Temporal Logics for Concurrent Recursive Programs: Satisfiability and Model Checking

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Motivation

- Verification of concurrent recursive systems against temporal logics.
- We make the verification generic wrt.
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  - the models: capture Words, Trees, Nested Words, Mazurkiewicz traces
  - the temporal logics: capture LTL, CTL, PDL (with $^{-1}$and $\cap$), XPath.
  - the decidabillity proofs: extend results to both concurrent and recursive
  - the complexity bounds for verifying such concurrent recursive systems.

Main Tools

1. Linearizations of concurrent recursive behaviors: multiply nested words.
2. Tree encoding of multiply nested words.
3. Adaptation of existing results for trees.

Recursive Systems

```
main():
    call a()
    call b()
    return
```

```
a():
    call b() or do c
    return
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b():
    call a() or do c
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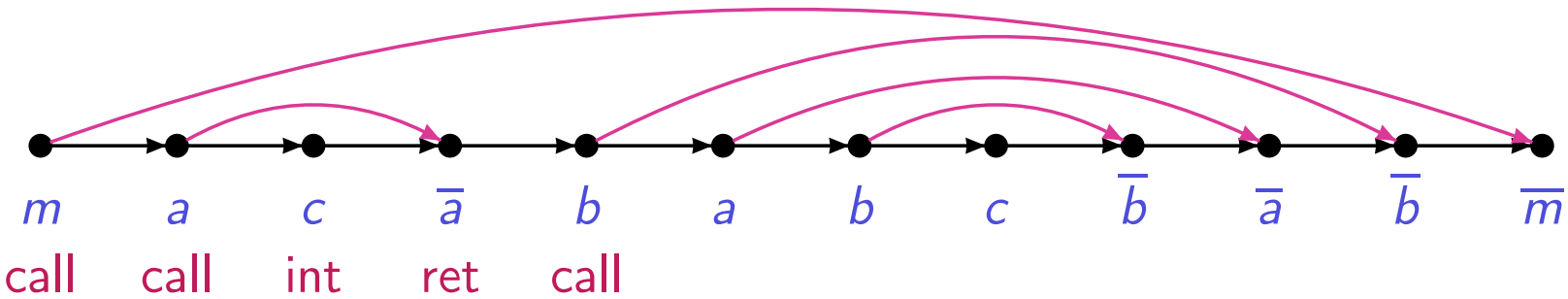

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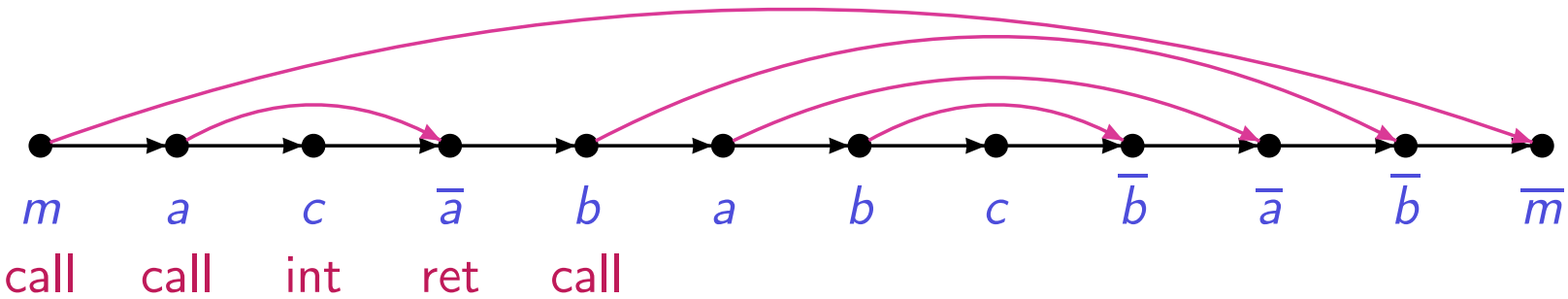


Nested Words [Alur, Madhusudan 04–] : $(V, \text{act}, \text{type}, \text{succ}, cr)$

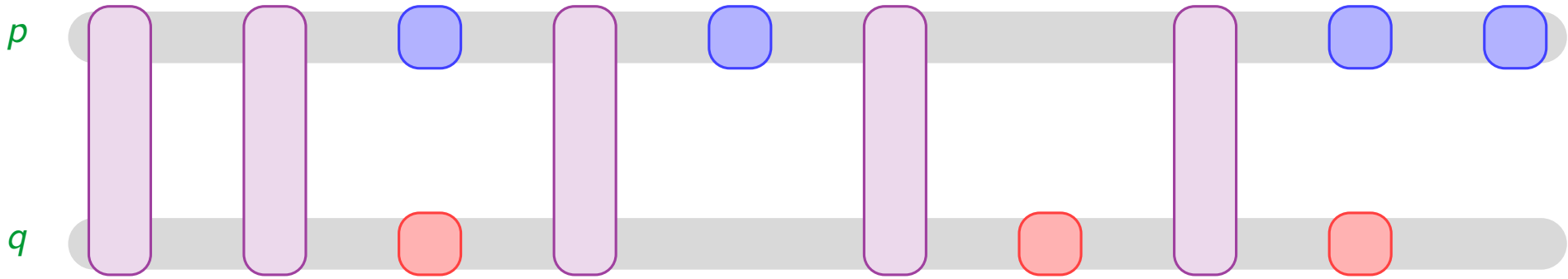
- V: set of nodes (events).
- $\text{act} : V \to \text{Labels}$
- $\text{type} : V \to \{\text{call}, \text{ret}, \text{int}\}$.
- $\text{succ} \subseteq V \times V$ is the linear successor.
- $cr \subseteq V \times V$ is the matching relation.

here, $\text{Labels} = \{a, b, c, m, r\}$.

calls/returns may be unmatched

Concurrent Systems

Concurrent Systems



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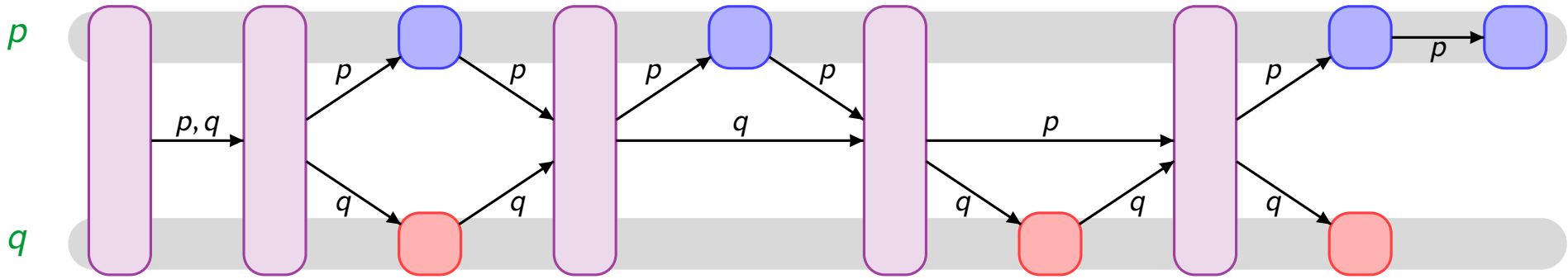

- Mazurkiewicz Trace: $(V, \text{act}, \text{proc}, <_p)$
- $proc : V \to \{p, q\}$, and $<_p$ order on process p. Yield partial ordering:
  - Two nodes sharing a process must be ordered.
