

# Introduction to the SageMath software



(<https://www.sagemath.org>)

slides available at

<http://www.labri.fr/perso/vdelecro/teaching.html>

June 1st 2017, Mini-course HSE Moscow

by Vincent Delecroix, CNRS researcher at LaBRI Bordeaux (France)






Why do you want (good) math softwares ?

# Why do you want (good) math softwares ?

- verify your small computations,

 jupyter SageMath Moscou Last Checkpoint: 4 hours ago (autosaved)

File Edit View Insert Cell Kernel Widgets Help

          Code   CellToolbar

In [34]: `1 + 1`

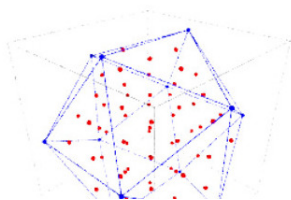
Out[34]: 2

In [ ]:

# Why do you want (good) math softwares?

- verify your small computations,
- make nice illustrations,

```
In [42]: P = 3 * polytopes.icosahedron()  
I = P.integral_points()  
G = P.plot(polygon=False) + point3d(I, color='red')  
G.show(viewer='tachyon', figsize=3)
```



# Why do you want (good) math softwares ?

- verify your small computations,
- make nice illustrations,
- make huge computations, test conjectures,

 jupyter SageMath Moscou Last Checkpoint: 5 hours ago (unsaved changes)

File Edit View Insert Cell Kernel Widgets Help

          Code  CellToolbar

```
In [68]: G = graphs.RandomGNP(150, 0.5)
         tsp = G.traveling_salesman_problem()
```

```
In [ ]:
```

# Why do you want (good) math softwares ?

- verify your small computations,
- make nice illustrations,
- make huge computations, test conjectures,
- develop new algorithms,

 jupyter SageMath Moscou Last Checkpoint: 5 hours ago (unsaved changes)

File Edit View Insert Cell Kernel Widgets Help


          Code   CellToolbar

```
In [63]: def is_prime(n):  
         if n < 2:  
             return False  
         for i in range(2, n):  
             if n % i == 0:  
                 return False  
         return True
```

In [ ]:

# Why do you want (good) math softwares ?

- verify your small computations,
- make nice illustrations,
- make huge computations, test conjectures,
- develop new algorithms,
- experimental physics or mathematics, make conjectures.

 jupyter SageMath Moscou Last Checkpoint: 4 hours ago (unsaved changes)

File Edit View Insert Cell Kernel Widgets Help

          Code  CellToolbar

```
In [57]: from scipy.integrate import ode
y0, t0 = [1.0jr, 2.0r], 0r
def f(t, y, arg1): return [1jr*arg1*y[0] + y[1], -arg1*y[1]**2r]
def jac(t, y, arg1): return [[1jr*arg1, 1r], [0r, -arg1*2r*y[1]]]
r = ode(f, jac).set_integrator('zvode', method='bdf', with_jacobian=True)
r.set_initial_value(y0, t0).set_f_params(2.0r).set_jac_params(2.0r)
t1 = 10r
dt = 1r
while r.successful() and r.t < t1:
    r.integrate(r.t+dt)
    print("%g %g" % (r.t, r.y[-1]))
```

# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .



# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .

- These softwares are expensive.

# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .

- These softwares are expensive.
- No way to verify how they work.

# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .

- These softwares are expensive.
- No way to verify how they work.
- Sometimes impossible to get bugs corrected.

# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .

- These softwares are expensive.
- No way to verify how they work.
- Sometimes impossible to get bugs corrected.
- They can disappear.

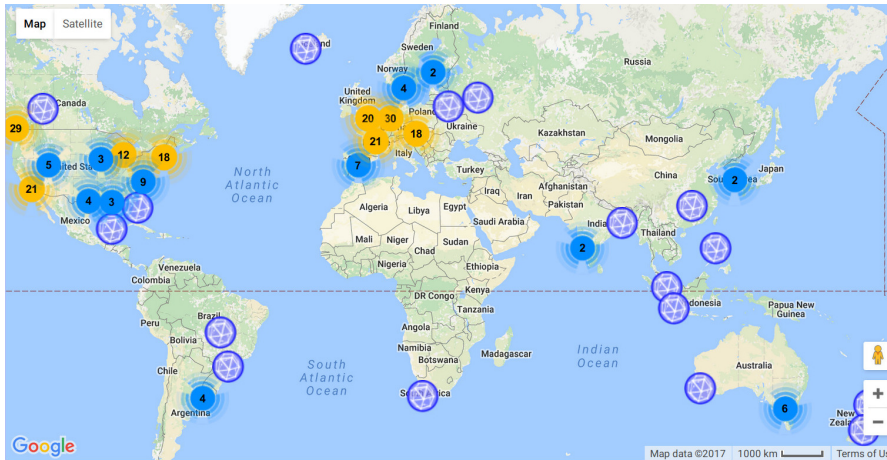
# Why SageMath

Four commercial softwares are dominating the field of mathematical computations : Mathematica, Maple, Matlab, Magma. . .

