

Forbidden Patterns in Temporal Graphs resulting from Encounters in a Corridor

ANR TEMPORAL meeting

Minh Hang Nguyen

joint work with Michel Habib, Mikaël Rabie, Laurent Viennot

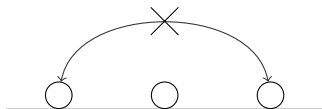
July 2023

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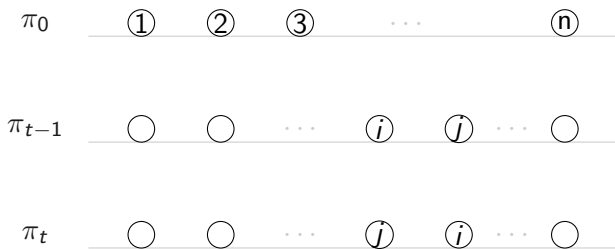
- 1 Mobility Graphs
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- 3 Counting and Spanner
- 4 Multi-Crossing



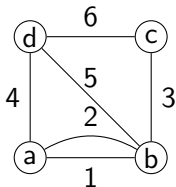
Model



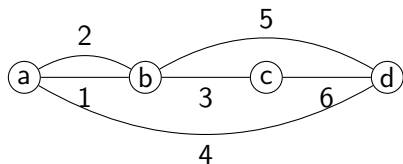
Model



Ordered Temporal Graph

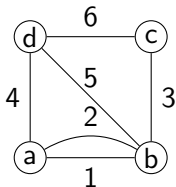


- $\mathcal{G} = (V, E, \lambda)$
 $\lambda : E \rightarrow 2^{\mathbb{N}}$



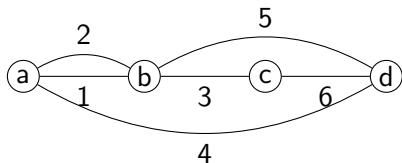
- (\mathcal{G}, π)
 π : vertex ordering

Ordered Temporal Graph



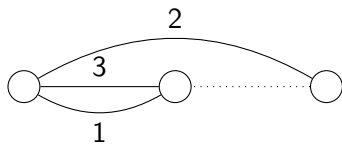
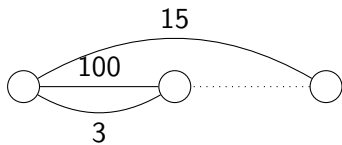
- $\mathcal{G} = (V, E, \lambda)$
 $\lambda : E \rightarrow 2^{\mathbb{N}}$

Incremental $\lambda(e) \cap \lambda(f) = \emptyset$

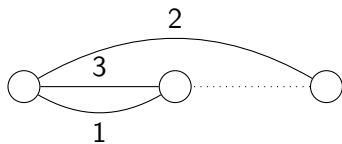
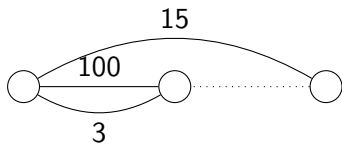


- (\mathcal{G}, π)
 π : vertex ordering

$$(\mathcal{H}, \pi) \cong (\mathcal{H}', \pi')$$



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$$\Phi: \quad V(\mathcal{H}) \quad \rightarrow \quad V(\mathcal{H}')$$

$$\text{Ordering } \pi(u) < \pi(v)$$

$$\pi'(\Phi(u)) < \pi'(\Phi(v))$$

$$\mathcal{R} \quad \dots, (u, v, t_i), \dots$$

$$\dots, (\Phi(u), \Phi(v), t'_i), \dots$$

n agents move on a line.

- $\pi_0 = 1, \dots, n$
- time t : i cross j
 $\rightarrow x_t = ij$

Schedule x

- from π_0
- $x = x_1, \dots, x_T$

Model

n agents move on a line.

