Parameterized Verification of Systems with Broadcast Communication

Arnaud Sangnier

IRIF - Université Paris Diderot-Paris 7

joint work with **Giorgio Delzanno & Gianluigi Zavattaro**

DRV - Bertinoro - 19th May 2016

Motivation

Verify network of processes of unbounded size

Why to consider such networks?

- Classical distributed algorithms (*mutual exclusion, leader election,...*)
- Telecommunication protocols (*routing,...*)
- Algorithms for ad-hoc networks
- Model for biological systems
- and many more applications ...

Hypothesis

All the processes have the same behavior

In [Esparza, STACS'14], such networks are called **crowd**

More precisely:

- Each process will follow the same protocol
- Process can communicate
- Communication way:
  - Message passing
  - Shared variable
  - **Broadcast communication**
  - **Multi-diffusion (selective broadcast)**

Question:
Is there a network with N processes which allows to reach a goal ?

Outline

1. Systems with broadcast communication
2. Ad Hoc Networks
3. Conclusion

Outline

Parameterized Networks with Broadcast

[Esparza et al., LICS'99]

Main characteristics

- No creation/deletion of processes
- Each process executes the same finite state protocol
- Synchronization through broadcast of a message
- **All the processes** receive the message

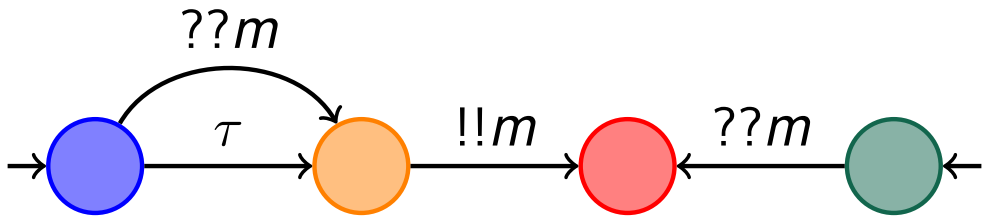
Broadcast Networks: syntax

A protocol $P = \langle Q, \Sigma, R, Q_0 \rangle$

Finite state system whose transitions are labeled with:

1. broadcast of messages - $!!m$
2. reception of messages - $??m$
3. internal actions - τ

where m belongs to the finite alphabet Σ

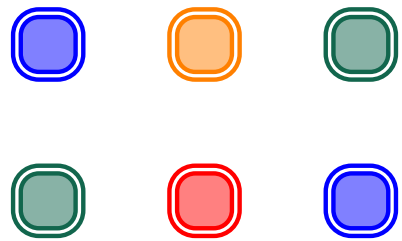


A protocol defines a Broadcast Network (BN)

Broadcast Networks: configurations

A configuration is a multiset $\gamma : Q \mapsto \mathbb{N}$

- Same as for Rendez-vous Networks



- **Initial configurations**: $\gamma(q) > 0$ iff $q \in Q_0$

Remarks:

- The size of configurations is not bounded
- Infinite number of configurations

$\Rightarrow$ **BN are infinite state systems**

Broadcast Networks: semantics

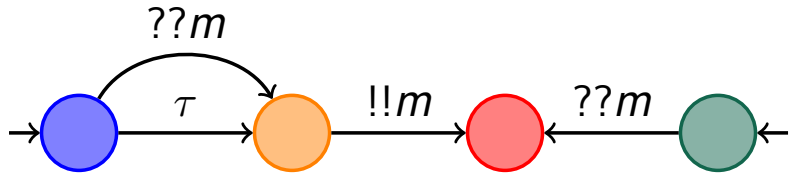
Transition system $BN(P) = \langle \mathcal{C}, \rightarrow, \mathcal{C}_0 \rangle$ associated to P

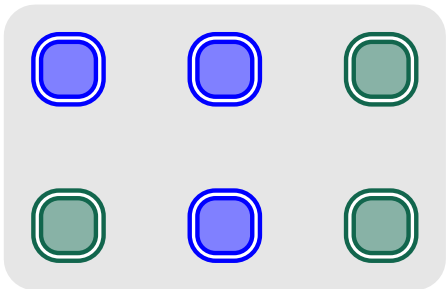
- $\mathcal{C}$: set of configurations
- $\rightarrow$: $\mathcal{C} \times \mathcal{C}$: transition relation
- $\mathcal{C}_0$: initial configurations

The relation $\rightarrow$ respects the following rules during an execution:

- The number of processes in an execution does not change
- Processes can only change their state
- Two kind of transitions according to the given process
  1. **local actions** - one process performs an internal action τ
  2. **broadcast** - one process emits a message with $!!m$, **all the processes** that can receive it with $??m$ have to receive it

Broadcast Networks: an example





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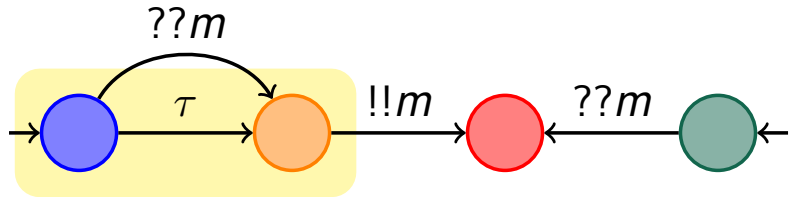

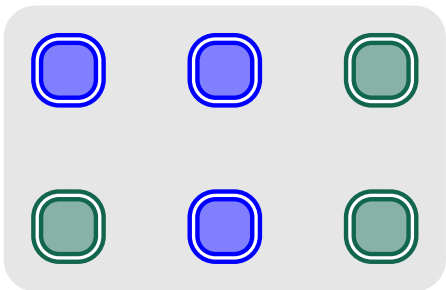

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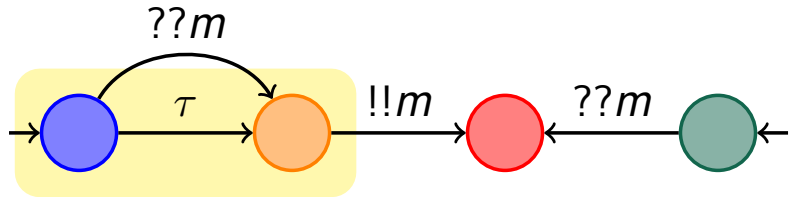

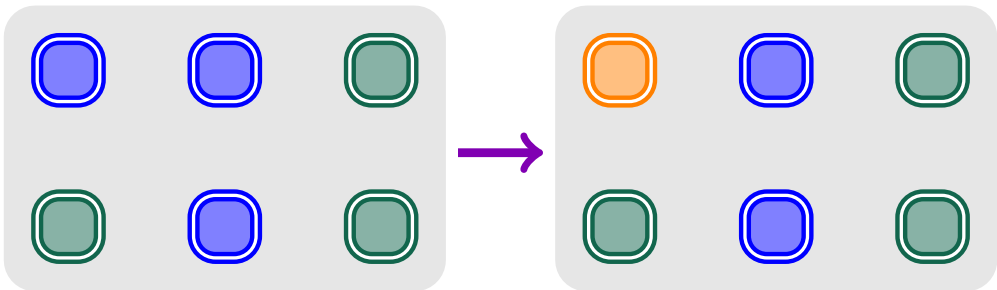

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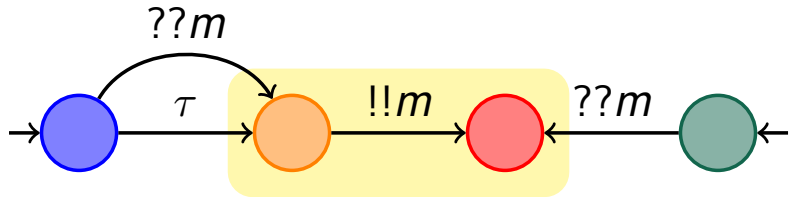