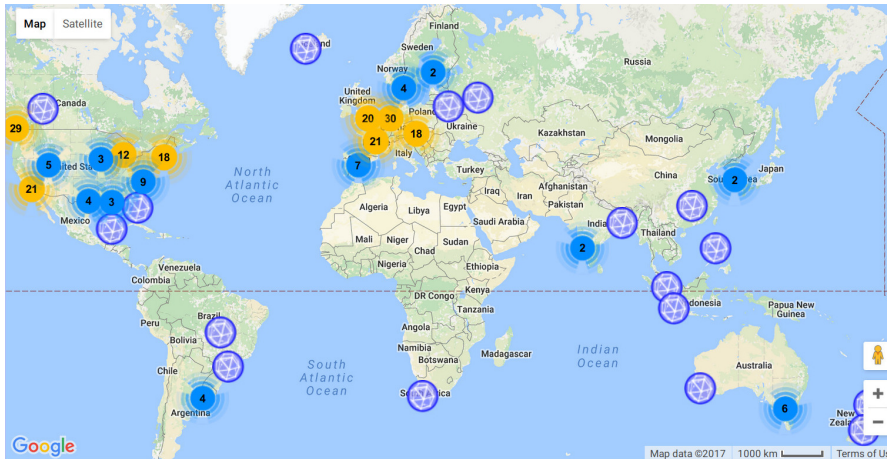
- These softwares are expensive.
- No way to verify how they work.
- Sometimes impossible to get bugs corrected.
- They can disappear.

Free mathematical softwares exists since the 80's like GAP, R, Maxima, PARI/GP. Each of them is specialized. SageMath is an international project started in 2005 and aims to cover a large range of mathematics.

# The developer map



# The developer map



One contributor from Russia!?! (Alexey U. Gudchenko, contributed 6 years ago)

## Four important ingredients



open source license



## Four important ingredients



open source license



popular programming language

# Four important ingredients



open source license



popular programming language



generic notebook interface

# Four important ingredients



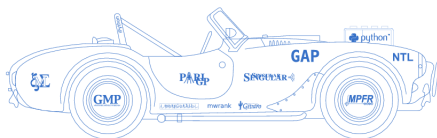
open source license



popular programming language



generic notebook interface



»Every free computer algebra system I've tried has  
reinvented many times the wheel without being able to build the car.«

on the shoulders of hundreds of free mathematical libraries



***Free as in Freedom***

- SageMath is free.
- Can be downloaded from internet <https://www.sagemath.org/>
- Source code at <https://git.sagemath.org/sage.git/>
- Contributions open to anyone <https://trac.sagemath.org>



***Free as in Freedom***

- SageMath is free.
- Can be downloaded from internet <https://www.sagemath.org/>
- Source code at <https://git.sagemath.org/sage.git/>
- Contributions open to anyone <https://trac.sagemath.org>

Freeness will remain forever and is guaranteed by the GNU GPL license.



Python is a very popular programming language that is easy to learn and close to mathematical notation.



Python is a very popular programming language that is easy to learn and close to mathematical notation. The set

$$\{x \in \{1, \dots, 20\} : 2x^2 - 1 \text{ is prime}\}$$

can be constructed in Sage as

```
[x for x in [1..20] if is_prime(2*x^2 - 1)]
```



Python is a very popular programming language that is easy to learn and close to mathematical notation. The set

$$\{x \in \{1, \dots, 20\} : 2x^2 - 1 \text{ is prime}\}$$

can be constructed in Sage as

```
[x for x in [1..20] if is_prime(2*x^2 - 1)]
```

Python is used for many other purposes : web programming, script language, biology, data analysis, etc





Jupyter is a generic web interface for programming language. It can be used with many different languages and softwares : Sage, PARI/GP, C, C++, etc. The list of kernels can be found at

<https://github.com/jupyter/jupyter/wiki/Jupyter-kernels>

A screenshot of a Jupyter notebook interface. At the top, it says "jupyter Untitled" and "Last Checkpoint: 9 minutes ago (unsaved changes)". Below that is a menu bar with "File", "Edit", "View", "Insert", "Cell", "Kernel", and "Help". A toolbar contains icons for file operations and a "Code" dropdown menu. The notebook content shows four input cells:

- In [1]: `from sympy import init_printing`  
`init_printing(use_latex='mathjax')`
- In [2]: `from sympy.abc import x,y,z,alpha,epsilon`  
`x ** 2 / (alpha + epsilon)`  
Out[2]: 
$$\frac{x^2}{\alpha + \epsilon}$$
- In [10]: `from sympy import Integral, integrate, sin, pi, Eq`  
`A = Integral(sin(x), (x,0,pi))`  
`B = integrate(sin(x), (x,0,pi))`  
`Eq(A, B)`  
Out[10]: 
$$\int_0^x \sin(x) dx = 2$$
- In [11]: `integrate(sin(x))`  
Out[11]:  $-\cos(x)$
- In [14]: `%matplotlib inline`  
`from sympy import plot`  
`plot(sin(x), (x,-2*pi, 2*pi))`

The output of the last cell is a plot of the sine function,  $y = \sin(x)$ , for  $x$  ranging from  $-2\pi$  to  $2\pi$ . The x-axis is labeled "x" and ranges from -6 to 6. The y-axis is labeled "y" and ranges from -1.0 to 1.0. The plot shows two full cycles of the sine wave.- Out[14]: `<sympy.plotting.plot.Plot object at 0x7f8daa3f2850>`



«Every fine computer algebra system I've tried has  
reversed many times the wheel without being able to build the car»

Sage is built on top of hundreds of scientific libraries and softwares.

- GMP or MPIR : C library for arbitrary precision integers
- flint : a C library for algebra
- linbox : C++ library for exact linear algebra
- PARI/GP : a CAS for number theory
- GAP : a CAS for group computations
- complete list at  
<http://www-ftp.lip6.fr/pub/math/sagemath/spkg/upstream/>