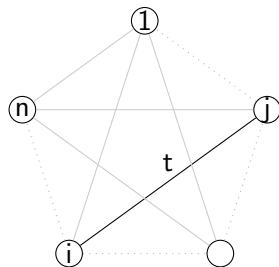
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Graph $\mathcal{G}_{\pi_0, x}$:

- $V = \{1, \dots, n\}$,
- $E = \{uv : \exists t, x_t = uv\}$,
- $\lambda(uv) = \{t : x_t = uv\}$.

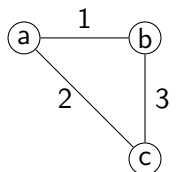


Mobility Graph

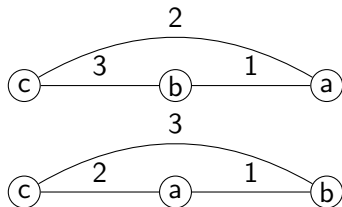
\mathcal{G} is a mobility graph if there are π and x s.t. $\mathcal{G} \equiv \mathcal{G}_{\pi,x}$.

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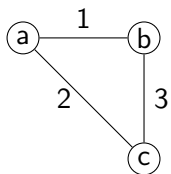


$x = ab, ac, bc$

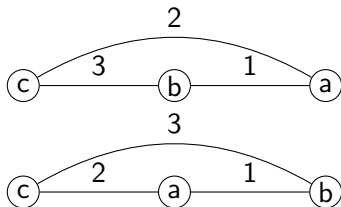
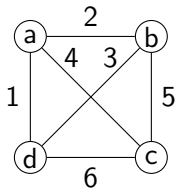


Mobility Graph

\mathcal{G} is a mobility graph if there are π and x s.t. $\mathcal{G} \equiv \mathcal{G}_{\pi,x}$.



$x = ab, ac, bc$



$\nexists \pi$

Additional assumption: Each pair of agents cross exactly once.

→ $\mathcal{G}_{\pi, X}$: mobility clique.

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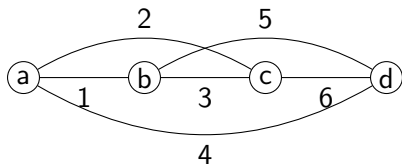
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Forbidden Patterns

(\mathcal{G}, π') excludes (\mathcal{H}, π) if $\mathcal{G} \not\subseteq \mathcal{H}'$ s.t. $(\mathcal{H}', \pi'|_{\mathcal{H}'}) \cong (\mathcal{H}, \pi)$.

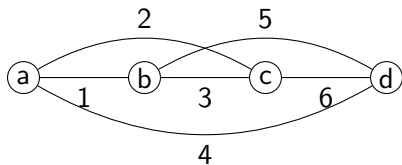
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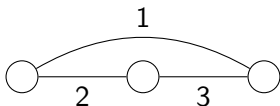


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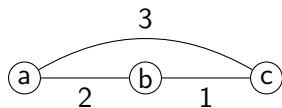
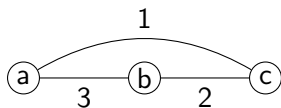
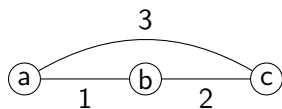
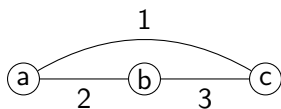
excludes



Characterization

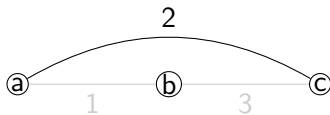
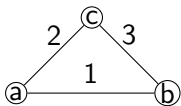
Theorem

A temporal clique is a mobility clique iff there exists an ordering of its nodes that excludes the patterns:

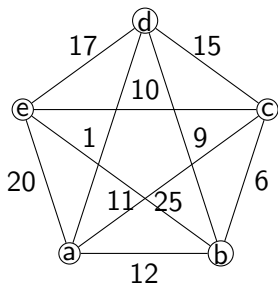


Recognize Algorithm

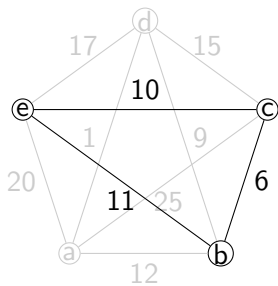
Given a temporal clique \mathcal{G} , potential vertex ordering π ?