  - Two consecutive nodes must share a process.

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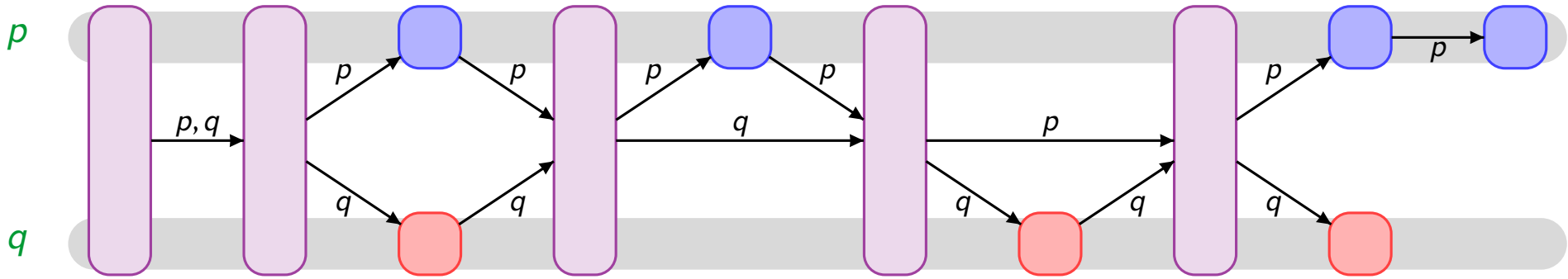

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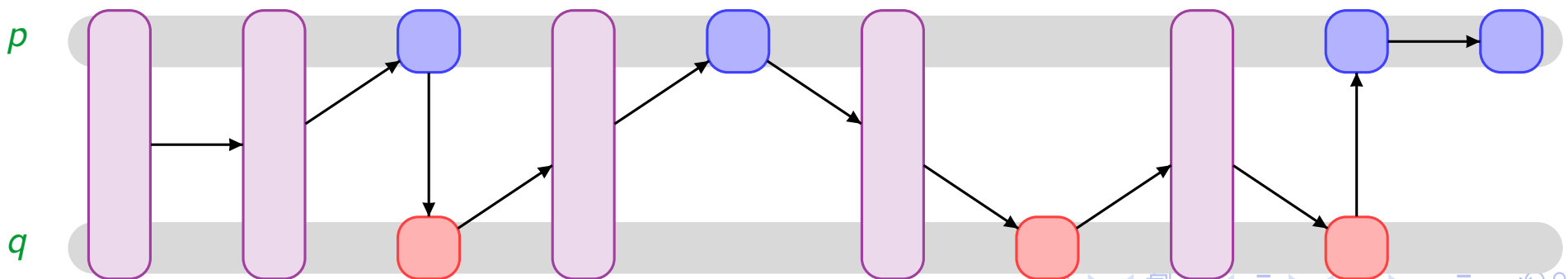
  - Two consecutive nodes must share a process.
- We use linearizations of such traces:



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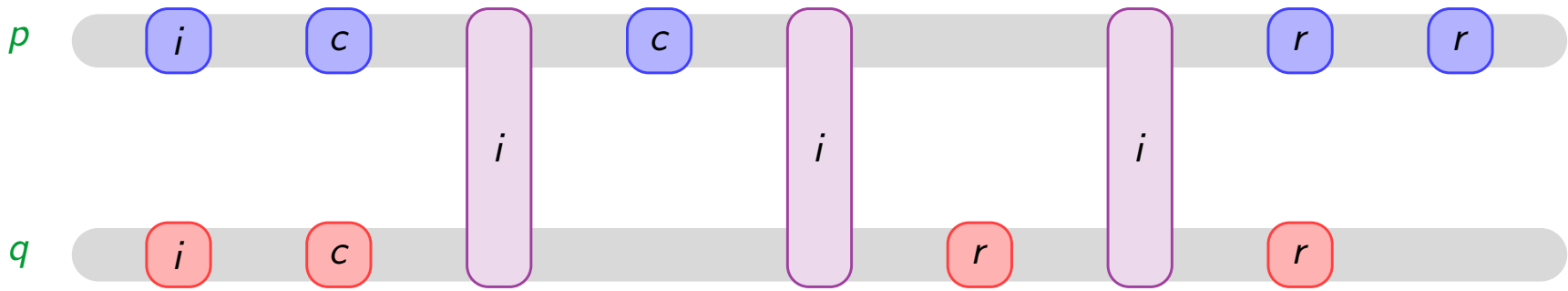

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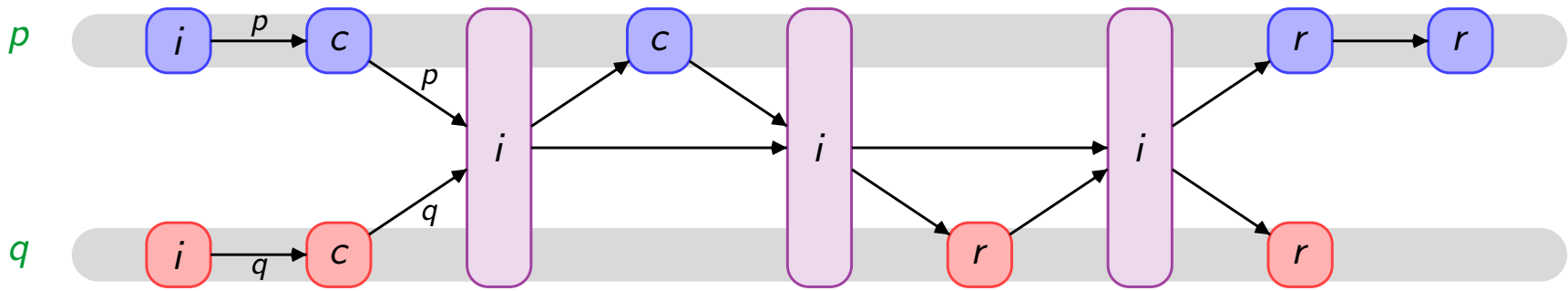

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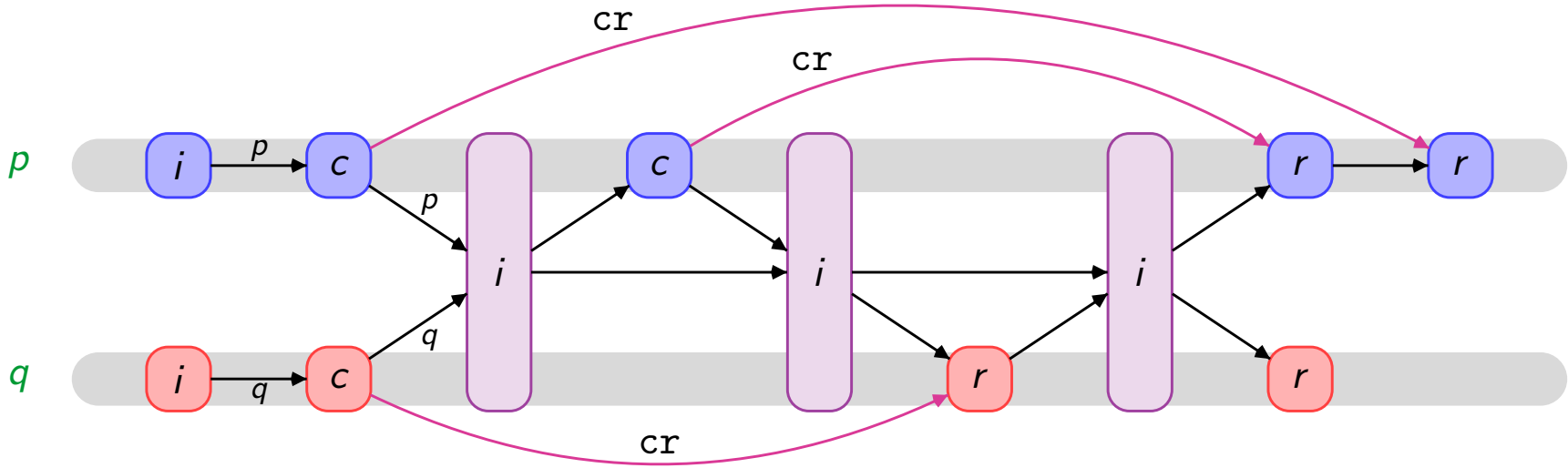

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- Calls and returns local to one process.