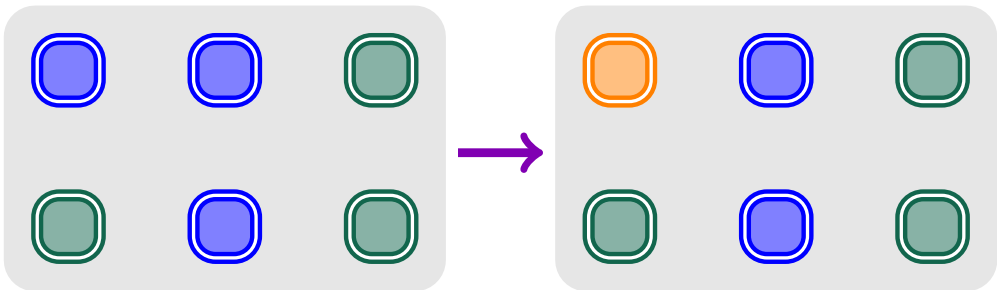

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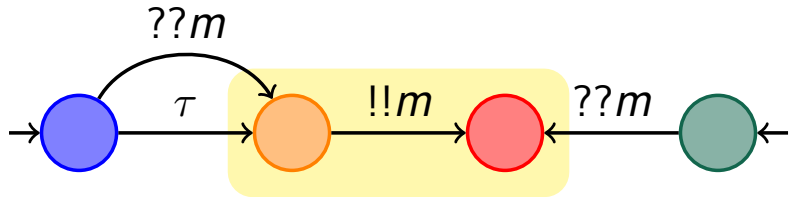

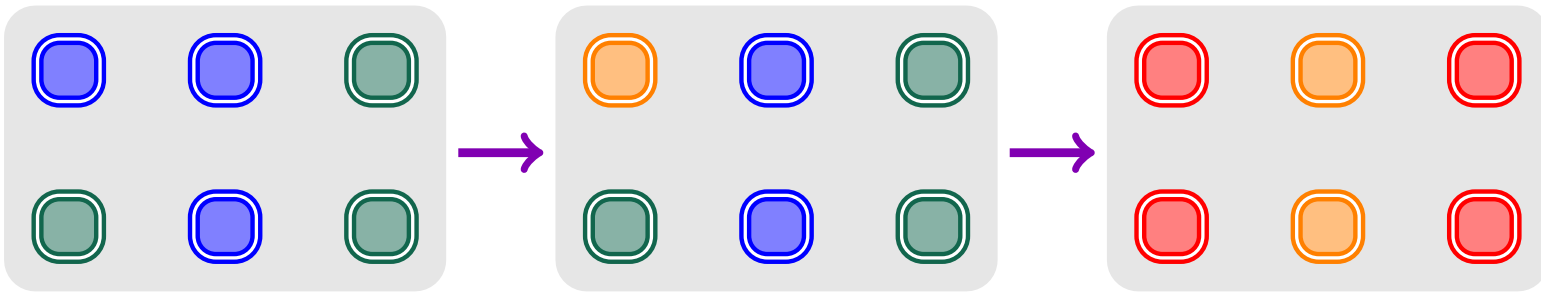

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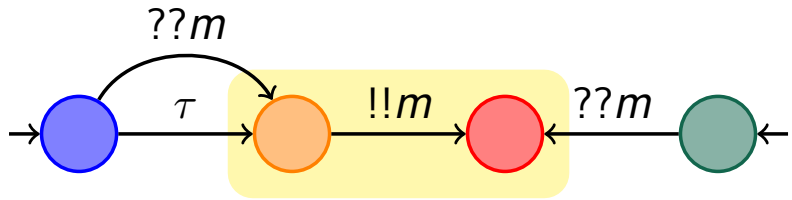

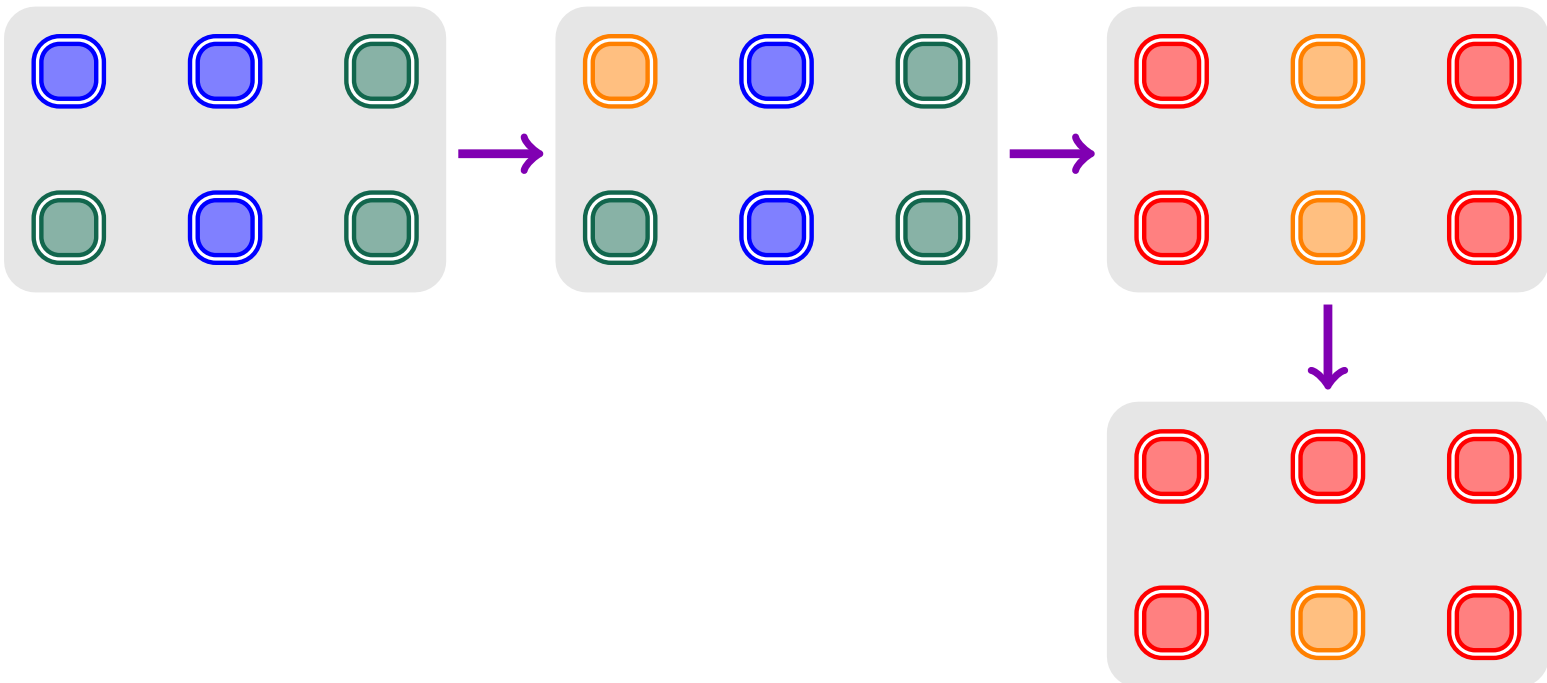

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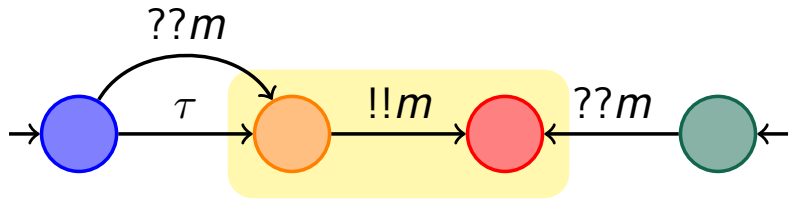