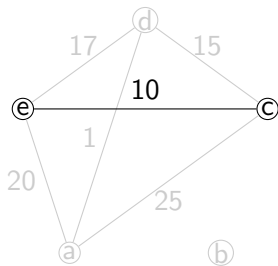
Vertex Sorting



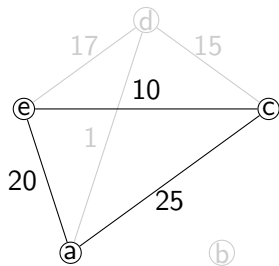
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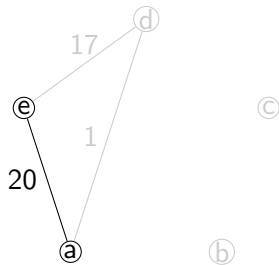
Vertex Sorting



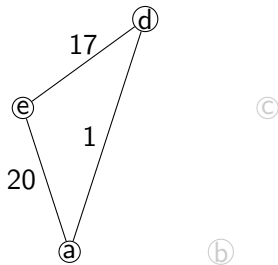
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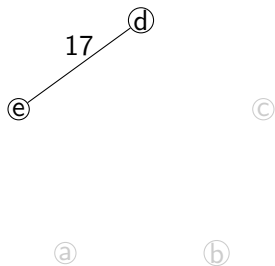
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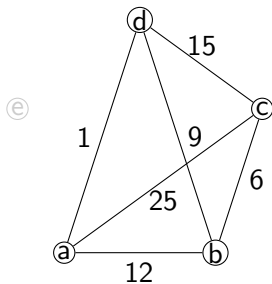


Vertex Sorting



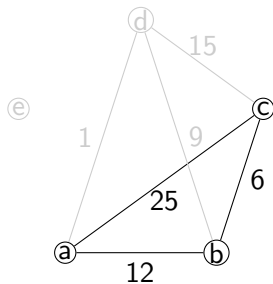
$\pi = e, \dots, d$

Vertex Sorting



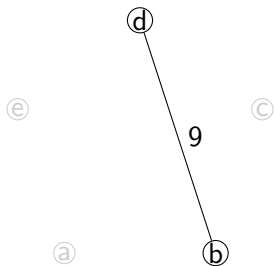
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Vertex Sorting



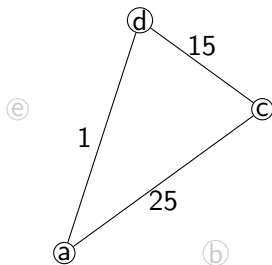
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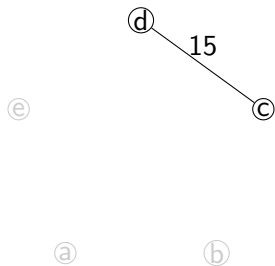
$\pi = e, b, \dots, d$

Vertex Sorting



$$\pi = e, b, \dots, d$$

Vertex Sorting



$$\pi = e, b, c, a, d$$

Recognition Algorithm

$$\mathcal{R} = \{(a, d, 1), (b, c, 6), (b, d, 9), (e, c, 10), \dots\}$$

$$\pi_0 = e, b, c, a, d$$

Recognition Algorithm

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π_0 e b c a d

π_1 e b c d a

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π_1 e b c d a

π_2 e c b d a

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$$\pi_0 = e, b, c, a, d$$

π_2 e c b d a

π_3 c e d b a

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π_3 c e d b a

π_4 e c b d a

Recognition Algorithm

$$\mathcal{R} = \{(a, d, 1), (b, c, 6), (b, d, 9), (e, c, 10), \dots\}$$

π_0	e	b	c	a	d
			...		
π_T	d	a	c	b	e

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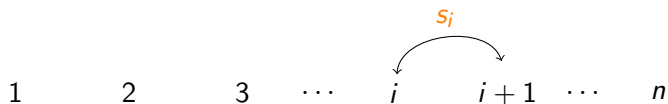
Reduced Decomposition