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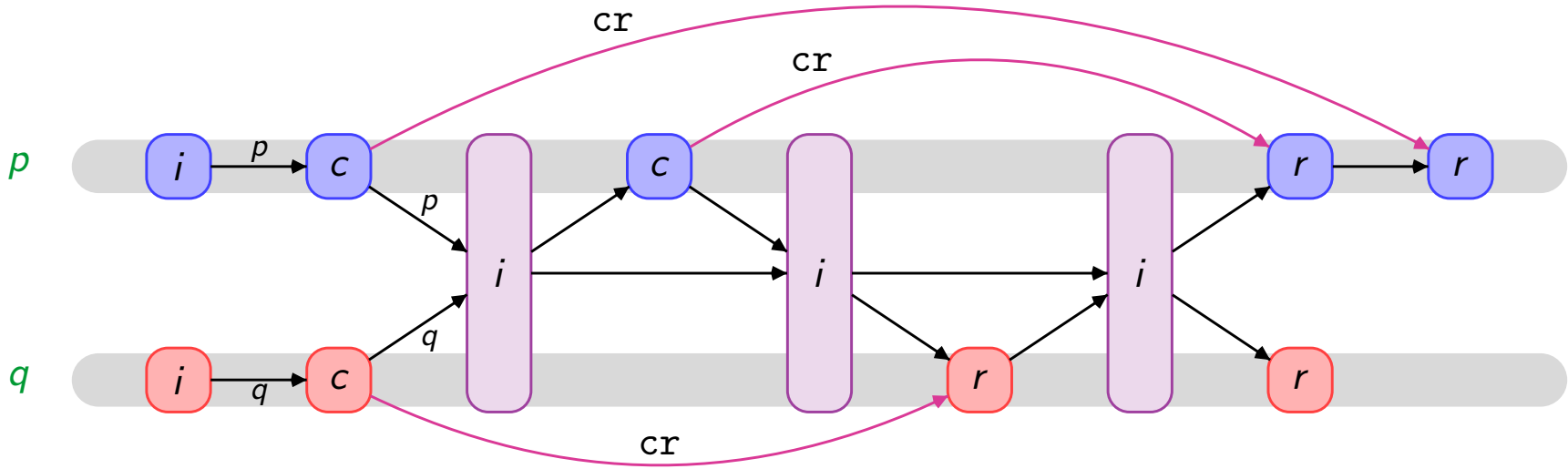
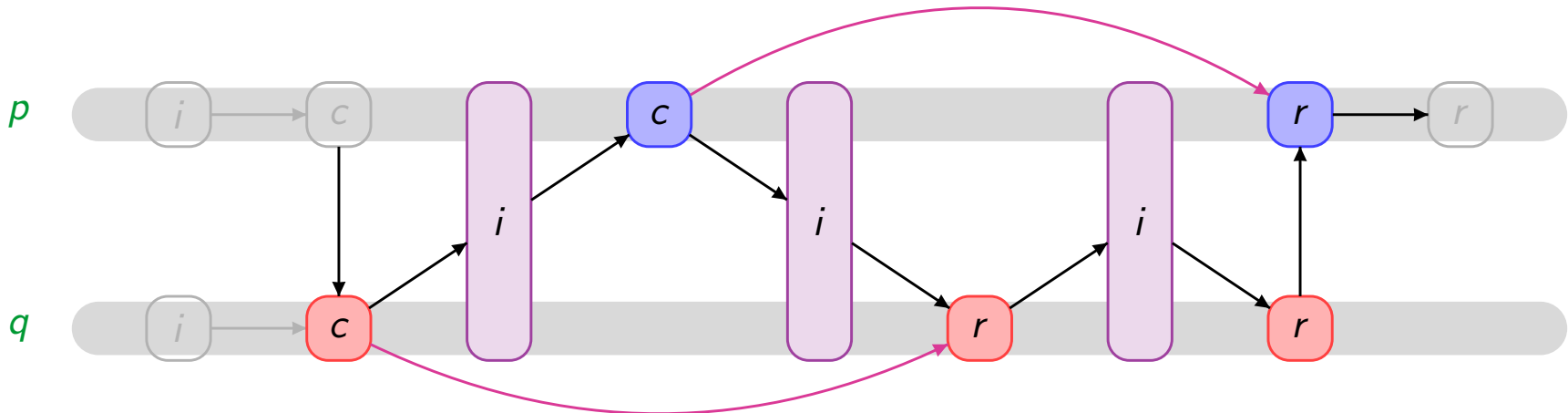
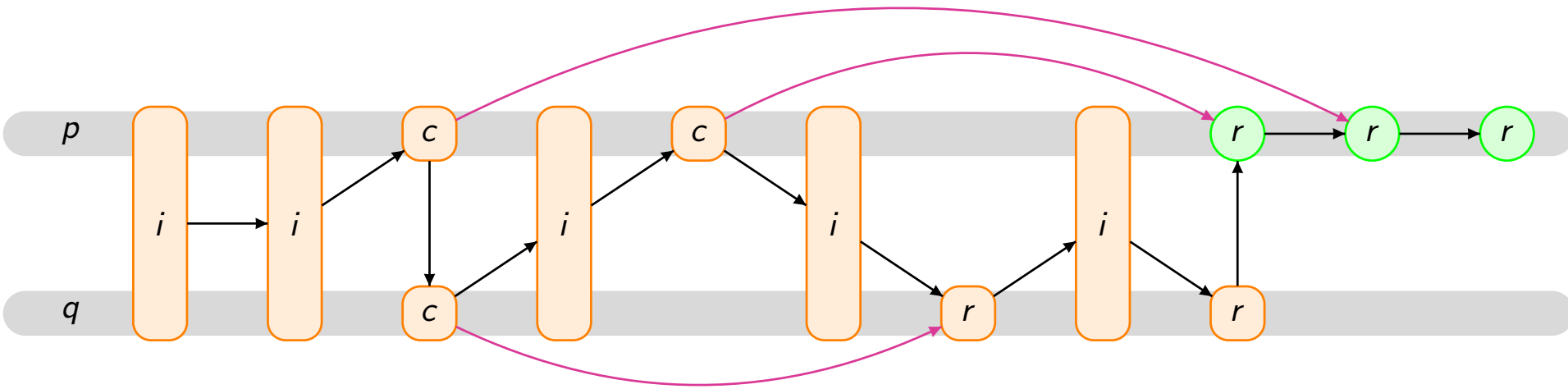


- Nested traces: $(V, \mathsf{act}, \mathsf{proc}, \mathsf{type}, <_p, <_q, \mathsf{cr})$.
