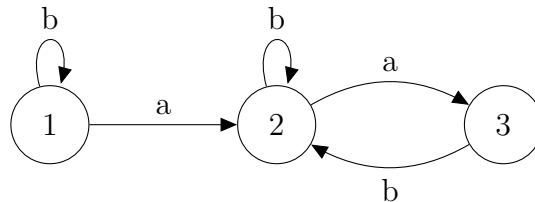


1 Parity and Büchi automata

1.1 Example

The automaton below accepts with the parity condition (maximal color visited infinitely often is even), with the color of a state being equal to the state number.



1. Which language is accepted by \mathcal{A} ?
2. Give an equivalent Büchi automaton.

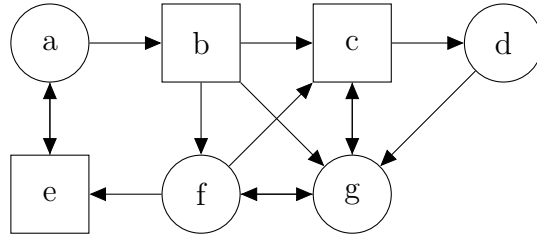
1.2 Parity and Büchi

1. Show how to transform a Büchi automaton into an equivalent parity automaton with the same number of states.
2. Show how to transform a parity automaton into an equivalent Büchi automaton of size polynomial in the size of the original automaton.

2 Parity games

2.1 Example

Consider the following parity game, with coloring $\chi(a) = \chi(e) = 0$, $\chi(b) = \chi(f) = 1$, $\chi(c) = \chi(g) = 2$, $\chi(d) = 3$:



Compute the winning region W_0 of P_0 , as well as a winning strategy.

2.2 Parity games: algorithms

Describe a polynomial algorithm for the following problem: given a finite arena and a positional strategy σ_0 for P_0 , check that σ_0 is winning for P_0 (starting from any vertex).

Deduce that parity games are in $\text{NP} \cap \text{co-NP}$.

2.3 Parity games and finite games

We fix a starting vertex v_0 in a finite (parity) game G and we define a new game G' where all plays are finite. The game stops as soon as a vertex is visited twice. A play is thus a finite path v_0, \dots, v_n such that v_0, \dots, v_{n-1} are pairwise distinct and $v_n = v_j$ for some $j < n$. Player P_0 wins if the maximal color of the loop $\max\{\chi(v_j), \dots, \chi(v_n)\}$, is even.

Show that the parity game G starting in v_0 and G' are *equivalent* (i.e., P_0 wins in G from v_0 iff he wins in G'):

1. Show that if P_i has a *positional* winning strategy σ in G , then σ is also winning in G' .
2. Show that if P_i has a winning strategy in G' , then he has also one in G .