

Contents

1	Introduction	11
1.1	What is a translation surface?	11
1.1.1	Three examples	11
1.1.2	Three definitions	12
1.1.3	Relations between the three definitions	15
1.1.4	Half-translation surfaces	17
1.1.5	Structures	20
1.2	Examples	24
1.2.1	Polygonal billiards	24
1.2.2	Baker's surfaces	28
1.2.3	The infinite staircase	29
1.2.4	Wind-tree models	31
1.2.5	Panov planes	32
1.2.6	Hooper-Thurston-Veech construction	37
1.2.7	Random models	41
1.2.8	Holomorphic foliations	45
2	Basic topological and geometrical aspects	49
2.1	Topological classification of surfaces	49
2.1.1	Ends of topological spaces	49
2.1.2	Topological classification of orientable surfaces	50
2.2	Covering spaces	53
2.3	Singularities and the space of linear approaches	57
2.3.1	The space of linear approaches	59
3	Symmetries	67
3.1	Analytic automorphisms: isometries and translations	67
3.2	Every group is a Veech group	71
3.3	Affine groups of coverings	75
3.3.1	Affine groups of Abelian coverings	77
3.3.2	Veech groups of Origamis	79
3.3.3	Affine symmetries of wind-tree models	82
3.4	Hooper-Thurston-Veech construction	86
3.5	Application to the baker's surface	93
4	Translation flows	97
4.1	Translation flows and interval exchange transformations	97
4.1.1	From finite to infinite interval exchange transformations	97
4.1.2	Skew-products of iet as return maps of G -coverings	101
4.2	Málaga maps	102
4.3	The jungle of infinite interval exchange transformations	102
4.3.1	Measurable construction	103
4.3.2	Topological constructions	107
4.4	Entropy	108
4.5	Parabolicity	111
4.5.1	Parabolicity via extremal length	111
4.5.2	Flat criterion for parabolicity	112

4.6	Criteria for completeness of the flow	114
5	Recurrence and diffusion	117
5.1	Recurrence in finite area	117
5.2	The boxes lemma	119
5.3	Recurrence in periodic and non-periodic windtree models with $a = b = 1/2$	120
5.4	Recurrence in G -coverings	123
5.4.1	The case of $G = \mathbb{Z}^d$	123
5.4.2	Avila-Hubert criterion	123
5.4.3	Example of recurrent G -covering for any group G	125
5.5	Hooper-Thurston-Veech	127
5.6	Diffusion rate and quantitative recurrence in G -coverings	128
5.6.1	Diffusion rate	128
5.6.2	Diffusion and quantitative recurrence	130
6	Invariant sets and invariant measures	133
6.1	Ergodicity and non-ergodicity of G -coverings	133
6.1.1	A criterion for non-ergodicity of \mathbb{Z}^d -coverings	134
6.1.2	Ergodic G -coverings	135
6.2	Hooper-Thurston-Veech	136
6.2.1	Homology, cohomology and measures	136
6.2.2	Pulling back measures from other Thurston-Veech constructions	137
6.2.3	Γ_λ action on the surface and induced action in homology	137
6.2.4	The subsequence decay property and measure classification	137
6.2.5	Application to G -coverings	138
6.3	Minimality	139
6.3.1	Limit sets	139
6.3.2	Two counterexamples to an infinite Keane theorem	139
6.3.3	An infinite translation surface without minimals	141
6.3.4	Panov's argument for transitive orbit	143
6.4	Typical minimality and ergodicity from approximation by compact surfaces	144
6.5	Remarks	146
6.5.1	Treviño extension of Masur criterion	146
A	Background on dynamical systems	147
A.1	Topological dynamics and measurable dynamics	147
A.2	Invariant measures of topological systems	147
A.3	Recurrence	148
A.4	Skew-products	149
A.4.1	Definitions	149
A.4.2	Ergodicity and essential values	150
A.4.3	Short note on cohomologies	151
B	Group action and the hyperbolic plane	153
B.1	Approximation in Fuchsian groups	153
B.2	Rosen continued fractions for G_λ	154
C	Compact translation surfaces	159
C.1	Interval exchanges and suspensions	159
C.2	Keane theorem and closed invariant sets	159
C.3	Deviation of Birkhoff averages and diffusion	160
C.4	Ergodicity and invariant measures	161
C.5	Oseledets theorem for KZ (Chaika-Eskin)	161
C.6	Veech groups	162
C.7	Dilation surfaces	163
C.8	Veech surfaces in $\mathcal{H}(2)$	164
D	Eigenvalues of infinite graphs	167

E	Discussion to have, open problems to solve	169
E.1	Possible places to cut	169
E.2	Ends of coverings	169
E.3	Non-commutative KZ cocycle	169
E.4	Potential examples for the introduction	169
E.4.1	Kernel Foliation	169
E.4.2	Affine structures	169
E.5	The metrics on flat surfaces of infinite type	170
E.6	Huge coverings	170
E.7	Panov plane	170
E.8	Ehrenfest	171
E.9	Non-abelian coverings	171
E.10	Further questions	171