Adjacent transposition

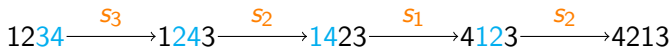


Reduced Decomposition

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Reduced decomposition



$$(3, 2, 1, 2) \in R(4231)$$

For $w \in S_n$, $R(w)$: set of all reduced decompositions of w .

Reduced Decomposition

Adjacent transposition



Reduced decomposition



$$(3, 2, 1, 2) \in R(4231)$$

For $w \in S_n$, $R(w)$: set of all reduced decompositions of w .
Stanley gave a way to calculate $|R(w)|$.

- $f : \{(\mathcal{G}, \pi_0)\} \rightarrow R(w_n)$ is a bijection.
 $w_n = n, (n-1), \dots, 1.$
- Each mobility clique, \mathcal{G} , has two possible vertex ordering.

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- Each mobility clique, \mathcal{G} , has two possible vertex ordering.
- The number of mobility cliques on n nodes is

$$\frac{|R(w_n)|}{2} = \frac{1}{2} \frac{\binom{n}{2}!}{1^{n-1} 3^{n-2} \dots (2n-3)^1}.$$

Temporal Spanner

Theorem

Given a mobility clique \mathcal{G} , let \mathcal{H} be the subgraph of \mathcal{G} consisting the $(2n - 3)$ edges which are adjacent with either node 1 or node n . \mathcal{H} is a temporal spanner of \mathcal{G} .

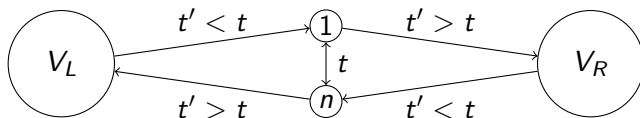
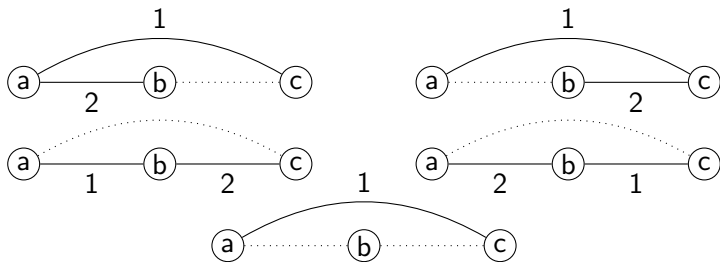


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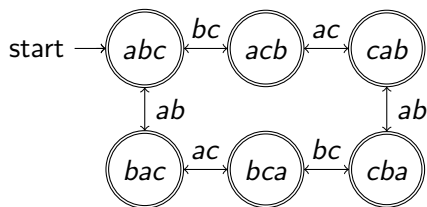
At most one



Theorem (Unordered Patterns)

For $n \in \mathbb{N}$, there exists a graph on n nodes, not be a mobility graph, s.t. no pattern on a subset of its nodes can be forbidden.

→ Forbidden ordered patterns on three nodes.



The ordered sequence of edges as a word in Σ^* with $\Sigma = \{ab, bc, ac\}$.

Open Questions

- Agents move with constant speeds?
- 2D mobility model?

This program has received funding from ANR project TEMPOGRAL (ANR-22-CE48-0001), and the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska -Curie grant No 945332.

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Thank you!