- Calls and returns local to one process.
- ⚠ Linearizations need not be nested words!



Phases

- We limit ourselves to k-phase nested traces.
- A trace is k-phase if
  - it has a linearization that can be split into k factors.
  - in each factor, only one single process can return.



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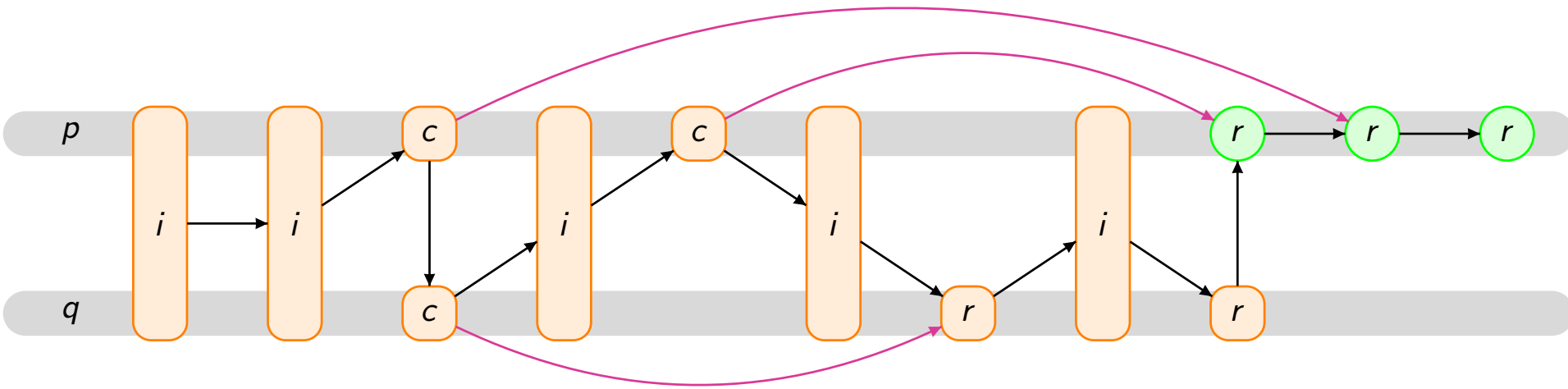

- For traces with unbounded phase, crossings of call-return edges of two stacks $\rightsquigarrow$ encoding of the grid.

A generic model: Labeled structures

- Examples models: Words, Trees, Mazurkiewicz traces, Nested traces.
- Models = labeled graphs.
- Signature $\mathcal{S} = (\Sigma, \Gamma)$. node labels Σ, edge labels Γ.
- $\mathcal{S}$-graph $G = (V, \lambda, \nu)$
- Node-labeling $\lambda : V \to 2^{\Sigma}$, and Edge-labeling $\nu : (V \times V) \to 2^{\Gamma}$.
  Graph induced $u \to v$ iff $\nu(u, v) \neq \emptyset$ well-founded and acyclic.

A generic temporal logic over $\mathcal{S}$-graphs

Node formulas $\varphi ::= \sigma \mid \neg\varphi \mid \varphi \vee \varphi \mid M(\underbrace{\varphi, \ldots, \varphi}_{arity(M)}) \mid \exists\pi$

Path expressions $\pi ::= ?\varphi \mid \gamma \mid \gamma^{-1} \mid \pi \cup \pi \mid \pi \cap \pi \mid \pi \circ \pi \mid \pi^*$

- M is an MSO($\mathcal{S}$) modality.
- Captures LTL, CTL, PDL (with $\cap$ and converse), regular XPath, NWTL

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$$M = \mathsf{X}(\alpha),\ \mathsf{U}(\alpha, \beta)$$
$$[\![\mathsf{X}]\!](x, Y) = \exists y\,(y \in Y \wedge \mathrm{succ}(x, y))$$
$$[\![\mathsf{U}]\!](x, Y, Z) = \exists z\,(x \leq z \wedge z \in Z \wedge \forall y(x \leq y < z) \Rightarrow y \in Y)$$

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  EGα = $\exists$ infinite path from the current node where α always holds.

$$[\![\mathrm{EG}]\!](x, X) = \exists Y\,(Y \subseteq X \wedge x \in Y \wedge \forall z\,(z \in Y \rightarrow \exists z'\,(z' \in Y \wedge z \text{ succ } z')))$$

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$$M = \mathsf{X}(\alpha), \mathsf{Y}(\alpha), \mathsf{X}^{\mathtt{cr}}(\alpha), \mathsf{Y}^{\mathtt{cr}}(\alpha), \mathsf{U}^{\mathtt{s}}(\alpha,\beta), \mathsf{S}^{\mathtt{s}}(\alpha,\beta)$$

Semantics

$$\varphi \quad ::= \quad \sigma \mid \neg\varphi \mid \varphi \vee \varphi \mid M(\underbrace{\varphi, \ldots, \varphi}_{\mathit{arity}(M)}) \mid \exists\pi$$

$$\pi \quad ::= \quad ?\varphi \mid \gamma \mid \gamma^{-1} \mid \pi \cup \pi \mid \pi \cap \pi \mid \pi \circ \pi \mid \pi^*$$

- A formula is evaluated in an $\mathcal{S}$-graph $G = (V, \lambda, \nu)$.
  Node formula φ: $[\![\varphi]\!]_G \subseteq V$. Path formula π: $[\![\pi]\!]_G \subseteq V \times V$.
- $?\varphi$ is the set of pairs (x, x) where φ holds at x.

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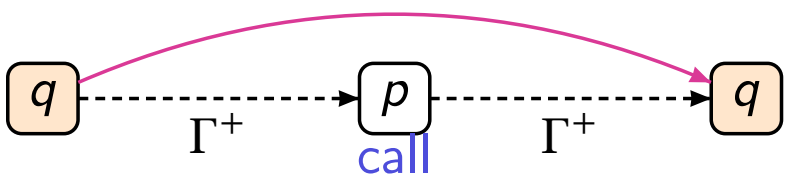
Example: Process p is not allowed to call a new procedure when it is in the scope of an active procedure call from q.



$$\varphi = \neg\exists(\mathtt{cr} \cap (?q \circ \Gamma^+ \circ ?(\mathsf{call} \wedge p) \circ \Gamma^+))$$

MSO modalities over nested traces: NTrLTL

- Logic generalizing both NTrL on traces and NWTL on nested words.

Ex. In the scope of the current procedure call p and with the same stack contents, there is a synchronizing action with process q.

- X^{cr}: we are at a call position and φ holds at the corresponding return.

