Message Sequence Charts

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Abstract. Message sequence charts (MSC) are a graphical notation standardized by the ITU and used for the description of communication scenarios between asynchronous processes. This survey compares MSCs and communicating finite-state automata, presenting two fundamental validation problems on MSCs, model-checking and implementability.

1 Introduction

Modeling and validation, whether formal or ad-hoc, are important steps in system design. Over the last couple of decades, various methods and tools were developed for decreasing the amount of design and development errors. A common component of such methods and tools is the use of formalisms for specifying the behavior and requirements of the system. Experience has shown that some formalisms, such as finite-state machines, are particularly appealing, due to their convenient mathematical properties. In particular, the expressive power of finite-state machines is identical to regular languages, an important and well-studied class of languages. Although their expressiveness is restricted, finite-state machines are used for the increasingly successful automatic verification of software and hardware, also called model-checking \cite{8,10}. One of the biggest challenges in developing new validation technology based on finite-state machines is to make this model popular among system engineers.

The Message Sequence Charts (MSC) model has become popular in software development throughout its visual representation, depicting the involved processes as vertical lines, and each message as an arrow between the source and the target processes, according to their occurrence order. An international standard \cite{1}, and its inclusion in the UML standard, has increased the popularity. The standard has also extended the notation to Message Sequence Graphs (MSGs), which consist of finite transition systems, where each state embeds a single MSC. Encouraged by the success of the formalism among software developers, techniques and tools for analyzing MSCs and MSGs have been developed.

In this survey we describe the formal analysis of MSCs and MSGs. The class of systems that can be described using this formalism does not directly correspond to a well-studied class such as regular languages. It turns out that MSGs are incomparable with the class of finite-state communication protocols. One thus needs to separately study the expressiveness of MSG languages, and
adapt the validation algorithms. Several new algorithms are suggested in order to check MSG properties, mostly related to an automatic translation from MSG specification into skeletons of concurrent programs. Our survey concentrates on the following subjects:

Expressiveness: Comparing the expressive power of MSGs to the expressive power of other formalisms, in particular communicating finite-state machines.

Verification: The ability to apply automatic verification algorithms on MSGs, and the various formalisms used to define properties of MSGs.

Implementability: The ability to obtain an automatic translation from MSG specification into skeletons of code.

Generalizations and Restrictions: Various extensions and restrictions of the standard notation are suggested in order to capture further systems, and on the other hand, to obtain decidability of important decision procedures.

Very recently, several MSC-based specification formalisms have been proposed, such as Live Sequence Charts [17], Triggered MSCs [30], Netcharts [25] and Template MSCs [12]. The motivation behind these models is to increase the expressiveness of the notation, and to make their usage by designers even more convenient.

2 Message Sequence Graphs and Communicating Finite-State Machines

We present in this section two specification formalisms for communication protocols, Message Sequence Charts and Communicating Finite-State Machines.

Message Sequence Charts (MSC for short) is a scenario language standardized by the ITU [1]. They are simple diagrams depicting the activity and communications in a distributed system. The entities participating in the interactions are called instances (or processes). They are represented by vertical lines, on which the behavior of each single process is described by a sequence of events. Message exchanges are depicted by arrows from the sender to the receiver. In addition to messages, atomic events, timers, local/global conditions can also be represented.

**Definition 1** A Message Sequence Chart (MSC for short) is a tuple \( M = (\mathcal{P}, E, \mathcal{C}, \ell, m, <) \) where:

- \( \mathcal{P} \) is a finite set of processes,
- \( E \) is a finite set of events,
- \( \mathcal{C} \) is a finite set of names for messages and local actions,
- \( \ell : E \rightarrow \mathcal{T} = \{p\mid q(a), p\mid q(a), p(a) \mid p \neq q \in \mathcal{P}, a \in \mathcal{C}\} \) labels an event with its type: in process \( p \), either a send \( p\mid q(a) \) of message \( a \) to process \( q \), or a receive \( p\mid q(a) \) of message \( a \) from process \( q \), or a local event \( p(a) \). The labeling \( \ell \) partitions the set of events by type (send, receive, or local), \( E = S \cup R \cup L \), and by process, \( E = \bigcup_{p \in \mathcal{P}} E_p \).
- \( m : S \rightarrow R \) is a bijection matching each send to the corresponding receive. If \( m(s) = r \), then \( \ell(s) = plq(a) \) and \( \ell(r) = p'q(a) \) for some processes \( p, q \in \mathcal{P} \) and some message name \( a \in \mathcal{C} \).