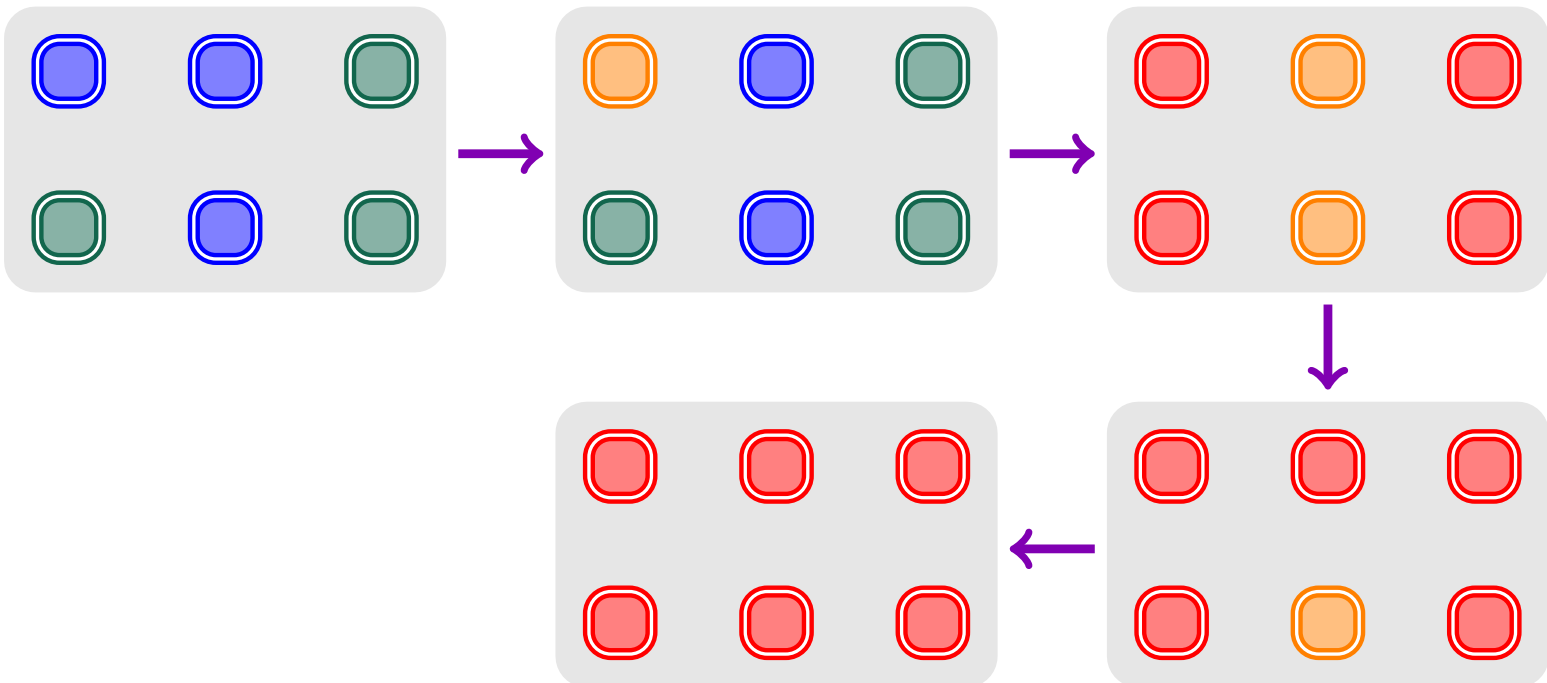

Reachability question

Parameters: Number of processes

Control State Reachability (REACH)

Input: A protocol and a control state $q \in Q$;

Output: Does there exist $\gamma \in \mathcal{C}_0$ and $\gamma' \in \mathcal{C}$ s.t. $\gamma \to^* \gamma'$ and $\gamma(q) > 0$?

Remarks:

- This problem considers an infinite number of possible initial configurations
- Reachability of a configuration γ' is easier, **the number of processes is in fact fixed**

WQO and upward closed sets

Well Quasi Ordering (wqo)

$(X, \leq)$ is a well-quasi ordering if for all infinite sequences $s_1, s_2, \ldots$, there exists $i < j$ such that $s_i \leq s_j$.

Upward closed set

A set $Y \subseteq X$ is upward closed w.r.t $(X, \leq)$ if $y \in Y$ and $y \leq y'$ implies $y' \in Y$.

- Upward closure of $Y \subseteq X$: $Y\uparrow = \{x \in X \mid \exists y \in Y \ \wedge \ y \leq x\}$

Lemma

If $(X, \leq)$ is a wqo and if $Y \subseteq X$ is upward closed w.r.t. $(X, \leq)$, then there exists a finite set $B \subseteq X$ s.t. $Y = B\uparrow$.

Well structured transition systems everywhere

$$\gamma \preceq \gamma' \text{ iff } \forall q \in Q, \text{ we have } \gamma(q) \leq \gamma'(q)$$

Theorem

$(\mathcal{C}, \preceq)$ is a well-quasi-ordering.

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Monotonicity lemma

For $\gamma_1, \gamma_1', \gamma_2 \in \mathcal{C}$, if

- $\gamma_1 \Rightarrow \gamma_1'$ and $\gamma_1 \preceq \gamma_2$

then there exists $\gamma_2' \in \mathcal{C}$ s.t.

- $\gamma_2 \Rightarrow \gamma_2'$ and $\gamma_1' \preceq \gamma_2'$

- BN are **Well Structured Transition Systems**
  [Abdulla et al., LICS'96; Finkel & Schnoebelen, TCS'01]

Deciding REACH in Broadcast Networks

Theorem [Esperza et al., LICS'99]

REACH is decidable for Broadcast Networks

Idea of the proof

- For $S \subseteq \mathcal{C}$, $pre(S) = \{\gamma \in \mathcal{C} \mid \gamma \Rightarrow \gamma' \ \wedge \ \gamma' \in S\}$

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- For S upward-closed, there exists $i \in \mathbb{N}$ s.t. $\Gamma^{i+1}(S) = \Gamma^{i}(S)$ and given a finite basis B of S, one can compute a finite basis B' of $\Gamma^{i}(S)$

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- Take for S the configuration γ such that $\gamma(q) = 1$ and $\gamma(q') = 0$ for all $q' \neq q$

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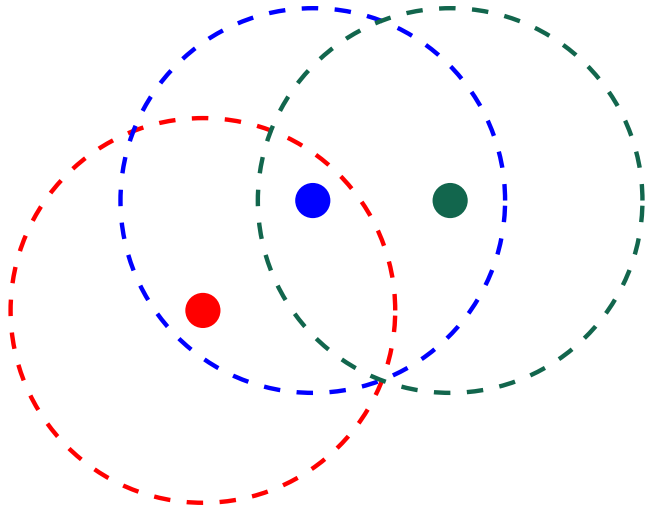
Theorem [Schmitz & Schnoebelen, CONCUR'13]

REACH for Broadcast Networks is Ackermann-complete.

Outline

Ad Hoc Networks



Main characteristics of Ad Hoc Networks

- Nodes can be mobile
- Topology is not known a priori
- Messages are broadcasted to the neighbours
- Problems linked to communication (collision, loss of messages, etc.)

Defining a model for Ad Hoc Networks

Main characteristics [Delzanno et al., CONCUR'10]

- No creation/deletion of nodes
- Each node executes the same finite state process
- Model based on the ω-calculus
- Broadcast of the messages to the neighbors
- Static topology represented by a connectivity graph

Ad Hoc Networks: syntax

A protocol $P = \langle Q, \Sigma, R, Q_0 \rangle$

Finite state system whose transitions are labeled with:

1. broadcast of messages - $!!m$
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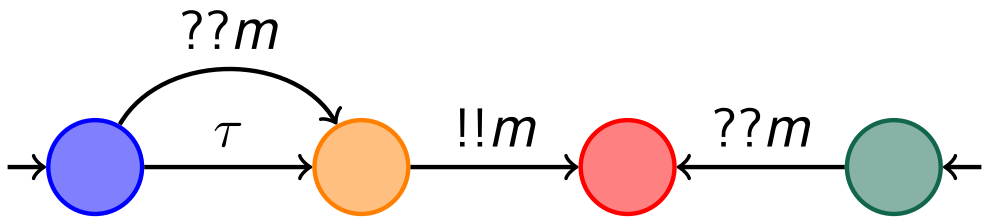
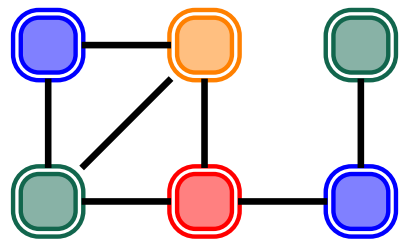


A protocol defines an Ad Hoc Network (AHN)

Ad Hoc Networks: configurations

A configuration is a graph $\gamma = \langle V, E, L \rangle$

- V : finite set of vertices
- $E : V \times V$: finite set of edges
- $L : V \to Q$: labeling function



- **Initial configurations**: **all** vertices are labeled with initial states
- *Notation : $L(\gamma)$ all the labels present in γ*