$$[\![X^{cr}]\!](x, Y) = \exists y\,(\mathtt{cr}(x, y) \wedge y \in Y)$$

- $X_p\,\varphi$: φ holds at the next p-position,

$$[\![X_p]\!](x, Y) = \exists y\,(p(y) \wedge x \prec y \wedge y \in Y \wedge \forall z\,(p(z) \wedge x \prec z \rightarrow y \preceq z))$$

- Binary future modalities $\{EU, AU, EU^a, EU^s\}$ and $\{U_p, U_p^a, U_p^s \mid p \in Proc\}$.
- $\varphi\,AU\,\psi$: $\exists$ a future node satisfying ψ, and φ holds on nodes in between.

$$[\![AU]\!](x, X_1, X_2) = \exists z\,(x \preceq z \wedge z \in X_2 \wedge \forall y\,(x \preceq y \prec z \rightarrow z \in X_1))$$

The satisfiability problem

- Problem Nested-Trace-SAT parametrized by Proc, act, k, and a temporal logic $\mathscr{L}$ definable in MSO over nested traces.

Nested-Trace-Sat $(\mathscr{L}, k)$

Instance: $\varphi \in \mathrm{Form}(\mathscr{L})$

Question: Is there $G \in \mathit{Traces}_k(\mathit{Proc}, \mathit{Act})$ and node u such that $G, u \models \varphi$?

Theorem: Nested-Trace-Sat decidability [Bollig, Cyriac, Gastin, Z.]

The problem Nested-Trace-Sat$(\mathscr{L}, k)$ is in 2EXPTIME.
Intersection free fragment: Nested-Trace-Sat$(\mathscr{L}^-, k)$ is EXPTIME-complete.

- Same holds for ordered ranked or unranked trees.
- Proof for nested traces: reduction to the case of trees.

Proof for trees

1. Precompile [Gastin, Kuske 05] MSO modalities into tree NFA [Rabin 69].
2. Inductively build an alternating 2-way tree automaton (A2A) equivalent to the input formula [Göller, Lohrey, Lutz 07].
3. Reduce SAT to emptiness of A2A, eg [Vardi 98].

Outline of the proof for Nested Traces

1. Encode k-phase linearizations by binary trees.
2. Translate the formula to be checked on k-phase linearizations into an MSO formula on binary trees.
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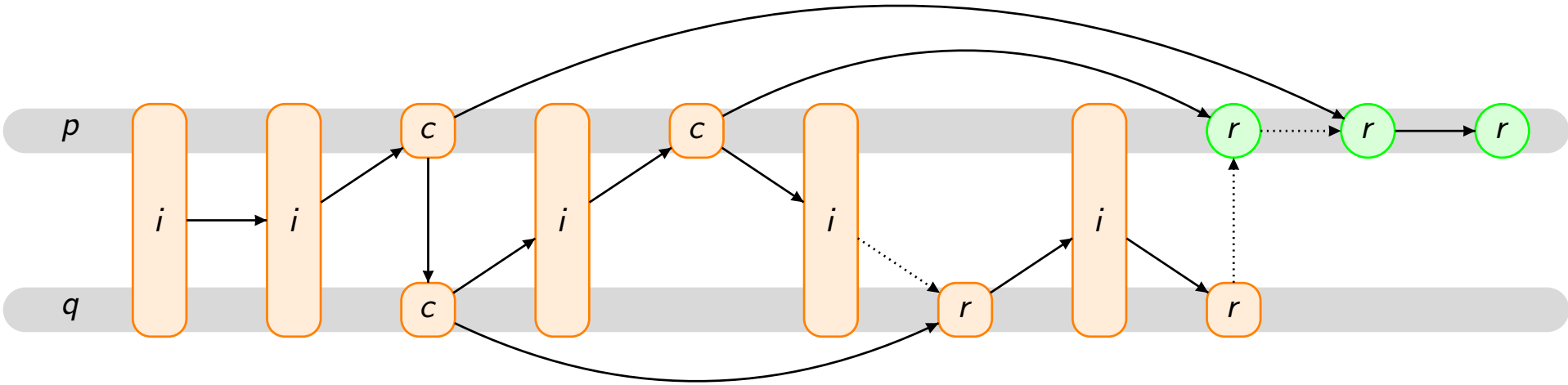
In the following, we fix the max phase number to k.

Encoding nested traces by trees



1. Remove edges leading to matched returns.

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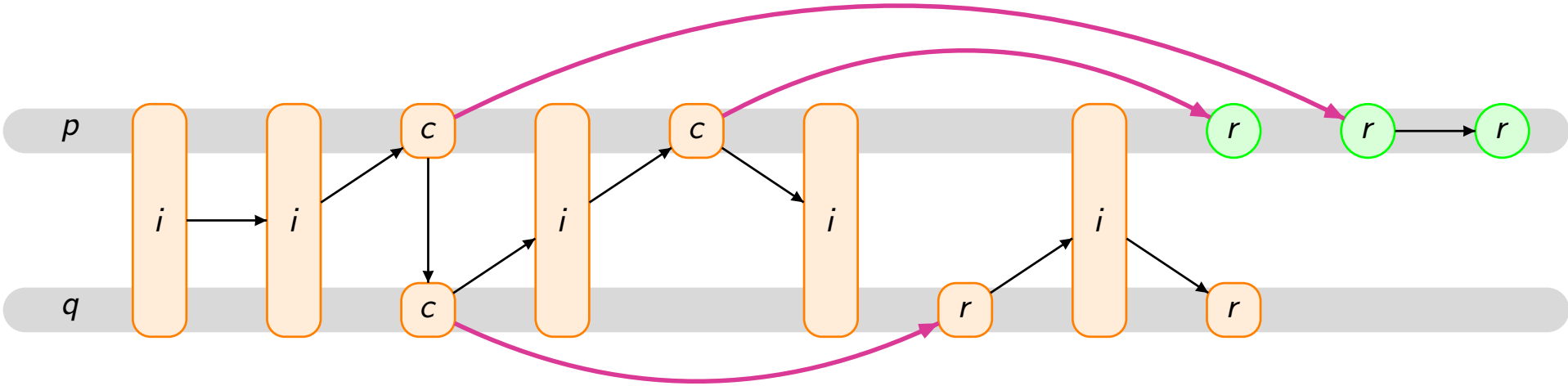

