On the size of good-for-games Rabin automata and its link with the memory in Muller games
Antonio Casares, Thomas Colcombet, Karoliina Lehtinen

Muller games

Players move a pebble producing an infinite word \( w \in C^\omega \).

Muller languages:
For \( \mathcal{F} \) a family of subsets of colours (for ex. \( \mathcal{F} = \{ \{a, b, c\}, \{a\}, \{b\} \} \)):
\[
\mathcal{L}_F = \{ w \in C^\omega \mid \text{Colours appearing infinitely often in } w \text{ form a set in } \mathcal{F} \}.
\]
Player 1 wins the Muller game if the word \( w \in C^\omega \) produced is in \( \mathcal{L}_F \).

Memory structures

Structure that tells Player 1 how to play. It is updated after each move in the game.

Two kinds of memory structures:
- General memory: Update transitions can depend on the specific edges of the game.
- Chromatic memory: Update transitions only depend on the colours of the condition (for example, the memory structure above).

Main results: Correspondance Memory ↔ Automata

Let \( \mathcal{L}_F \) be a Muller language.

Rabin automata

Autornaton recognising \( \mathcal{L}_F \) for \( \mathcal{F} = \{ \{a, b\}, \{a, c\}, \{b\} \} \).

Rabin condition:
Output alphabet: \( \Gamma = \{*, \bullet, \oplus\}^k \) (\( k = 2 \) in the example). A run in \( \mathcal{A} \) produces an infinite sequence of arrays \( v_1, v_2, \ldots \in \Gamma^\omega \).

A run is accepting if for some component \( x \in \{1, \ldots, k\} \), \( v_i[x] = \bullet \) infinitely often and \( v_i[x] \neq \bullet \) from some point onwards.

Good-for-Gameness (GFG)
A non-deterministic automaton \( \mathcal{A} \) is good-for-games (GFG) if there exists a strategy resolving its non-determinism 
\[
\sigma : \Sigma^* \rightarrow \Delta,
\]
such that:
\[
w \in \Sigma^* \text{ is accepted by the automaton} \iff \text{The run over } w \text{ obtained following } \sigma \text{ is accepting}.
\]
\( \Sigma = \) Input alphabet. \( \Delta = \) Transitions of the automaton.

For example, the Rabin automaton on the left is GFG (but it is not deterministic).