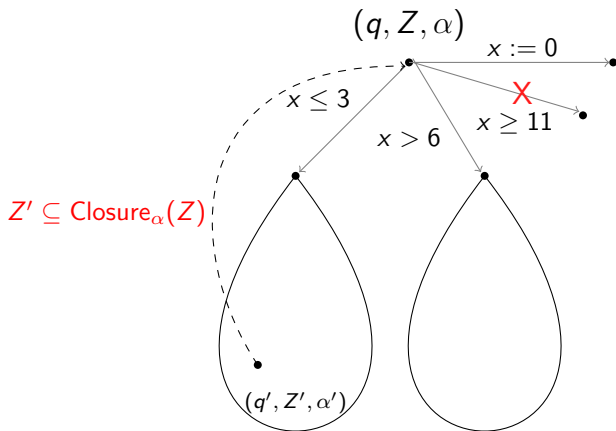


Constant propagation

$$\alpha(x) = 11$$

All **tentative nodes** consistent
+ No more **exploration**

→ **Terminate!**



Invariants on the bounds

- ▶ Non tentative nodes: $\alpha = \max\{\alpha_{succ}\}$ (modulo resets)
- ▶ Tentative nodes: $\alpha = \alpha_{covering}$

Invariants on the bounds

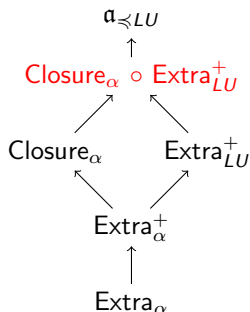
- ▶ Non tentative nodes: $\alpha = \max\{\alpha_{succ}\}$ (modulo resets)
- ▶ Tentative nodes: $\alpha = \alpha_{covering}$

Theorem (Correctness)

An accepting state is reachable in $ZG(\mathcal{A})$ iff the algorithm reaches a node with an accepting state and a non-empty zone.

Overall algorithm

- ▶ Compute $ZG(\mathcal{A})$: $Z \subseteq \text{Closure}_{\alpha'}(Z')$ for **termination**
- ▶ **Bounds** α calculated **on-the-fly**
- ▶ Abstraction Extra_{LU}^+ can **also** be **handled**:



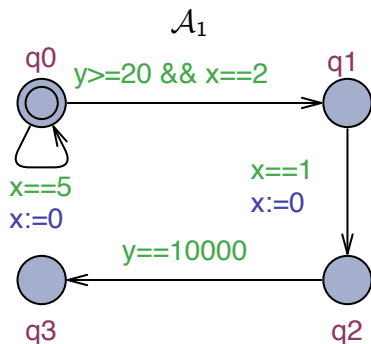
An **efficient** $\mathcal{O}(|X|^2)$ procedure for $Z \subseteq \text{Closure}_{\alpha}(\text{Extra}_{LU}^+(Z'))!$

Benchmarks

Model	Our algorithm		UPPAAL's algorithm		UPPAAL 4.1.3 (-n4 -C -o1)	
	nodes	s.	nodes	s.	nodes	s.
CSMA/CD7	5031	0.32	5923	0.27	—	T.O.
CSMA/CD8	16588	1.36	19017	1.08	—	T.O.
CSMA/CD9	54439	6.01	60783	4.19	—	T.O.
FDDI10	459	0.02	525	0.06	12049	2.43
FDDI20	1719	0.29	2045	0.78	—	T.O.
FDDI30	3779	1.29	4565	4.50	—	T.O.
Fischer7	7737	0.42	18374	0.53	18374	0.35
Fischer8	25080	1.55	85438	2.48	85438	1.53
Fischer9	81035	5.90	398685	12.54	398685	8.95
Fischer10	—	T.O.	—	T.O.	1827009	53.44

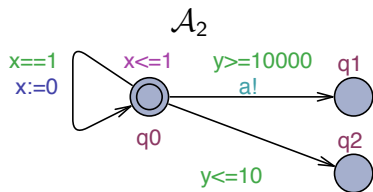
- ▶ **Extra_{LU}⁺** and **static** analysis bounds in UPPAAL
- ▶ **Closure_α(Extra_{LU}⁺)** and **otf** bounds in our algorithm

Experiments I



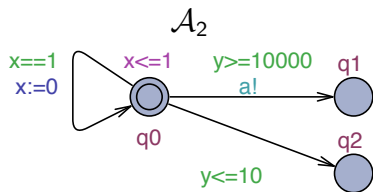
\mathcal{A}_1	nodes	s.
Our algorithm	7	0.0
UPPAAL's algorithm	2003	0.60
UPPAAL 4.1.3	2003	0.01

Experiments II



\mathcal{A}_2	nodes	s.
Our algorithm	2	0.0
UPPAAL's algorithm	10003	0.07
UPPAAL 4.1.3	10003	0.07

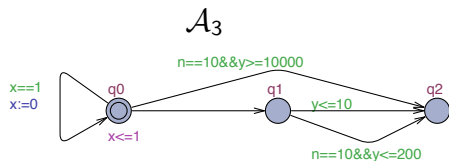
Experiments II



\mathcal{A}_2	nodes	s.
Our algorithm	2	0.0
UPPAAL's algorithm	10003	0.07
UPPAAL 4.1.3	10003	0.07

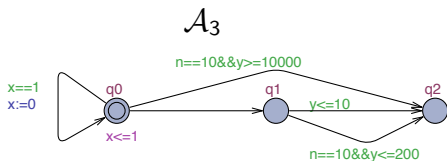
Occurs in **CSMA/CD!**

Experiments III



\mathcal{A}_3	nodes	s.
Our algorithm	3	0.0
UPPAAL's algorithm	10004	0.37
UPPAAL 4.1.3	10004	0.32

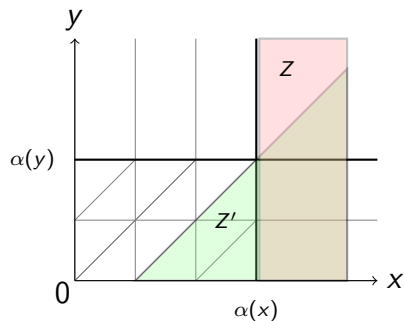
Experiments III



\mathcal{A}_3	nodes	s.
Our algorithm	3	0.0
UPPAAL's algorithm	10004	0.37
UPPAAL 4.1.3	10004	0.32

Occurs in **Fischer!**

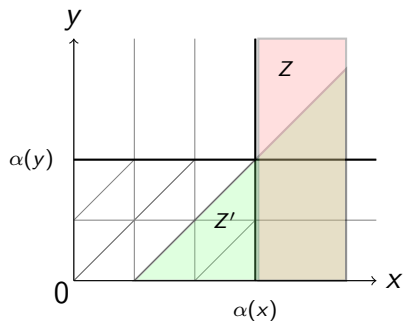
Experiments IV



$$Z' : x - y \geq 1$$

$$Z : x > \alpha(x)$$

Experiments IV



$$Z' : x - y \geq 1$$

$$Z : x > \alpha(x)$$

Occurs in **FDDI!**

Conclusions & Perspectives

- ▶ **Efficient implementation** of a non-convex approximation that **subsumes** current ones in use
- ▶ **On-the-fly learning** of bounds that is **better** than the current static analysis

- ▶ More **sophisticated** non-convex approximations
- ▶ Propagating **more** than constants
- ▶ Automata with **diagonal** constraints

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