1. Remove edges leading to matched returns.
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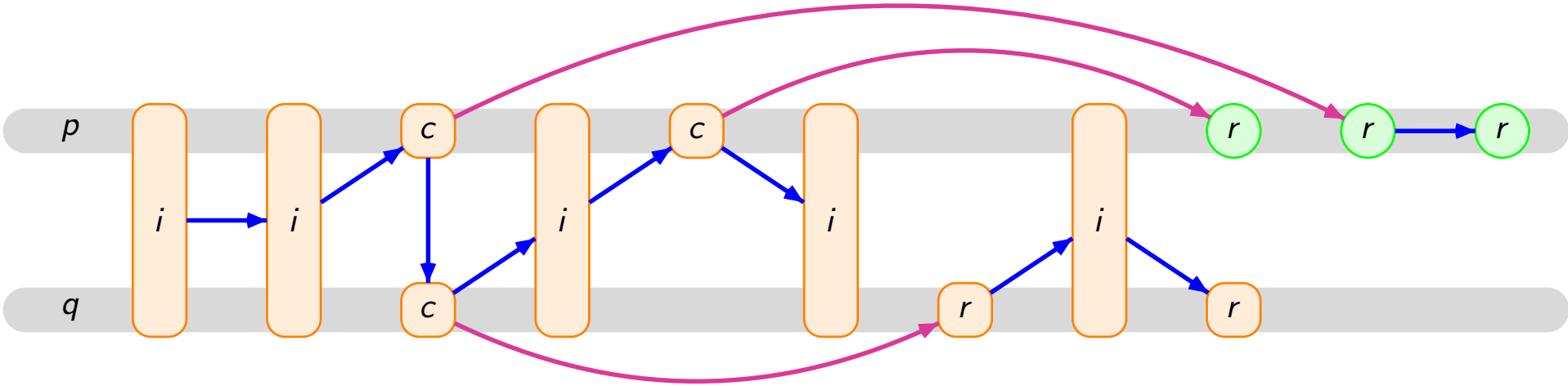

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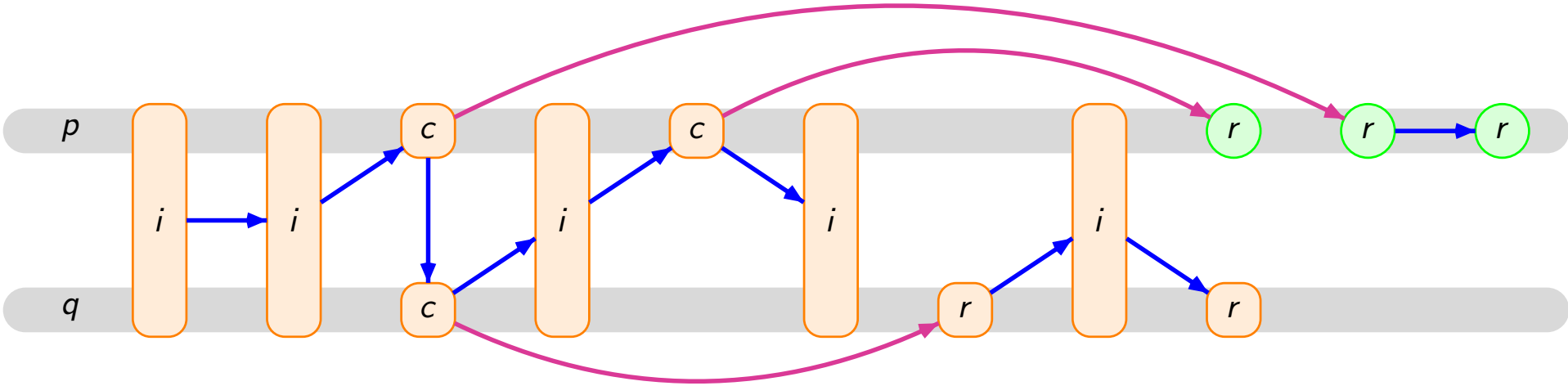

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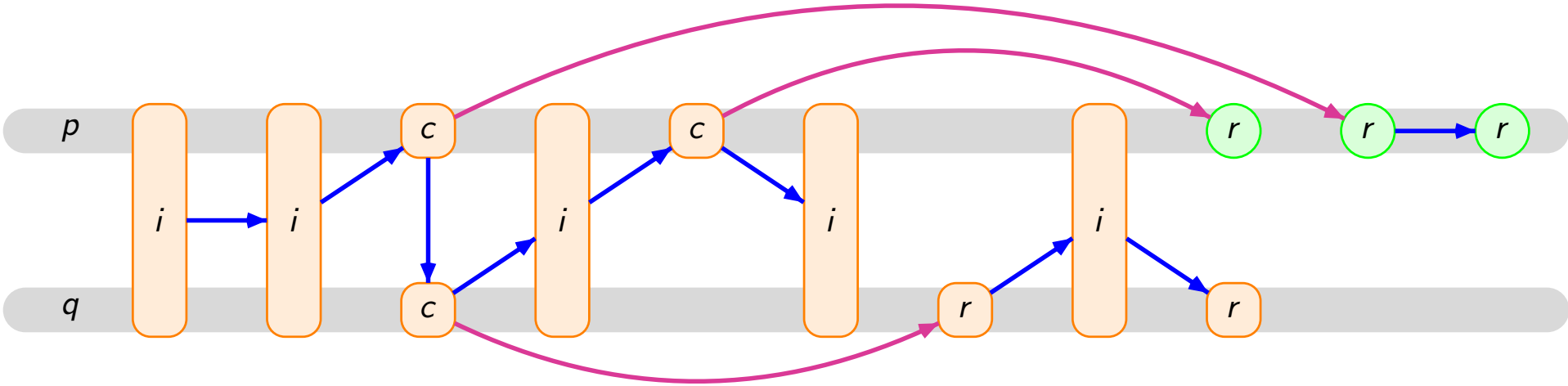

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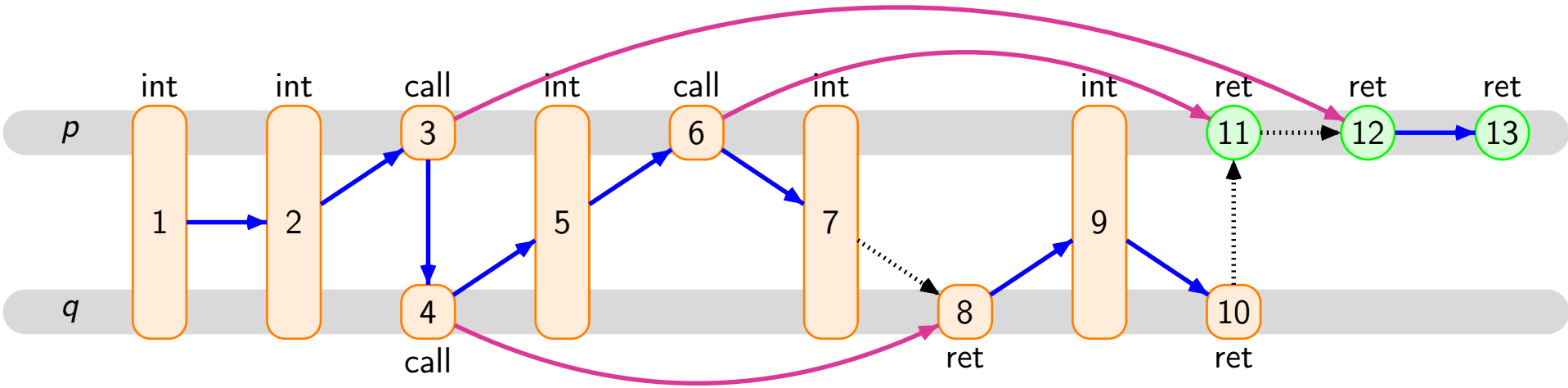
We get trees over an enlarged signature $\mathcal{S}'$.

Encoding nested traces by trees



- From linearization w, tree t_k^w over extended signature $\mathcal{S}'$.

