Parallel Sparse Linear Algebra and Application to Structural Mechanics *

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This presentation deals with the parallelization of the OSSAU structural mechanics code being developped at CEA/CESTA (Le Barp, France).

In computational structural mechanics, the handling of complex 3D objects whose meshes consist in a large number of finite elements yields several problems. Since the matrix of the system does not have good properties (mainly due to the use of serendipity family finite elements), iterative methods do not behave well. Therefore, one must use direct linear out-of-core solvers that take into account the sparsity of the system. When the number of degrees of freedom becomes too large (typically above 100 000 dofs), sequential out-of-core solvers are outperformed too, and parallel solving is required. In that case, sparsity and irregularity is still a concern, all the more that the problem to solve is often non-linear (because of plasticity, large displacement, contact, etc.).

The OSSAU application computes the response of a structure to various constraints (external forces, pressures, thermic dilatations, accelerations). For each increment, one has to nullify a vector function of the unknown nodal displacement vector, which amounts to solving a sparse linear system. From the displacement vector, we compute structure constraints; in case of large displacements, plasticity occurs and the problem becomes non-linear. A potential destruction of the structure can be predicted thanks to the constraint vector.

One of the goals of the collaboration between CEA/CESTA and the ALiENor team of LaBRI is to parallelize the OSSAU code. More generally, our purpose is to develop a whole parallel processing chain for the assembly and solving, by direct methods, of huge sparse linear systems, for MIMD calculators with distributed memory. The final objective is a realistic validation of these tools for the OSSAU application.

In our presentation, we will describe our parallel processing chain, which integrates all of the state-of-the-art techniques used in parallel sparse linear algebra for direct methods:

- ordering, based on a tight coupling of the Nested Dissection and Approximate Minimum Degree followed by amalgamation;
- block symbolic factorization, in linear time and space complexity;
- block partitioning and mapping, based on a logical block simulation;
- parallel block solving, with local aggregations (Fan-In effect) and BLAS computations;
- distribution of elements, induced by the one of the matrix blocks.

Performance results will be shown, that have been obtained on IBM SP2 machines located at LaBRI (Bordeaux, France) and at CNUSC (Montpellier, France), for a large class of sparse matrices.

* This work is supported by the French *Commissariat à l'Énergie Atomique* CEA/CESTA under contract No. 7V1555AC, and by the GDR ARP (iHPerf group) of the CNRS.

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