Remarks:

- The size of the considered graphs is not bounded
- Infinite number of configurations

$\Rightarrow$ **BN are infinite state systems**

Ad Hoc Networks: semantics

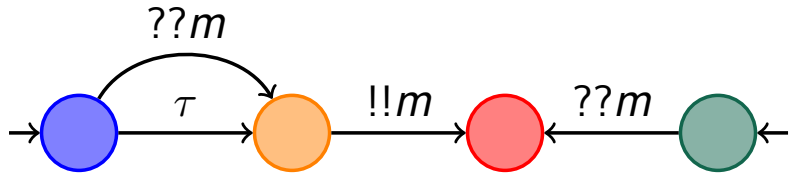
Transition system $BN(P) = \langle \mathcal{C}, \rightarrow, \mathcal{C}_0 \rangle$ associated to P

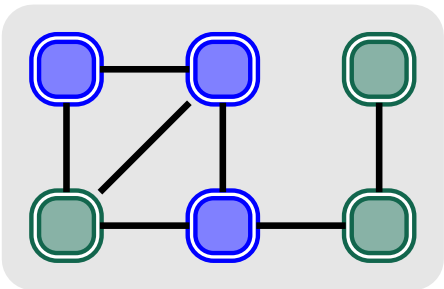
- $\mathcal{C}$: set of configurations
- $\rightarrow$: $\mathcal{C} \times \mathcal{C}$: transition relation
- $\mathcal{C}_0$: initial configurations

The relation $\rightarrow$ respects the following rules during an execution:

- The topology remains **static**
  - The number of vertices does not change
  - The edges do not change
  - Only the labels of the vertices can evolve
- Two kind of transitions according to the given protocol
  1. **local actions** - one process performs an internal action τ
  2. **broadcast** - one process emits a message with $!!m$, all its neighbors that can receive it with $??m$ have to receive it

Ad Hoc Networks: an example





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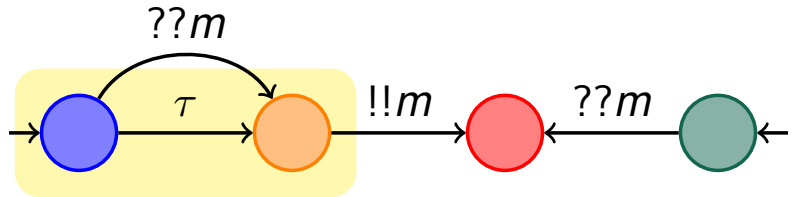

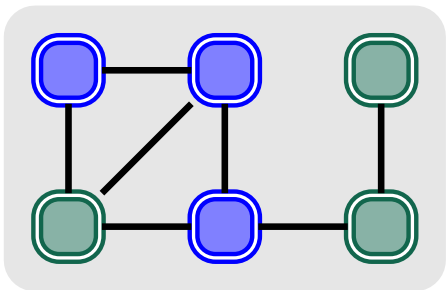

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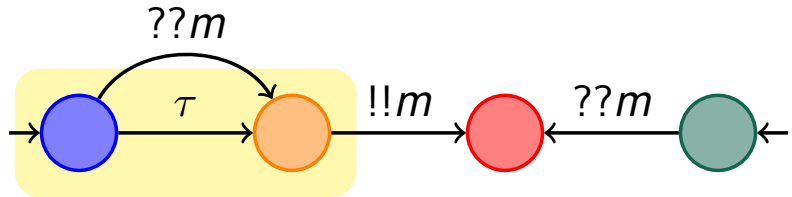

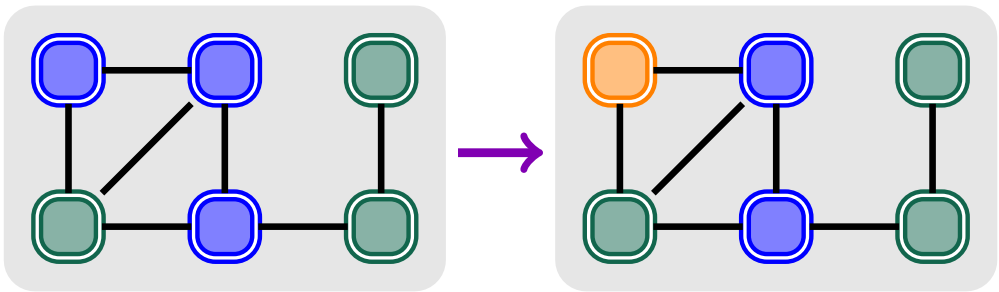

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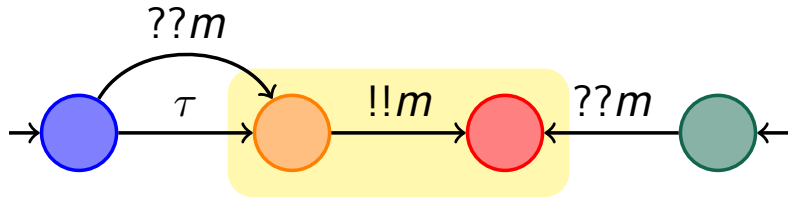

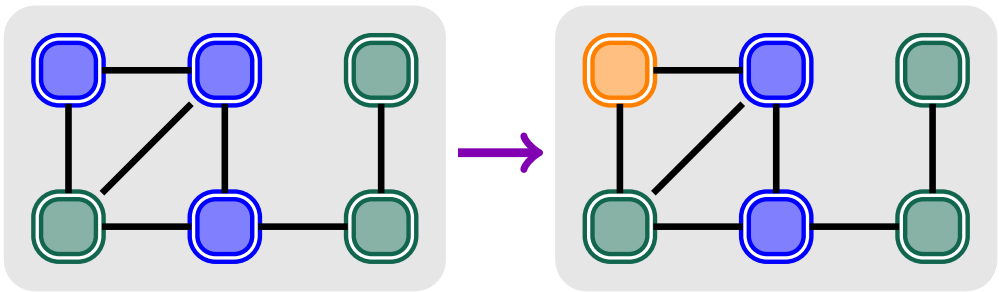

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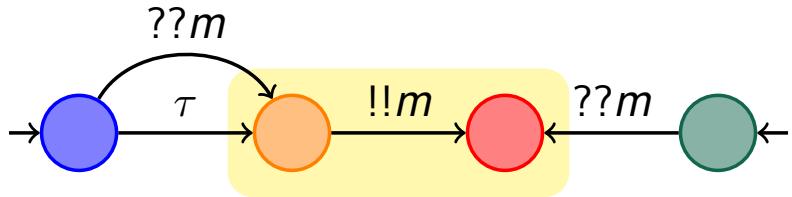

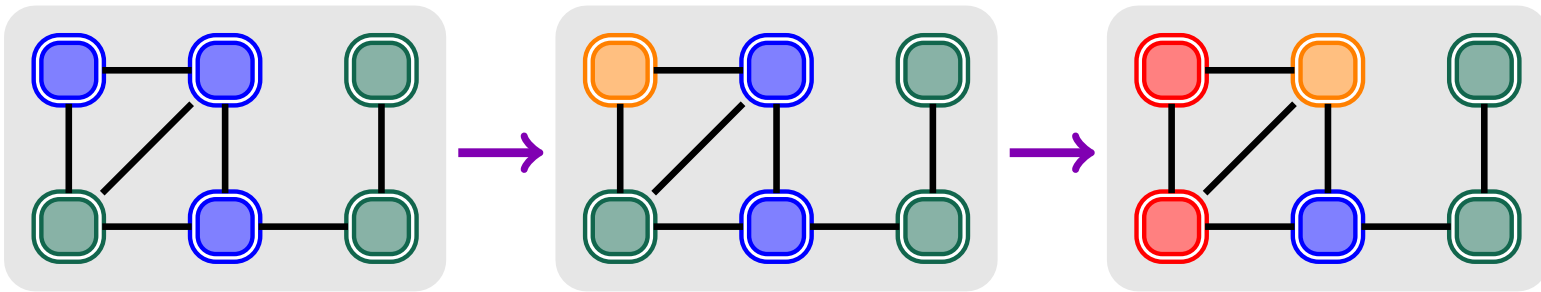

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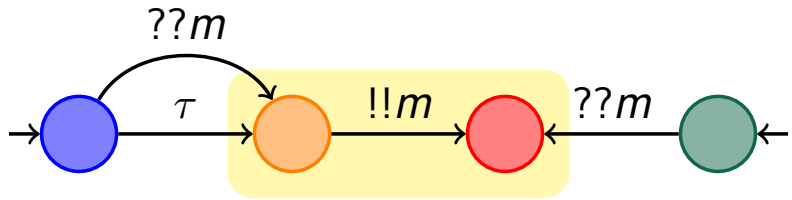