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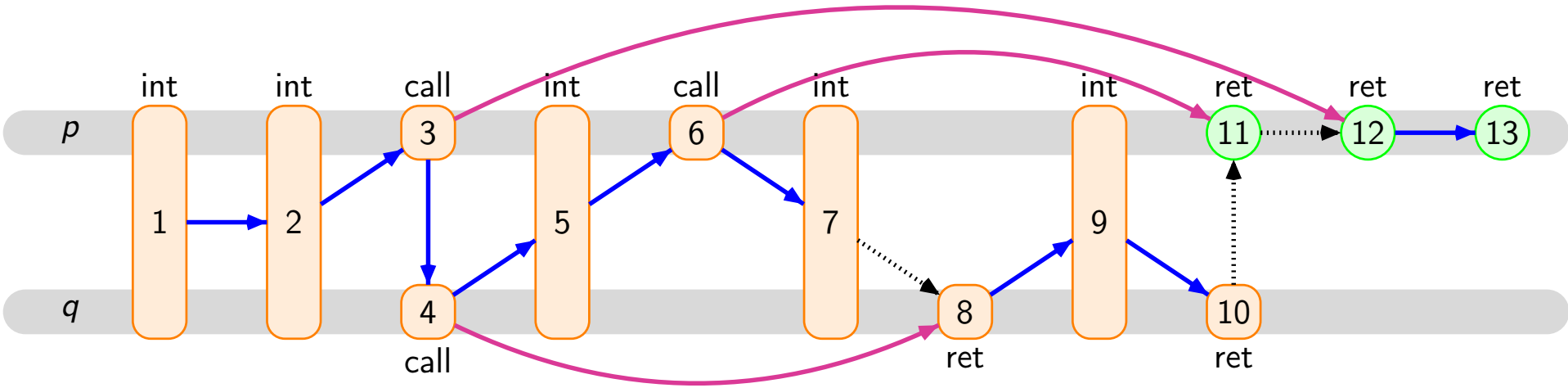

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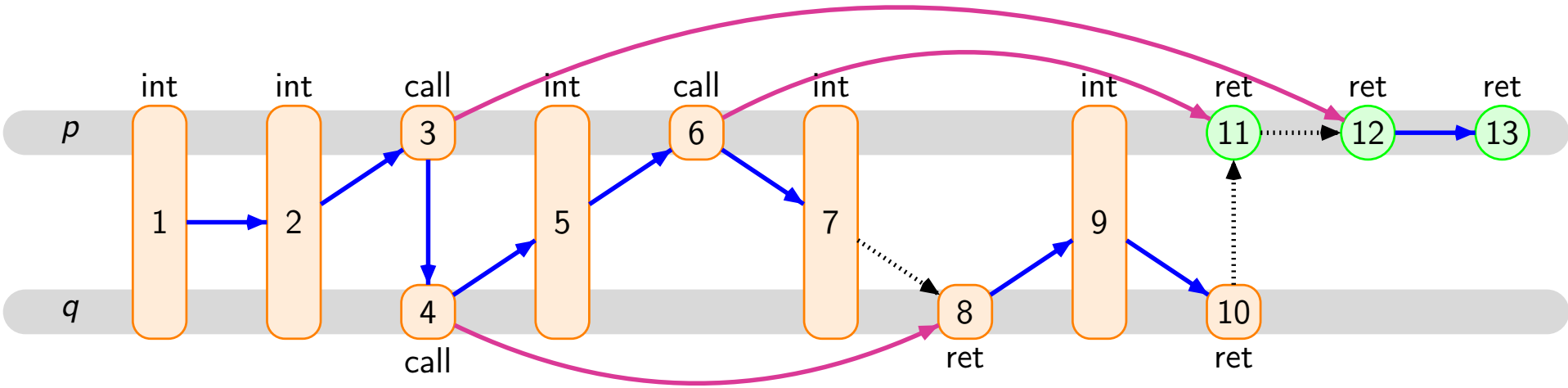

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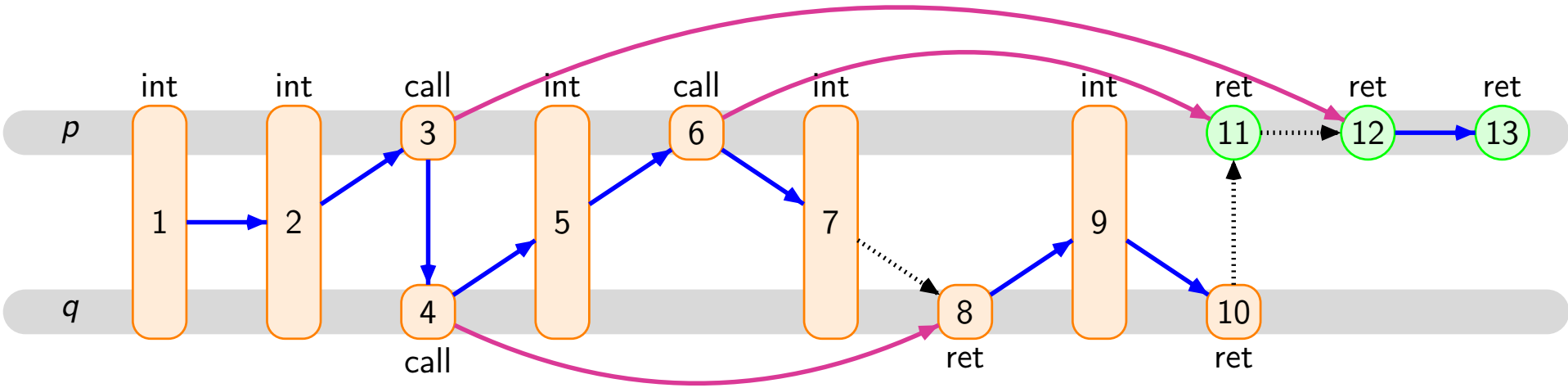
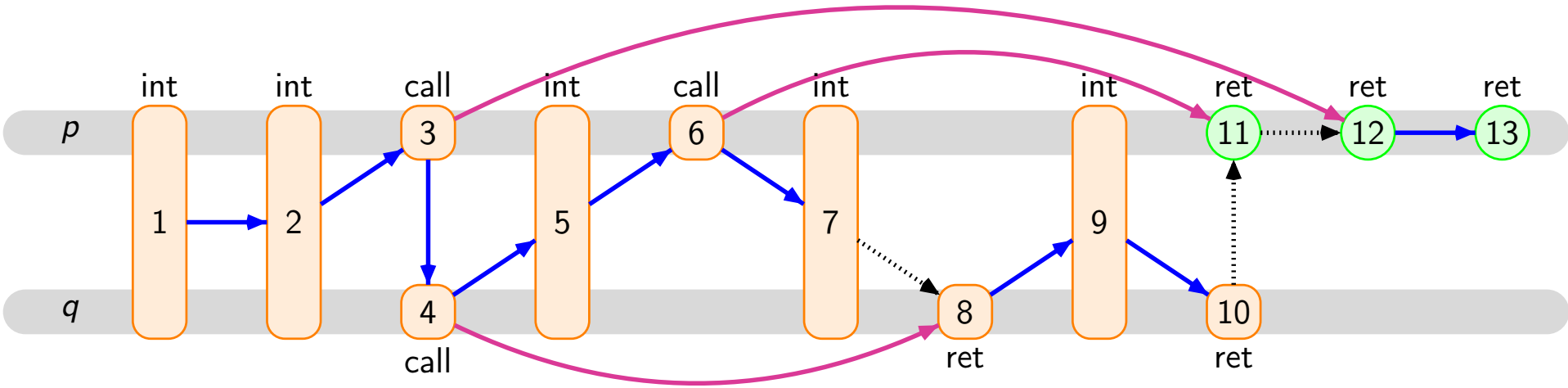


- From linearization w, tree t_k^w over extended signature $\mathcal{S}'$.