- \( \prec \subseteq E \times E \) is an acyclic relation between events consisting of:
  1. a total order on \( E_p \), for every process \( p \in \mathcal{P} \), and
  2. \( s \prec r \), whenever \( m(s) = r \).

The event labeling \( \ell \) implicitly defines the process \( pr(e) \) for each event \( e \)
as \( pr(e) = p \) if \( e \in E_p \) (equivalently, \( \ell(e) \in \{plq(a), p'q(a), p(a)\} \) for some \( q \in \mathcal{P}, a \in \mathcal{C} \)). Since point-to-point communication is usually FIFO (first-in-first-out)
we make in the following the same assumption for MSCs. That is, we assume that whenever \( m(s_1) = r_1 \), \( m(s_2) = r_2 \) holds with \( pr(s_1) = pr(s_2), pr(r_1) = pr(r_2) \) and \( s_1 < s_2 \), then we also have \( r_1 < r_2 \).

The example in figure 1 is an MSC \( M \) with messages sent between two
processes \( p_1, p_2 \). It corresponds to a scenario of the alternating bit protocol, in
which the sender \( p_1 \) is forced to resend the message to the receiver \( p_2 \), since \( p_2 \)'s
acknowledgments arrive too late.

![Fig. 1. MSC execution of the alternating bit protocol.](image)

The relation \( \prec \) is called the visual order on the MSC, since it corresponds
to its graphical representation. It is comprised of the process ordering and
the message ordering, pairwise between send and matching receive. Since \( \prec \)
is required to be acyclic, its reflexive-transitive closure \( \prec^* \) is a partial order
on the set \( E \) of events, which we will denote for simplicity also by \( \leq \). Any
extension of \( \leq \) to a total order on \( E \) is called a linearization of \( M \). We
denote by Lin(\( M \)) the set of all labeled linearizations of an MSC \( M \), Lin(\( M \)) =
\( \{\ell(e_1) \cdots \ell(e_n) \mid e_1 \cdots e_n \text{ is a linearization of } M \} \).
Since the specification of a communication protocol consists of many scenarios, either in positive or in negative form, a high-level description is needed for combining them together and defining infinite sets of (finite or infinite) scenarios. The Z/120 standard description introduces high-level MSCs using non-deterministic branching, concatenation and iteration of finite MSCs. The semantics is provisional, that is, the high-level MSC usually describes possible behaviors of the system. Formally, a Message Sequence Graph (MSG for short) \( G = (V, R, v^0, V_f, \lambda) \) consists of a finite transition system \( (V, R, v^0, V_f) \) with set of nodes \( V \) and set of transitions \( R \subseteq V \times V \), initial node \( v^0 \in V \) and terminal nodes \( V_f \subseteq V \). In pictures, the initial node is marked by an incoming arrow, and final nodes by outgoing arrows. Each node \( v \) is labeled by the finite MSC \( \lambda(v) \). For instance, the MSG in figure 2 describes the possible runs of a protocol for connecting a user \( U \) with a server \( S \) through a firewall \( F \). After a connection request (initial node \( A \)) either the server accepts the user and the firewall grants the access (final node \( B \)), or else the server’s accept arrives too late (after the firewall denied the access, node \( C \)). This negative behavior can repeat (loop between \( A \) and \( C \)) and leads eventually to an error (final node \( D \)).

![Diagram of a communication protocol represented by an MSG.](image)

An execution of an MSG \( G \) is the labeling \( \lambda(v_0)\lambda(v_1)\cdots\lambda(v_k) \) of some accepting path \( v^0 = v_0, v_1, \ldots, v_k \in V_f \) of \( G \), i.e., \((v_i, v_{i+1}) \in R\) for every \( 0 \leq i < k \). For example, \( ACAB \) in figure 2 is the execution of \( G \) in which the connection fails once, but the second request succeeds. The set of executions of \( G \) is denoted by \( L(G) \), the set of linearizations of executions of \( G \) is denoted by \( \text{Lin}(G) \). The size of a MSG \( G \) (denoted \(|G|\)) is the sum of the sizes of its nodes.