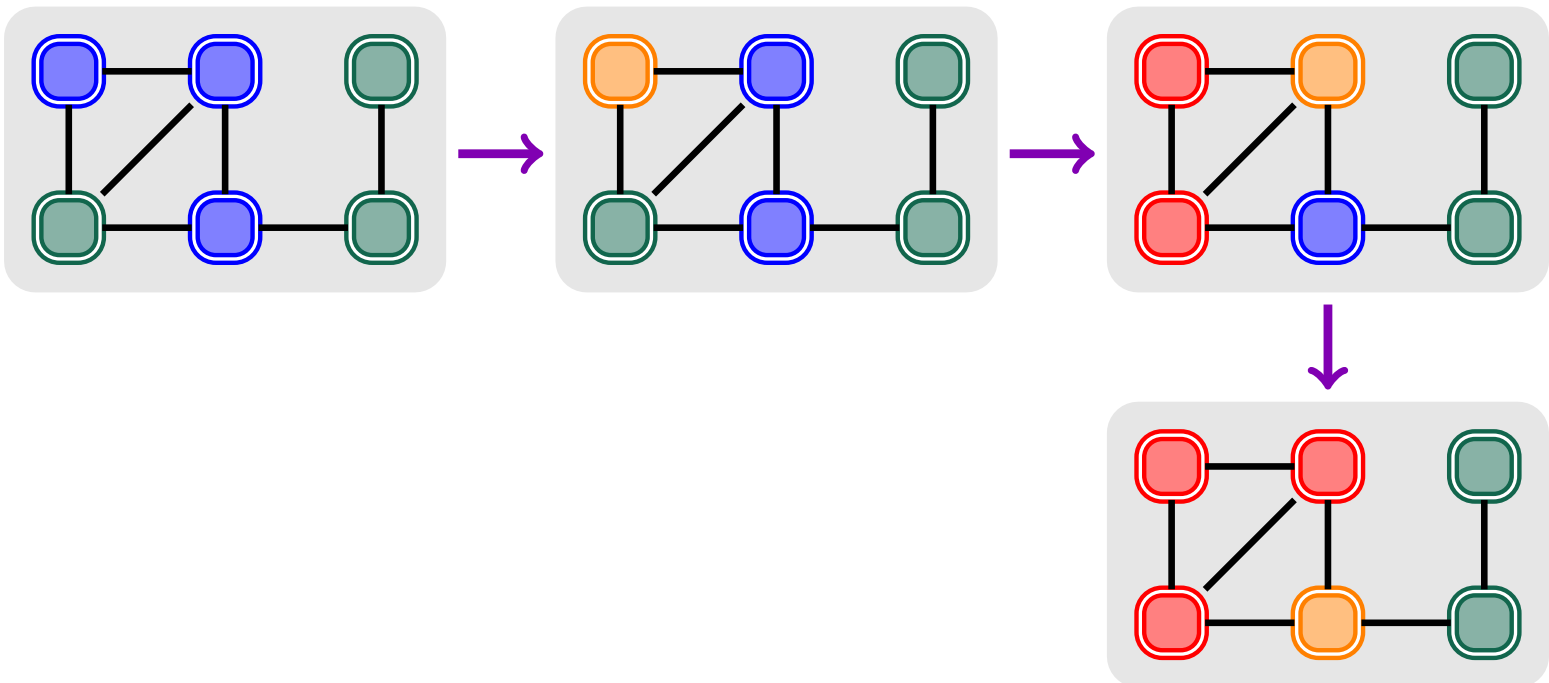

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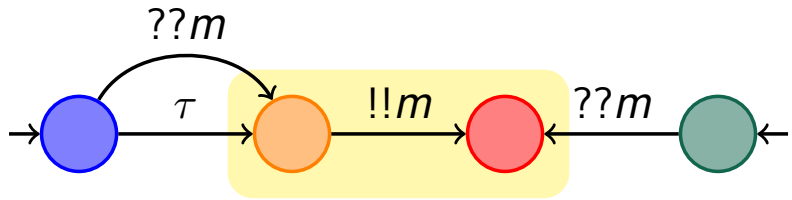
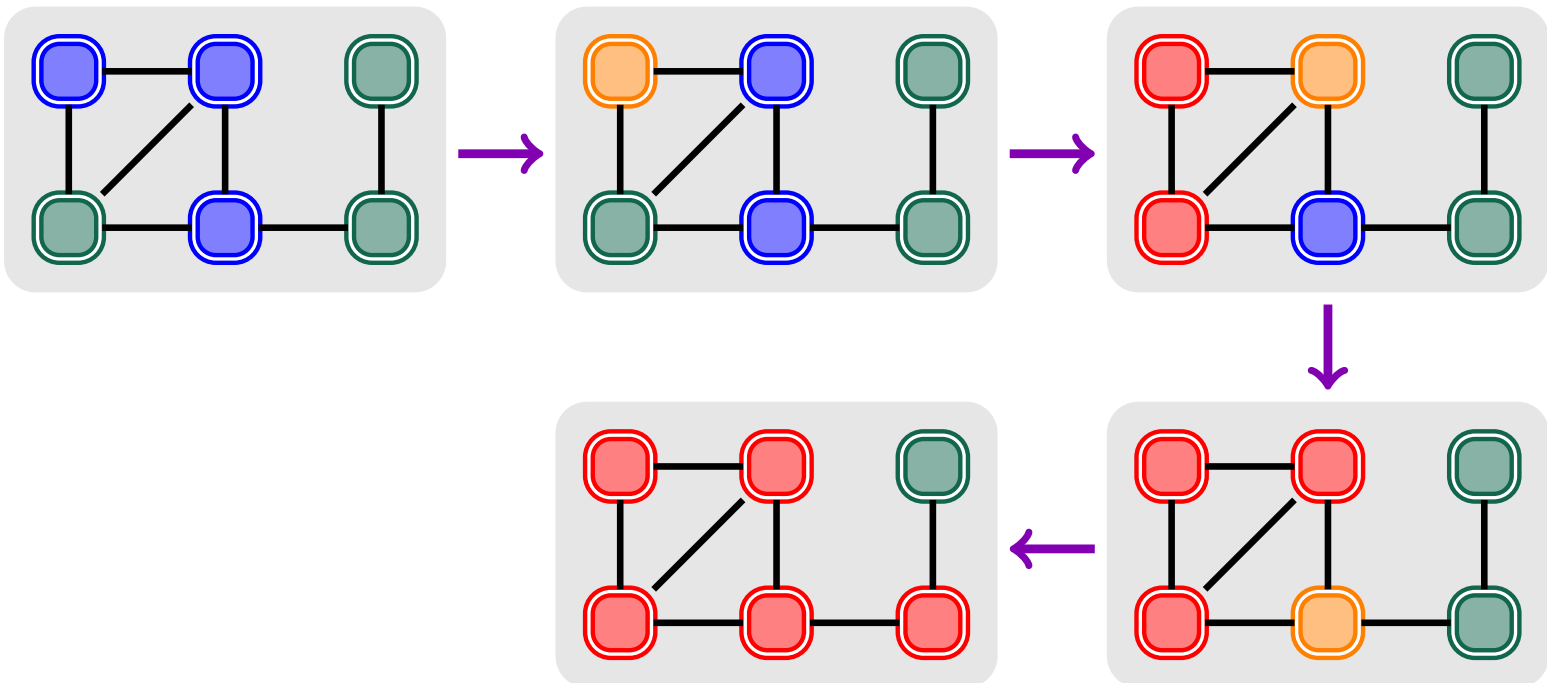
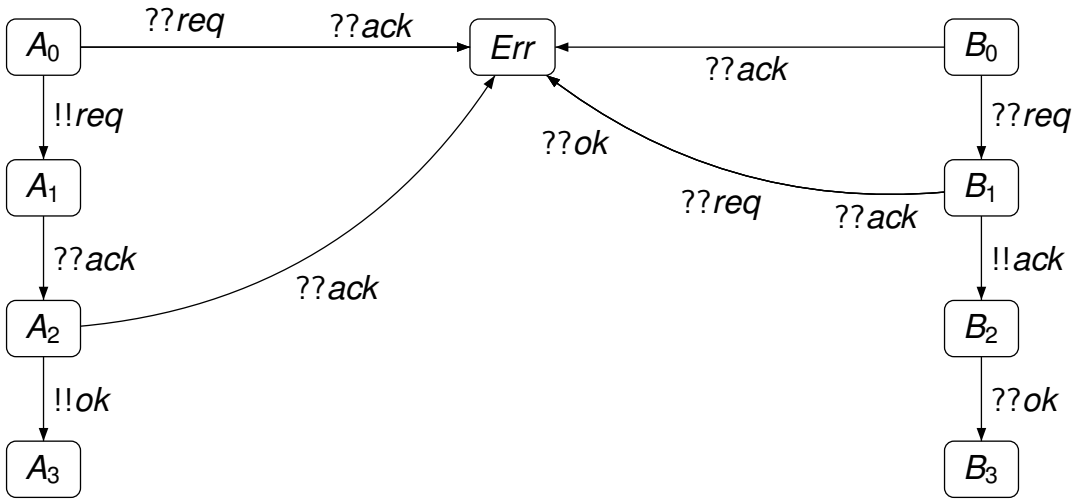




