

# IMA2017\_continued\_fractions

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## 1 Continued fractions

The continued fraction is a current way of working with [Computable numbers](#) in Sage. We demonstrate how to create an infinite precision real numbers using them.

Ideally, it should also be possible to construct real numbers by specifying a sequence of digits.

### 1.1 Finite continued fractions

```
In [1]: cf = continued_fraction([1, 3, 5, 2])
```

```
In [2]: cf
```

```
Out[2]: [1; 3, 5, 2]
```

```
In [3]: ascii_art(cf)
```

```
Out[3]:
```

$$1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{2}}}$$

```
In [1]: unicode_art(cf)
```

```
In [5]: show(cf)
```

```
[1; 3, 5, 2]
```

```
In [ ]:
```

```
In [9]: cf.value()
```

```
Out[9]: 46/35
```

```
In [ ]:
```

## 1.2 The infinite continued fraction of $\pi$

(and how to make  $\pi$  rational)

```
In [2]: cf = continued_fraction(pi)
```

```
In [3]: cf
```

```
Out[3]: [3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, ...]
```

```
In [4]: q = cf.convergent(20)
        q
```

```
Out[4]: 21053343141/6701487259
```

```
In [5]: q.denominator() * pi == q.numerator()
```

```
Out[5]: 6701487259*pi == 21053343141
```

```
In [6]: bool(q.denominator() * pi == q.numerator())
```

```
Out[6]: True
```

```
In [ ]:
```

## 1.3 Infinite continued fraction from its partial quotients

```
In [8]: w = words.ThueMorseWord([1,2])
        w
```

```
Out[8]: word: 12212112211212212112122112212112211221121221...
```

```
In [9]: # this is infinite!
        print w.length()
        print w[50000]
```

```
+Infinity
```

```
1
```

```
In [10]: cf = continued_fraction(w)
```

```
In [11]: cf
```

```
Out[11]: [1; 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2...]
```

```
In [12]: show(cf)
```

```
[1; 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2...]
```

```
In [13]: cf.numerical_approx()
```

```
Out [13]: 1.42238873688279
```

```
In [14]: cf.numerical_approx(digits=1000)
```

```
Out [14]: 1.422388736882785488341547116024565825306879108991711829311892452916456747
```

```
In [17]: parent(cf.value()) # not ideal...
```

```
Out [17]: Real Lazy Field
```

```
In [16]: cf.value()
```

```
Out [16]: 1.422388736882786?
```

```
In [ ]:
```