Interval exchange transformations
Part IV: Generic properties

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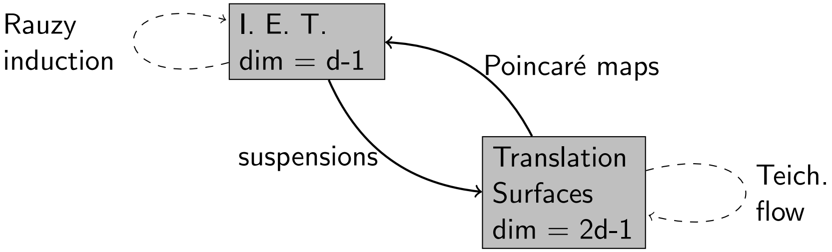
Rauzy induction and Teichmüller flow



Rauzy(-Veech) induction = first return map of the Teichmüller flow

Correspondence

Let $X \subset \mathcal{A}^{\mathbb{Z}}$ be a minimal shift with an invariant measure μ. We define

$$\varepsilon_n(X,\mu) = \min_{u \in \mathcal{L}_{X,n}} \mu([u]).$$

(symbolic)		(geometric)	
$n\varepsilon_n(X,\mu)$	Rauzy induction	Teichmüller flow	best approximations
$\liminf > 0$ (LR)	finite time positive-ness	bounded g_t-orbit	$\liminf q_n\{q_n\alpha\} > 0$
$\limsup > 0$ (Bosh. cond.)	?	non-divergent g_t-orbit	$\limsup q_{n+1}\{q_n\alpha\} > 0$

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open question 1: Exact conditions on $n\varepsilon_n$ (+ Rauzy graphs) for unique ergodicity?

Theorem

Let S be a translation surface and ϕ_t^θ its family of translation flows.

condition on the dynamics	parameters θ that satisfies the condition	who
minimal	countable complement	Keane 1975
uniquely ergodic	full Lebesgue measure	Kerckhoff-Masur-Smillie 1986
linearly recurrent	thick but 0 measure	Kleinbock-Weiss 2004, Chaika-Cheung-Masur 2013

Big questions

Theorem

Let $\mathcal{H}(\alpha)$ be a stratum of translation surfaces. For **almost every surfaces** (S, ϕ_t^θ) *in $\mathcal{H}(\alpha)$ we have the following*

- $\#V(S,R) \sim c_s R^2$ *(Eskin-Masur 2001)*

If moreover $\mathcal{H}(\alpha)$ is not a stratum of tori (i.e. $\alpha \neq (2\pi, \ldots, 2\pi)$)

- *weak-mixing for a.e. θ (Avila-Forni 2007)*

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open question 3 (4): What is the set of surfaces for which the translation flow is weak-mixing (topologically mixing) in almost every direction?

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Complete results for the so called Veech surfaces (Veech 1989, Avila-Delecroix 2014).

Some others...

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open question 6: (Veech 1982) Are almost every i.e.t. prime?