Ensuring the form of a topology

The Req/Ack/Ok-protocol



Properties

B_3

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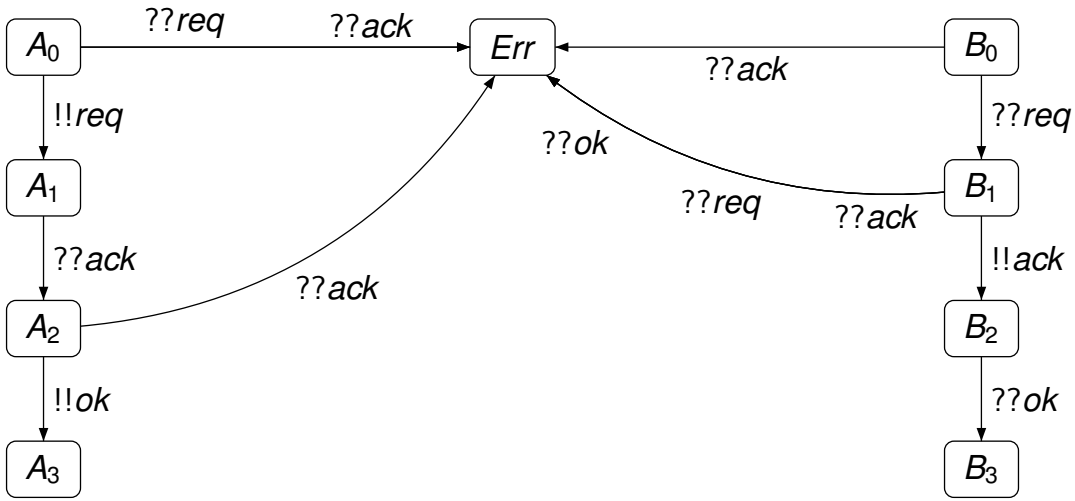

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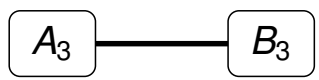

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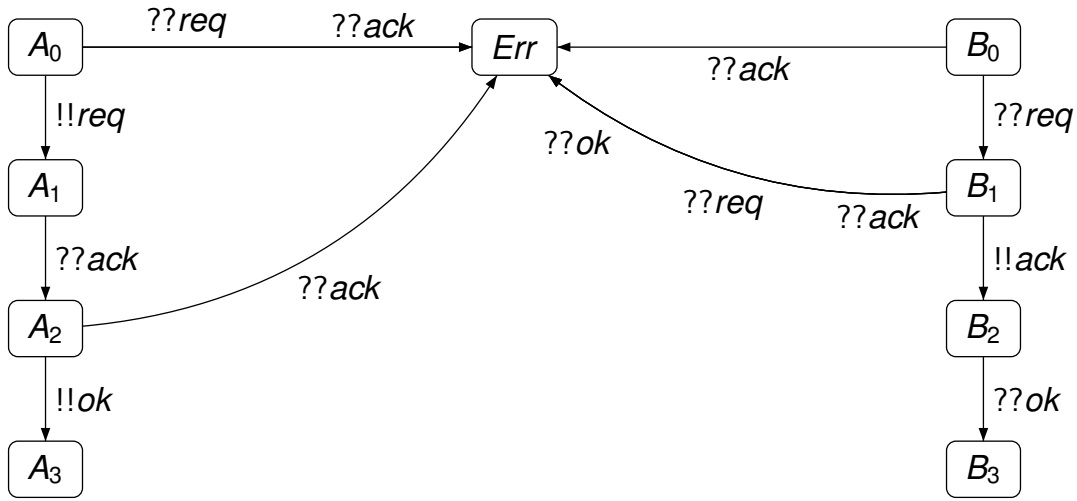
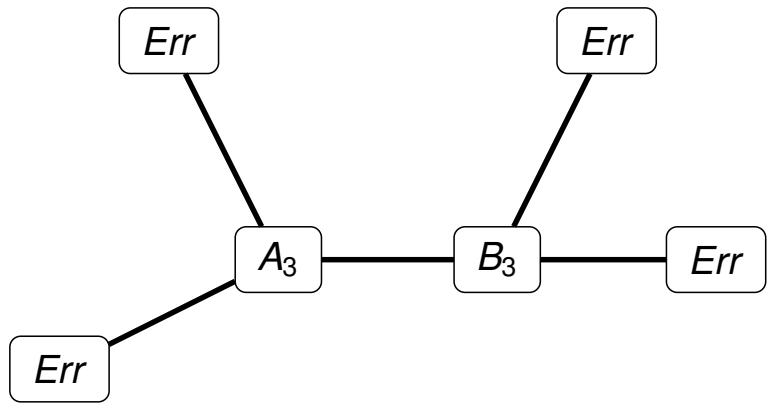
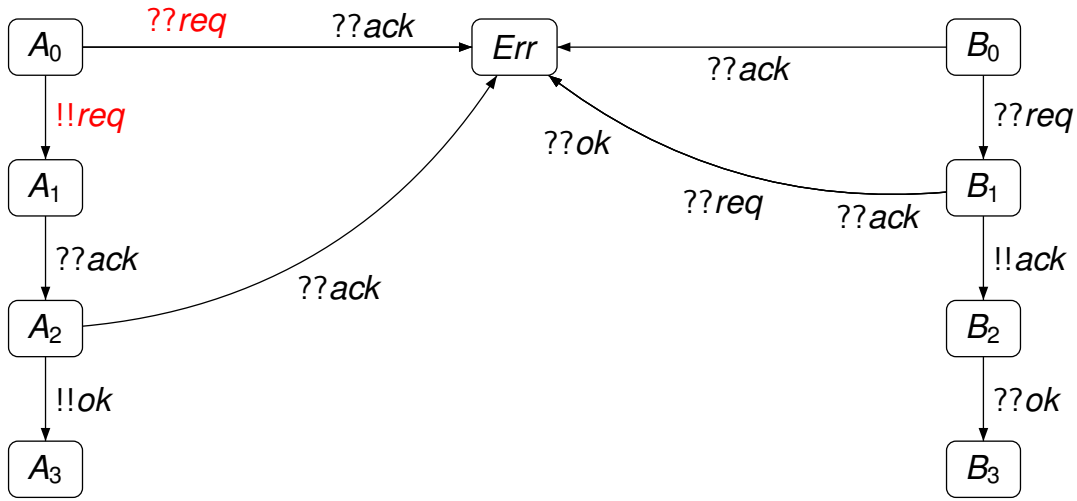


Properties



Ensuring the form of a topology

The Req/Ack/Ok-protocol



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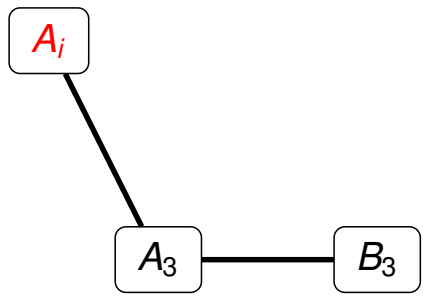

