

Contents

<i>Preface</i>	<i>page ix</i>
1 Recurrence	1
1.1 Invariant measures	2
1.2 Poincaré recurrence theorem	4
1.3 Examples	10
1.4 Induction	23
1.5 Multiple recurrence theorems	29
2 Existence of invariant measures	35
2.1 Weak* topology	36
2.2 Proof of the existence theorem	45
2.3 Comments in functional analysis	49
2.4 Skew-products and natural extensions	53
2.5 Arithmetic progressions	58
3 Ergodic theorems	64
3.1 Ergodic theorem of von Neumann	65
3.2 Birkhoff ergodic theorem	70
3.3 Subadditive ergodic theorem	78
3.4 Discrete time and continuous time	87
4 Ergodicity	93
4.1 Ergodic systems	94
4.2 Examples	100
4.3 Properties of ergodic measures	116
4.4 Comments in conservative dynamics	120
5 Ergodic decomposition	142
5.1 Ergodic decomposition theorem	142
5.2 Rokhlin disintegration theorem	150

6 Unique ergodicity	157
6.1 Unique ergodicity	157
6.2 Minimality	159
6.3 Haar measure	162
6.4 Theorem of Weyl	173
7 Correlations	181
7.1 Mixing systems	182
7.2 Markov shifts	190
7.3 Interval exchanges	200
7.4 Decay of correlations	208
8 Equivalent systems	213
8.1 Ergodic equivalence	214
8.2 Spectral equivalence	216
8.3 Discrete spectrum	222
8.4 Lebesgue spectrum	225
8.5 Lebesgue spaces and ergodic isomorphism	233
9 Entropy	242
9.1 Definition of entropy	243
9.2 Theorem of Kolmogorov–Sinai	254
9.3 Local entropy	262
9.4 Examples	268
9.5 Entropy and equivalence	274
9.6 Entropy and ergodic decomposition	286
9.7 Jacobians and the Rokhlin formula	294
10 Variational principle	301
10.1 Topological entropy	302
10.2 Examples	313
10.3 Pressure	325
10.4 Variational principle	338
10.5 Equilibrium states	345
11 Expanding maps	352
11.1 Expanding maps on manifolds	353
11.2 Dynamics of expanding maps	362
11.3 Entropy and periodic points	374
12 Thermodynamic formalism	380
12.1 Theorem of Ruelle	381
12.2 Theorem of Livšic	399
12.3 Decay of correlations	402
12.4 Dimension of conformal repellers	417

Appendix A Topics in measure theory, topology and analysis	430
A.1 Measure spaces	430
A.2 Integration in measure spaces	443
A.3 Measures in metric spaces	453
A.4 Differentiable manifolds	460
A.5 $L^p(\mu)$ spaces	469
A.6 Hilbert spaces	473
A.7 Spectral theorems	477
<i>Hints or solutions for selected exercises</i>	482
<i>References</i>	504
<i>Index of notation</i>	511
<i>Index</i>	515

