Verification of Population Protocols

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Joint work with Pierre Ganty, Jérôme Leroux,
and Rupak Majumdar

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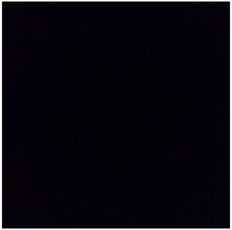

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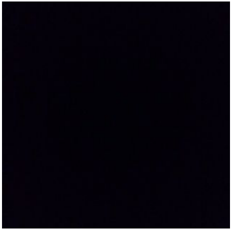
- How can they conduct the vote?



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- Each Ninja stores their current estimation of the final outcome of the vote (Yes or No). Additionally, it is Active or Passive.
- Initially all Ninjas are Active, and their initial estimation is their own vote
- Ninjas follow this protocol:

$$(YA, NA) \rightarrow (NP, NP) \quad \text{(opposite votes "cancel")}$$

$$\begin{array}{lcll} (YA, NP) & \rightarrow & (YA, YP) & \text{(active "survivors" tell} \\ (NA, YP) & \rightarrow & (NA, NP) & \quad \text{outcome to passive Ninjas)} \end{array}$$

$$(NP, YP) \rightarrow (NP, NP) \quad \text{(to deal with ties)}$$

Population protocols (PP)

Theoretical model for distributed computation

Proposed in 2004 by Angluin *et al.*

Designed to model collections of

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- ... and Ninjas

Syntax

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- Q is a finite set of states, and
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Intuition:

if $(q_1, q_2) \mapsto (q_1', q_2') \in \Delta$ **and**
two agents in states q_1 and q_2 "meet",

then the agents can interact and
change their states to q_1', q_2'.

Assumption: at least one interaction for each pair of states (possibly $(q_1, q_2) \mapsto (q_1, q_2)$)

Semantics

Configuration: mapping $C\colon Q \to \mathbb{N}$, where $C(q)$ is the current number of agents in state q.

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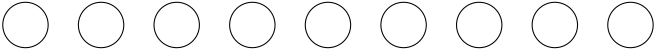
Execution: infinite sequence $C_0 \to C_1 \to C_2 \to \cdots$ of steps

Population protocols (PPs)

A population protocol (PP) consists of

- A PP-scheme (Q, Δ)

Q: 

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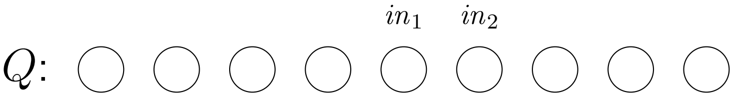

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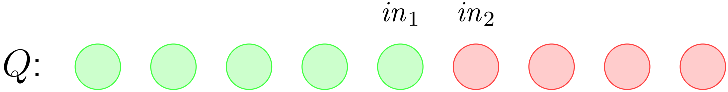
- A partition of Q into 1-states (green) and 0-states (pink)



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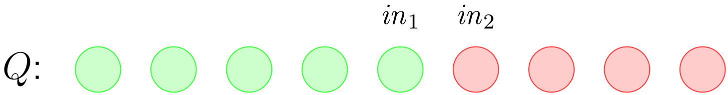
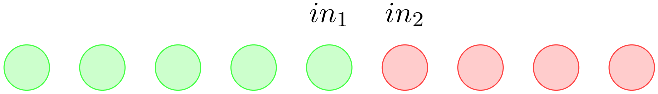


An execution reaches consensus $b \in \{0, 1\}$ if from some point on every agent stays within the b-states.

Computing with PPs

A PP computes the value b for input $(n_1, n_2, \ldots, n_k)$ if executions starting at the configuration



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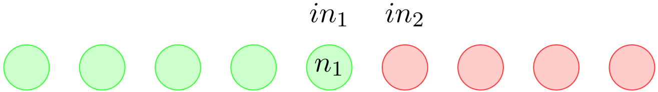

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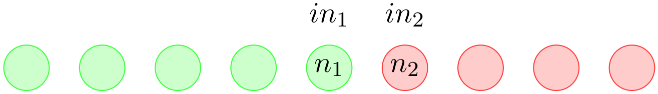

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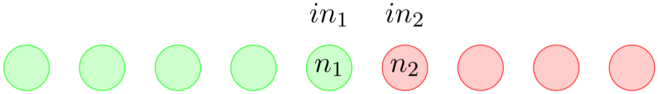

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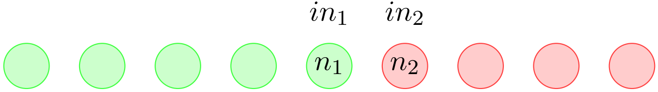

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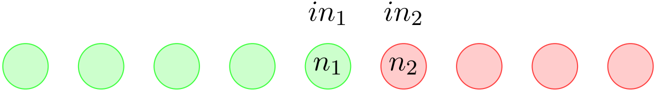

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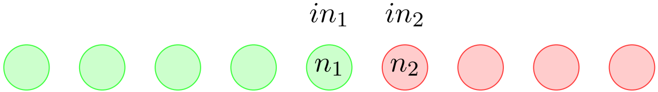

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A PP computes $P(x_1, \ldots, x_n) \colon \mathbb{N}^n \to \{0, 1\}$ if it computes $P(n_1, \ldots, n_k)$ for every input $(n_1, \ldots, n_k)$

Previous work

Expressive power thoroughly studied:

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- PPs compute exactly the Presburger predicates (Angluin *et al.* 2007)
- Probabilistic PPs (Angluin *et al.* 2004-2006, Chatzigiannakis and Spirakis, 2008)
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- Private computation in PPs (Delporte-Gallet *et al.* 2007)
- PPs with identifiers (Guerraoui *et al.* 2007)
- PPs with a leader (Angluin *et al.* 2008)
- Mediated PPs (Michail *et al.*, 2011)
- Trustful PPs (Bournez *et al.*, 2013)

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Well-specification problem: Given a protocol, decide if it is well-specified.

Correctness problem: Given a protocol and a Presburger predicate, decide if the protocol is well-specified and computes the predicate.

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Theorem: The reachability problem for Petri nets can be reduced to the well-specification and correctness problems for PPs.

From PPs to Petri nets

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PP	Net $+$ family of initial markings

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Theorem [Easy generalization]: Given two Presburger sets of markings $\mathcal{M}_1$, $\mathcal{M}_2$, it is decidable if some marking of $\mathcal{M}_2$ is reachable from some marking of $\mathcal{M}_1$

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Our main technical result: The set of all configurations belonging to all bottom SCCs of a PP, for all initial configurations, is effectively Presburger.

Fact: A PP is ill-specified iff there is an initial configuration C and

- a bottom SCC reachable from C with agents in both 0- and 1-states; or
- two bottom SCCs, one with only 0-states and the other with only 1-states, both reachable from C.

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- Check if $\mathcal{B}_{\mathrm{true}} \times \mathcal{B}_{\mathrm{false}}$ is reachable from $\mathcal{I}_2$ (using reachability in Petri nets)

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