Of course, the semantics of MSGs depends on the definition of the MSC product. We consider the usual weak product of MSCs, that concatenates MSCs along
the process lines. Let $M_1 = \langle \mathcal{P}, E_1, C_1, \ell_1, m_1, <_1 \rangle$ and $M_2 = \langle \mathcal{P}, E_2, C_2, \ell_2, m_2, <_2 \rangle$ be MSCs over the same set of processes $\mathcal{P}$. The product $M_1 \cdot M_2$ is the MSC $\langle \mathcal{P}, E_1 \cup E_2, C_1 \cup C_2, \ell_1 \cup \ell_2, m_1 \cup m_2, < \rangle$ over the disjoint union of events, with the visual order given by:

$$<_1 \cup <_2 \cup \{(e, f) \in E_1 \times E_2 \mid \mathcal{P}(e) = \mathcal{P}(f)\}.$$ 

Note that there is no synchronization between different processes when moving from one node to the next one (\textit{weak product}). Hence, it is possible that one process is still involved in some actions of $M_1$, while another process has advanced to an event of $M_2$.

A related standardized specification notation for telecommunication applications is SDL (Specification and Description Language, ITU Z.100). SDL is dedicated to the design of real-time, distributed systems and involves complex features as hierarchy, procedure calls and abstract data types. The basic theoretical model behind SDL are nested communicating finite-state machines. We recall the definition of (flat) communicating finite-state machines (CFM for short).

A CFM $A = (A_p)_{p \in \mathcal{P}}$ consists of finite-state machines $A_p$ associated with processes $p \in \mathcal{P}$, which communicate over unbounded, error-free, FIFO channels. The content of a channel is a word over a finite alphabet $\mathcal{C}$. With each pair $(p, q) \in \mathcal{P}^2$ of distinct processes we associate a channel $C_{p,q}$. Each finite-state machine $A_p$ is described by a tuple $A_p = (S_p, A_p, \rightarrow_p, F_p)$ consisting of a set of local states $S_p$, a set of actions $A_p$, a set of final states $F_p$ and a transition relation $\rightarrow_p \subseteq S_p \times A_p \times S_p$. The computation begins in an initial state $s^0 \in \prod_{p \in \mathcal{P}} S_p$. The actions of $A_p$ are either local actions or sending/receiving a message. We use the same notations as for MSCs. Sending message $a \in \mathcal{C}$ from process $p$ to process $q$ is denoted by $p \cdot q(a)$ and it means that $a$ is appended to the channel $C_{p,q}$. Receiving message $a$ by $p$ from $q$ is denoted by $p \cdot q'(a)$ and it means that $a$ must be the first message in $C_{q,p}$, which will be then removed from $C_{q,p}$. A local action $a$ on process $p$ is denoted by $l_p(a)$. We denote a run of the CFM as successful, if each process $p$ finishes the execution in some final state and all channels are empty. The set of successful runs of $A$ is denoted $\mathcal{L}(A)$. The size of $A$ is $|\sum_p |A_p|$ and is denoted $|A|$.

Note that each successful run of a CFM defines an MSC. Conversely, with each MSC $M = \langle \mathcal{P}, E, C, \ell, m, < \rangle$ we can associate an equivalent CFM, by defining the behavior of process $p$ as the (ordered) sequence of events $E_p$. However, the two formalisms MSG and CFM are incomparable in general, as discussed in the next section.

3 Comparing MSG and CFM

Comparing the expressivity of MSG and CFM is interesting for at least two reasons. First, both formalisms are heavily used in protocol design, sometimes for specifying different parts of a system at different stages of the design process. Second, MSCs are usually intended as early requirements, for a rough description of the desired/undesired behavior. Thus, the question whether the described
behavior can be turned into a protocol (implementability/realizability problem) is an important validation step in the design process.

A qualitative comparison between MSG and CFM concerns two important parameters, control and channels. Control in a CFM is inherently local, since it corresponds to local transition functions. The control structure of an MSG is global, since the branching from a node concerns all processes occurring in the future execution. The global control mechanism of an MSG is actually imposed by the visual character of the diagram graph, in which MSCs are composed sequentially. One problem arising from the global control is that an MSG G might be non-implementable, i.e., no CFM A exists with \( L(A) = L(G) \). For a simple example, consider the MSG G consisting of a single node v with a self-loop, labeled by a message from \( p_1 \) to \( p_2 \) and another message from \( p_3 \) to \( p_4 \). Since the MSCs in \( L(G) \) must contain equally many messages from \( p_1 \) to \( p_2 \) and from \( p_3 \) to \( p_4 \), there can be no equivalent CFM.

