On-line Multiplication

Athasit SURARERKS
LIAFA, Universite Pierre et Marie Curie
athasit@liafa.jussieu.fr

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1 Abstract

On-line multiplication, based on the incremental multiplication technic, was first introduced in 1977 by Ercegovac and Trivedi. During on-line computations, the operands as well as the results flow through arithmetic units serially, digit by digit, starting from the most significant digit. On-line systems are characterized by their *on-line delay*(i.e., the number δ such that n digits of the result are deduced from $n+\delta$ digits of the input values). The SDN (Signed-Digit Number) system representation of Avizienis was used to reduce the complexity.

First, we consider the on-line multiplication of two numbers in the classical radix-β numeration system, where β is an integer ≥ 2, into the SDN system(i.e., the digit set of input operands are the carnonical set $\{0, 1, ..., \beta - 1\}$, but the digits of output are in the balanced set $\{-a, ..., a\}$, a being an integer such that $\beta/2 < a \leq \beta - 1$. An algorithm can run with on-line delay 2 in binary system and on-line delay 1 if radix $\beta \geq 3$. We are interested in finding an algorithme that can be realized with on-line delay 1 for $\beta = 2$. We now propose the *negative error* on-line multiplication for the binary system with on-line delay 1.

The product of $X = 0.x_1x_2x_3...$ and $Y = 0.y_1y_2y_3...$ such that x_n, y_n are 0 or 1. We will denote $X_n = 0.x_1x_2x_3...x_n$ and $Y_n = 0.y_1y_2y_3...y_n$, means the first n digits of X and Y. The problem becomes to manipulate a sequence of W_n defined by :

$$W_{n+1} = \beta(W_{n-1} - z_{n-1}) + X_{n+1}y_{n+1} + Y_nx_{n+1}$$

Instead of choosing the output digit(z_n) that is close to the exact product(W_n), we take the z_n which is always greater than or equal to W_n. By this method, the output value is always greater than or equal to the exact product, in the case that we take only first n digits of the output value (the error is always negative value). But the exact result can be obtained with a double length of inputs.

Second, we focus on the on-line multiplication in the Signed-Digit Binary Number System(SDBN). An algorithm has been proposed with on-line delay 2. We find that the notion of Window is one solution to save time. In this case, we use a window of length 2. Only during the computation, we replace the digit set $\{-1, 0, 1\}$ by the rational digit set $\{0, 1/2, 1\}$ (The input and output always have digits in $\{-1, 0, 1\}$). This algorithm, applying negative error technic, can run with on-line delay 1. One condition is supposed, two input values should have the same sign. But it is easy to change a sign in the SDBN system.

Last, we consider the complex number system where the radix is a complex number which is a squar root of a negative integer ($\beta = \sqrt{-b}$). On-line multiplication can be taked place with delay 7 for minimumly redundant and 5 otherwise. We also consider the problem of on-line muliplication by digit that can be done by on-line finite automaton.