Some aspects of symbolic dynamics of Feigenbaum limit sets

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Abstract

We have applied the technique of symbolic dynamics to the study of one-dimensional maps in discrete time. Symbolic dynamics presents the great merit that it can also be applied successfully to the cases of period doubling and intermittency, where other techniques known in literature (as the zeta functions and the thermodynamic formalism in general) prove to be inadequate. We are thus able to study the symbolic dynamics of the logistic map at the accumulation point of the 2^k cycles (Feigenbaum point), with a new systematic method that we developed, thereby generalizing previous work by the Berlin and Brussels groups. Moreover, we proceeded to the same numerical investigations for the accumulation point of the $3 \cdot 2^k$ cycles of the logistic map. Our effort for a theoretical explanation of our numerical findings led us to introduce a new, original scheme for the "generation" and for the "reading" of symbolic sequences for every $m \cdot 2^\infty$ cycle.

The new method reveals the deep, self-similar structure of the attractor and allows for a calculation of all coarse-grained properties of the system from first principles.

With the aid of these considerations we can retrieve in a much more compact manner some well-known results of the literature, but also to realize original and accurate predictions for each of the $m \cdot 2^\infty$ cycles (that is for an infinite countable new category of cases that was not covered by the existing literature). We also realized the quantitative connection between symbolic dynamics and recurrence times.

We have next explored the connection between one dimensional Chaotic Dynamics and Number Theory. In particular we have shown that if in the place of the usual symbolic dynamics one uses instead of abstract symbols digits (for instance 0 and 1 instead of R and L), thus introducing a binary *digital approach*, then the numbers that are

generated at the Feigenbaum points (the accumulation points of the $m \cdot 2^k$ cycles) are necessarily *transcendental*: they cannot result as roots of algebraic equations with rational coefficients.

Furthermore, we show that this last conclusion holds for the same reasons for the accumulation points of the m^k cycles. For different reasons related to the theory of *normal* numbers, the same conclusion seems also to hold for chaotic systems with non-zero memory, for intermittent chaotic systems and for the case of developed chaos with zero memory. In the case of fully developed chaos, any outcome compatible with dynamics is possible: we can have any irrational number, either transcendental or algebraic irrational. In the case of periodic orbits finally, the outcome of symbolic dynamics are rational numbers.

Finally, the repercussions of the above results in the theory of automatic sequences have been assessed.

References

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