Minimisation of Automata

State of the art minimisation of co-Büchi automata:

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<th>State-based</th>
<th>Transition-based</th>
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<td>Deterministic</td>
<td>NP-complete (Schewe 2010)</td>
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<td>Good-For-Games</td>
<td>NP-complete (Schewe 2020)</td>
<td>Polynomial (Abu Radi-Kupferman 2019)</td>
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Our contributions:

**Theorem**
Minimisation of transition-based Rabin automata is NP-complete.

**Remark 1.** For the reduction we use a language that is a Muller condition: whether a word $w \in \Sigma^\omega$ belongs to the language only depends on $\text{Inf}(w)$.

**Proposition**
Minimisation of parity automata recognising Muller conditions can be done in polynomial time.

Muller games

Players move a pebble in turns producing an infinite word $w \in \{a, b, c\}^\omega$.

**Muller condition:** Family of subsets of colours $F = \{\{a, b, c\}, \{a\}, \{b\}\}$. Player 1 wins if the set of colours produced infinitely often is in $F$.

Memory structures

3 kinds of memory structures:

- **General memory:** Update transitions can depend on the specific edges of the game.
- **Chromatic memory:** Update transitions only depend on the colours of the condition (for example, the memory structure above).
- **Arena-independent memory:** Memory structure that can be used in any game where the winning condition is $F$.

**Question (Kopczyński 2006)**
Is it true that, if Player 1 wins a game using a given winning condition, she requires the same amount of “general memory” than “chromatic memory” to win it?

**Answer**
No. There are Muller conditions for which Player 1 requires strictly more “chromatic memory” than “general memory” to win.

Memory and Rabin automata

In the following, “automaton” stands for “deterministic transition-based automaton”.

**Theorem**
Let $F$ be a Muller condition. Any Rabin automaton recognising $F$ can be used as an arena-independent memory for $F$-games. Conversely, any arena-independent memory for $F$-games can be interpreted as a Rabin automaton recognising $F$. In particular, the following quantities coincide:
- The size of a minimal Rabin automaton recognising $F$.
- The smallest amount of chromatic memory required to win $F$-games.
- The size of a minimal arena-independent memory for $F$-games.

**Corollary**
Determining the amount of chromatic memory needed in games using a Muller condition $F$ is NP-complete, even if the condition is represented by its Zielonka tree.

References