- Branching nodes are calls.
- Erased edges may be recovered.
- Phase numbers increase along branches.

Lemma: k-phase trees are MSO-definable [La Torre, Madhusudan, Parlato]

The set of tree encoding of k-phase nested traces is MSO($\mathcal{S}'$)-definable.

Recovering the linear order



- Goal: reduce SAT problem for nested traces to SAT problem for trees.

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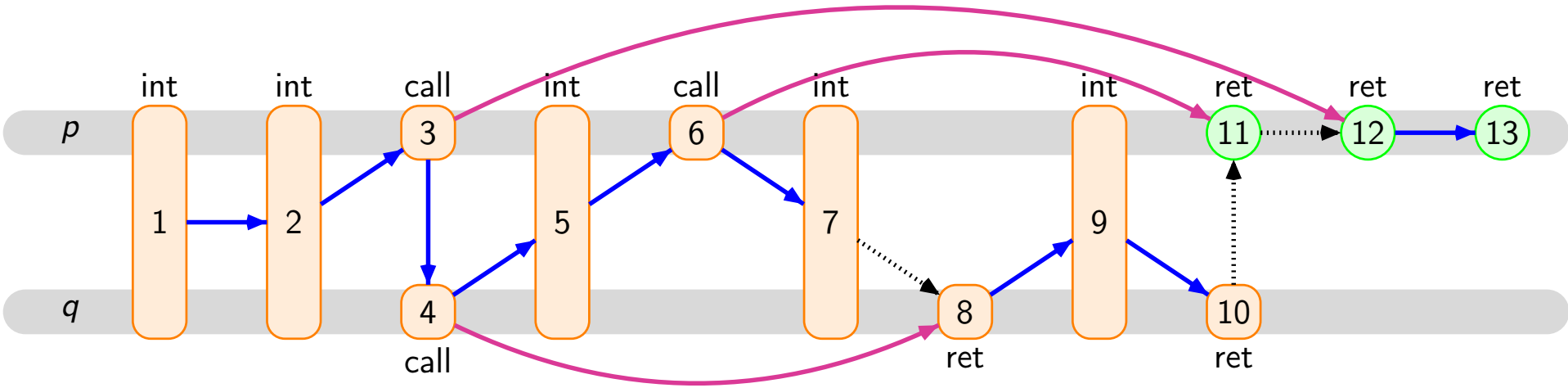

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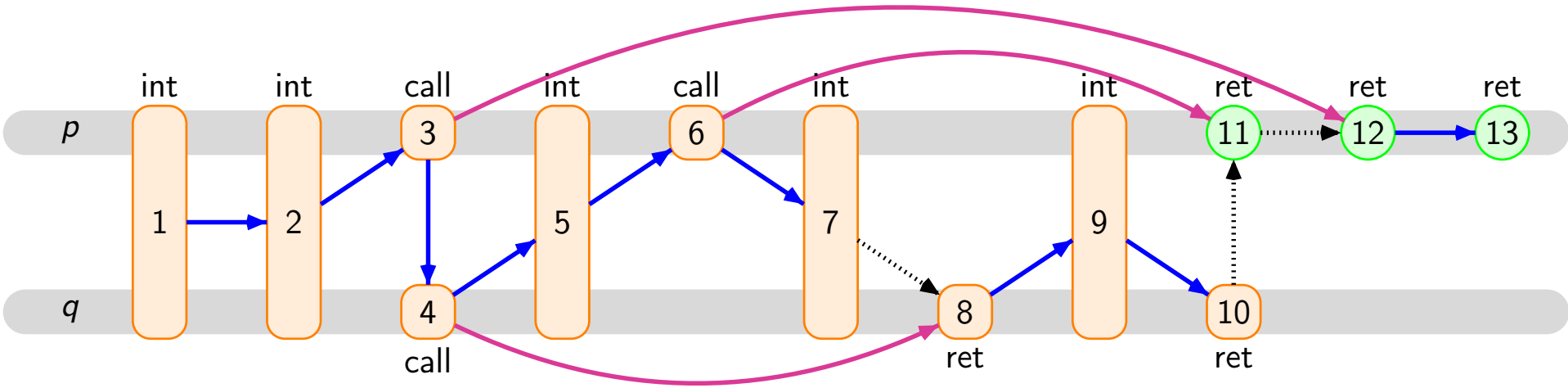

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- Need: express in MSO($\mathcal{S}'$) the successor relation $\lessdot$ of the linearization.
- ie, recover in MSO all dotted edges.

Lemma: Successor in linearization is expressible in MSO($\mathcal{S}'$)

For all k-phase linearizations $w = (V, \lambda, \leq)$: $[\![\mathrm{succ}_{\leq k}]\!]_{t_k^w} = \{(u, v) \in V^2 \mid u \lessdot v\}$.
The length of $\mathrm{succ}_{\leq k}$ is exponential in k.

Recovering the linear order

- Proof: Induction on the "phase prefix m".

$$[\![\mathtt{succ}_{\leq m}]\!]^w_{t_k} = \left\{(u,v) \in V^2 \mid u \lessdot v,\ \mathit{phase}_w(u) \leq m,\ \mathit{phase}_w(v) \leq m\right\}$$

- Distinguish cases:
  1. the successor remains in the current phase.
  2. the successor belongs to the next phase

$$\mathtt{succ}_{\leq m} \quad = \quad \mathtt{succ}_{\leq m-1} \cup \underbrace{\mathtt{succ}_{m,m}}_{(1)} \cup \underbrace{\mathtt{succ}_{m-1,m}}_{(2)}$$

Recovering the linear order

- Proof: Induction on the "phase prefix m".

$$[\![\mathtt{succ}_{\leq m}]\!]^w_{t_k} = \left\{(u,v) \in V^2 \mid u \lessdot v,\ \mathit{phase}_w(u) \leq m,\ \mathit{phase}_w(v) \leq m\right\}$$

- Distinguish cases:
  1. the successor remains in the current phase.
  2. the successor belongs to the next phase

$$\mathtt{succ}_{\leq m} \quad = \quad \mathtt{succ}_{\leq m-1} \cup \underbrace{\mathtt{succ}_{m,m}}_{(1)} \cup \underbrace{\mathtt{succ}_{m-1,m}}_{(2)}$$

- For (2), the successor is the first node of next phase m, hence a return.
- If it is unmatched: $?(m-1) \circ \mathsf{left} \circ ?m$.
- If it is matched, find the most recent call whose return is in phase m

$$\mathtt{prev\text{-}call\text{-}ret}_m \quad = \quad (?\neg\exists(\mathsf{right} \circ ?m) \circ \mathtt{succ}_{\leq m-1}{}^{-1})^* \circ \mathsf{right} \circ ?m$$

Model Checking

- The result goes through for model checking as well.
- The notion of concurrent recursive Kripke Systems.
- A CRK can be described by a path expression.

Further work

- Expressive power, FO completeness?
- Handle dynamic creation of threads (eg, dynamic MSCs).

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Thank you!