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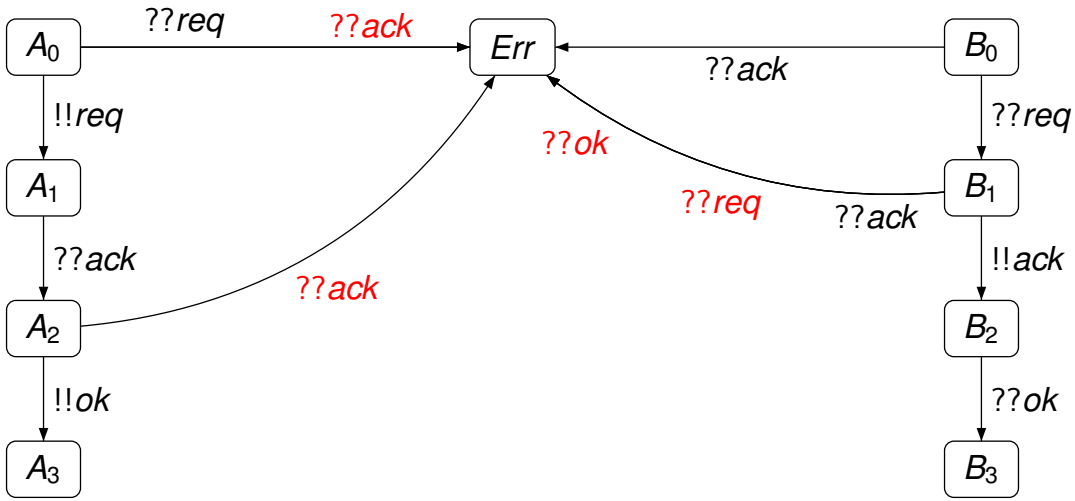

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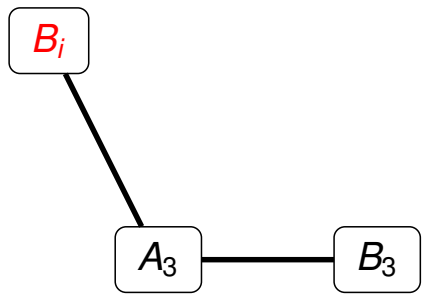

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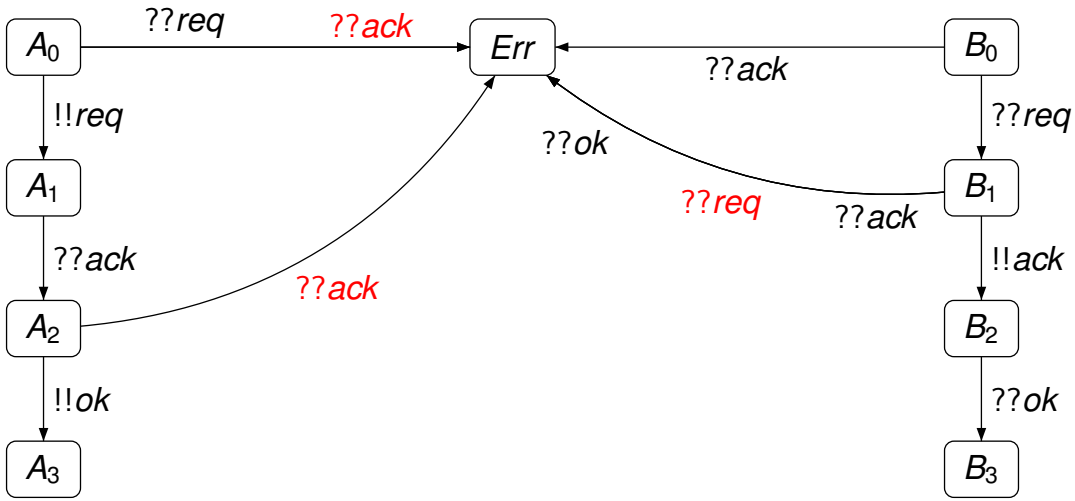

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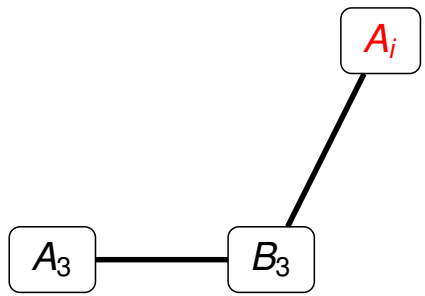

Ensuring the form of a topology

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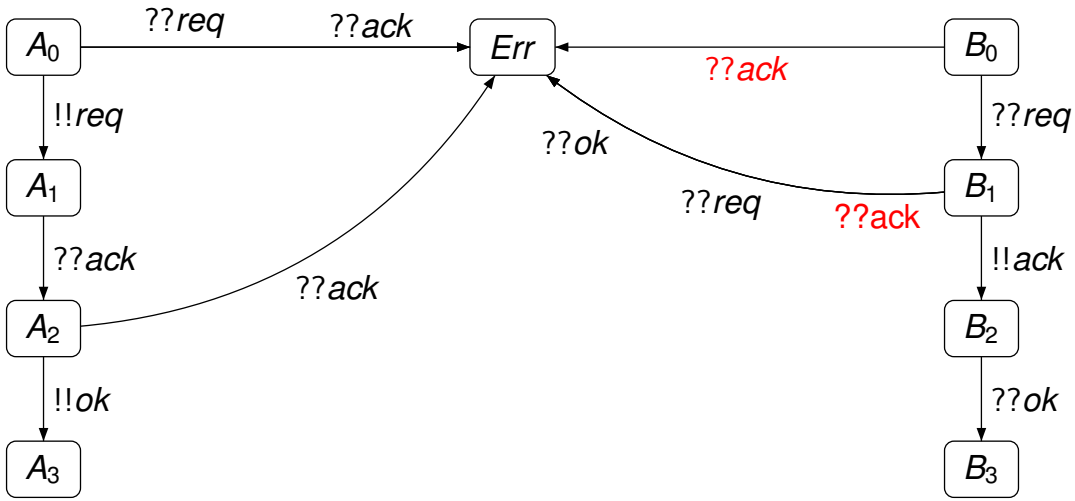

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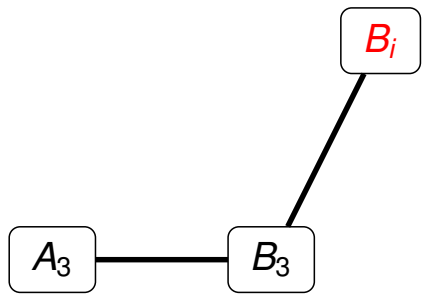

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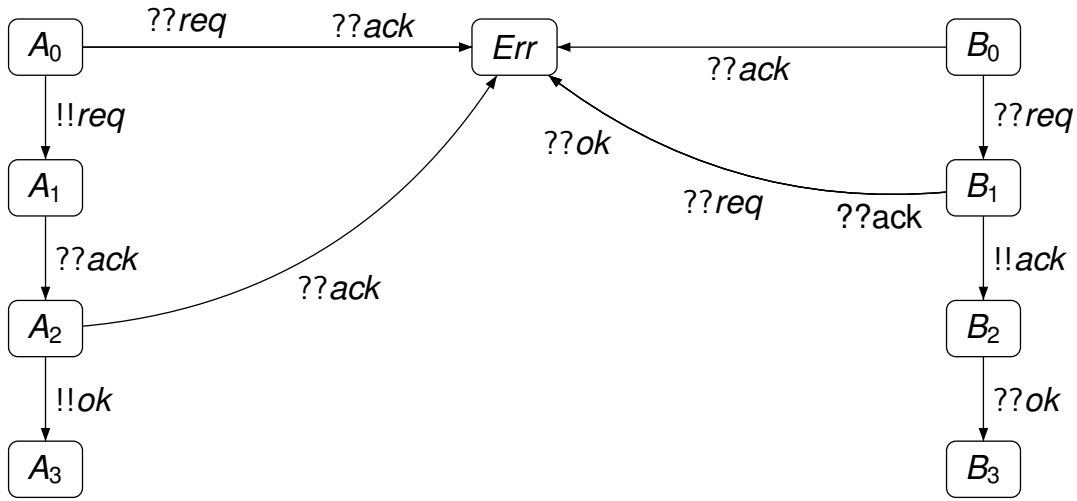

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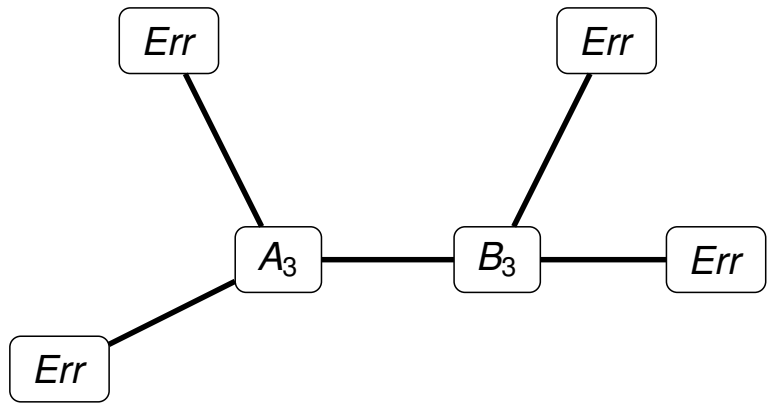


Undecidability result

Theorem [Delzanno et al, CONCUR'10]

REACH for Ad Hoc Networks is undecidable.

Undecidability result

Theorem [Delzanno et al, CONCUR'10]

REACH for Ad Hoc Networks is undecidable.

One way to regain decidability:
restrict the considered graphs

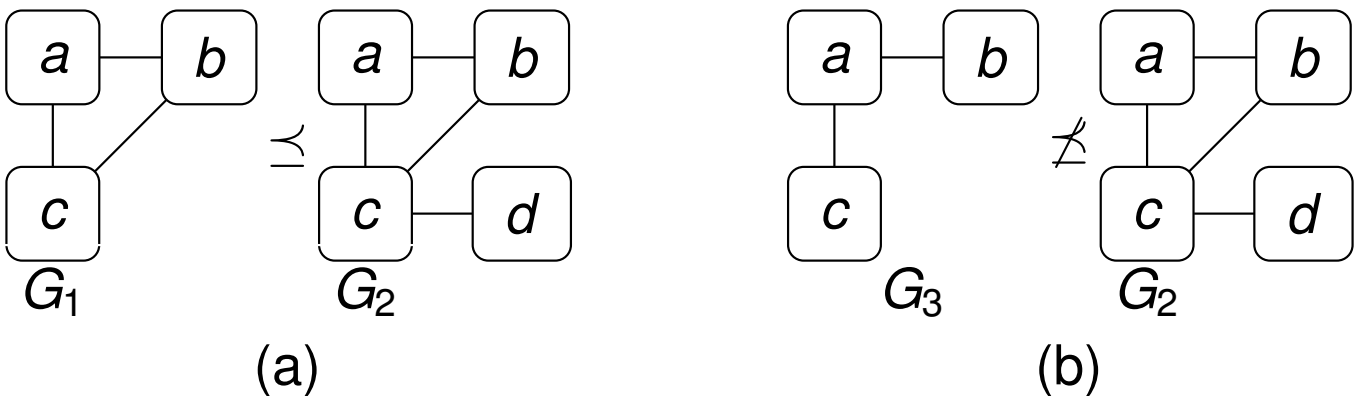
Considered order on graphs

- Given $\gamma \in \mathcal{C}$, $G(\gamma)$ is the associated graph

Induced subgraph relation

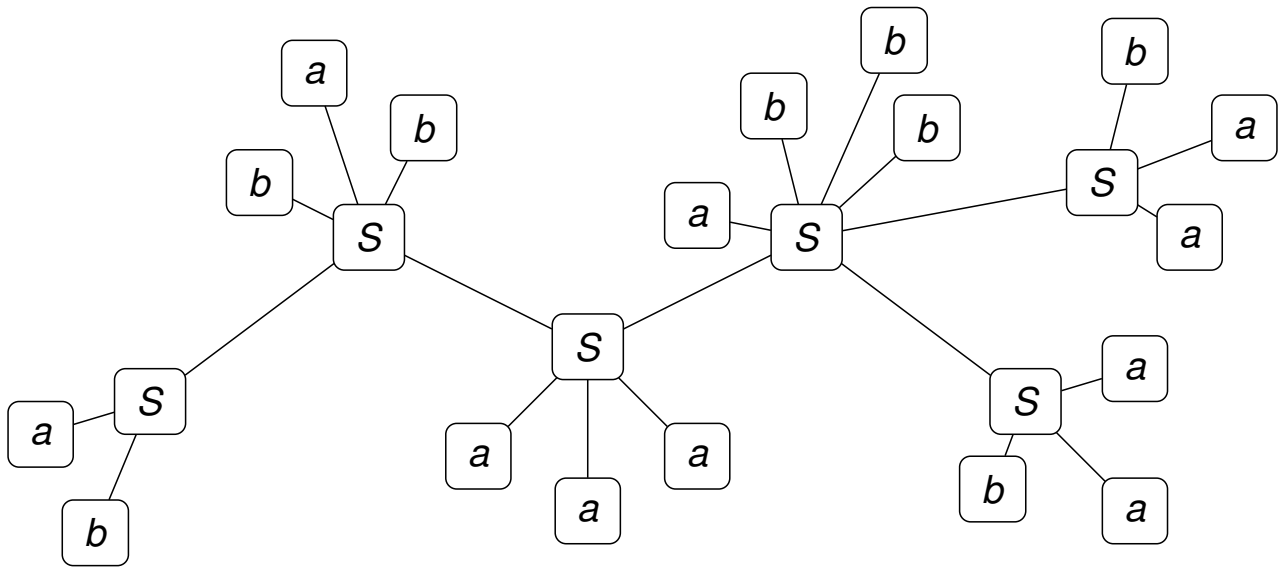
Given $\gamma_1, \gamma_2 \in \mathcal{C}$, $\gamma_1 \preceq \gamma_2$ if there exists a label preserving injection h from nodes of $G(\gamma_1)$ to nodes of $G(\gamma_2)$ s.t.:

- (n, n') is an edge in $G(\gamma_1)$ **if and only if** $(h(n), h(n')$ is an edge in $G(\gamma_2)$



Bounded path configurations

- $\mathcal{P}^K$: set of configurations $\gamma \in \mathcal{C}$ s.t. the length of the longest simple path in $G(\gamma)$ is smaller than K



Theorem [Ding, J. of Graph Theory'92]

For all $K \in \mathbb{N}$, $(\mathcal{P}^K, \preceq)$ is a well-quasi-ordering

Well structured transition systems everywhere

Monotonicity lemma

For $\gamma_1, \gamma_1', \gamma_2 \in \mathcal{P}^{\mathcal{K}}$, if

- $\gamma_1 \Rightarrow \gamma_1'$ and $\gamma_1 \preceq \gamma_2$

then there exists $\gamma_2' \in \mathcal{P}^{\mathcal{K}}$ s.t.

- $\gamma_2 \Rightarrow \gamma_2'$ and $\gamma_1' \preceq \gamma_2'$

- AHN restricted to K-bounded path configurations are **Well Structured Transition Systems**

Remark:

- This is true with induced subgraph but not with subgraph (*Node c broadcast a message received by node a and b)*

Well structured transition systems everywhere

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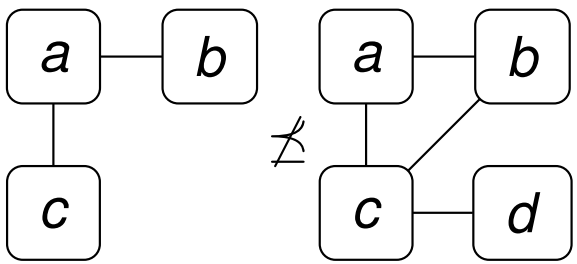

Decidability result

Theorem [Delzanno et al., CONCUR'10]

REACH is decidable for AHN restricted to K-bounded path configurations

Idea of the proof

- For $S \subseteq \mathcal{P}^K$, $pre_K(S) = \{\gamma \in \mathcal{P}^K \mid \gamma \Rightarrow \gamma' \ \wedge \ \gamma' \in S\}$
- if S is upward-closed, then $pre_K(S)$ is upward closed
- let $\Gamma : \mathcal{P}^K \mapsto \mathcal{P}^K$ s.t. $\Gamma(S) = S \cup pre_K(S)$
- For S upward-closed, there exists $i \in \mathbb{N}$ s.t. $\Gamma^{i+1}(S) = \Gamma^i(S)$ and given a finite basis B of S, one can compute a finite basis B' of $\Gamma^i(S)$
- Take for S the graph with a single node labelled with q

Outline

Conclusion

Complexity result for REACH in parameterized networks

Communication	Complexity
Broadcast	Ackermann-complete
Ad Hoc	Undecidable
Ad Hoc over K-bounded path configurations	Decidable