We turn now to the second parameter, namely channels. Although none of the models impose any (universal) bound on the channel capacity, validation tasks such as model-checking tend to be “more” decidable for MSGs than for CFMs that are Turing complete, see [9]. The reason is that MSGs have existentially-bounded channels, i.e., for each MSG G there exists an integer \( b \) such that every MSC in \( L(G) \) can be executed with channels of size at most \( b \). Formally, a set \( X \) of MSCs is called existentially-bounded if there exists some \( b \) such that every MSC \( M \in X \) has some linearization \( w \in \text{Lin}(M) \) satisfying the following property: for every pair of distinct processes \( p, q \) and every prefix \( v \) of \( w \), it holds that \( 0 \leq \sum_{a \in C} |v|v \langle a \rangle - |w|w \langle a \rangle \leq b \). For an MSG G the bound \( b \) is linear in the maximal size of the MSCs labeling the nodes of G. For an example of property that is undecidable for CFM (but not for MSG) one can consider the question whether a CFM generates at least one MSC [9]. A less trivial example is pattern-matching: given an MSC \( M \) and an MSG G, we ask whether there is some execution \( N \in L(G) \) and a factorization \( N = N_1 M N_2 \), where \( N_1, N_2 \) are both MSCs. The pattern-matching algorithm described in [12, 13] uses heavily the fact that MSGs are existentially-bounded (with an priori known bound).

More generally, some CFMs cannot be transformed into MSGs since MSGs are finitely generated. That is, for any MSG G there exists a finite set X of finite MSCs such that any execution \( M \in L(G) \) can be written as a (finite or infinite) product \( M = M_1 M_2 \cdots M_k \) of factors from X, \( M_i \in X \) for all i. A typical example of CFM that is not finitely generated corresponds to the alternating bit protocol. The executions of this protocol include the family of MSCs that generalize the pattern of the MSC shown in figure 1 with n crossing messages for every n. None of these MSCs M can be decomposed as \( M = M_1 M_2 \) with \( M_1, M_2 \) non-empty MSCs, since including a send in \( M_1 \) forces to add another send on the other process (the one preceding the corresponding receive). More generally, an MSC \( M \) is called atomic (or atom), if for any decomposition \( M = M_1 M_2 \) where both \( M_1, M_2 \) are MSCs, at most one is non-empty. For another example of atomic MSC, consider the MSC \( M_3 \) in figure 4. The set of atoms generating the MSC executions of an MSG G is denoted \( \text{At}(G) \). It is a finite set and it
represents a canonical set of generators of $\mathcal{L}(G)$. Moreover, it can be computed by a simple linear-time algorithm, see [19].

On the potentially infinite alphabet $\Delta$ of atomic MSCs, we can define an independence (commutation) relation $I \subseteq \Delta \times \Delta$ by letting $AI A'$ iff $\mathcal{P}(A) \cap \mathcal{P}(A') = \emptyset$. Notice that $AI A'$ implies that $A, A'$ commute, $AA' = A'A$, and that the decomposition of any MSC into atoms is unique up to commuting adjacent atoms $A, A'$ with $AI A'$.

Returning to the alternating bit example, it is easily seen that the set of linearizations $\text{Lin}(M)$ of the represented MSC $M$ is regular. Note that in this particular example every linearization of $M$ has channel bound at most 3. We call a set $X$ of MSCs universally-bounded if for every MSC $M \in X$, every linearization $w \in \text{Lin}(M)$, every prefix $v$ of $w$ and every pair of distinct processes $p, q$ we have $0 \leq \sum_{a \in C} |v|_{p \leftarrow a} - \sum_{a \in C} |v|_{q \leftarrow a} \leq 1$. Notice also that such a universal channel bound for an MSG $G$ does not suffice for $\text{Lin}(G)$ being a regular set. Hence, even if the specification is given as finite-state automaton $A$, we cannot automatically transform $A$ into an equivalent MSG $G$. This led to an extension of the MSG formalism, namely to Compositional Message Sequence Graphs (CMSG, for short) [16]. A compositional MSC (CMSC, for short) is defined as an MSC, except that the message function $m$ is partially defined. A send that does not belong to the domain of the message function $m$, or a receive not belonging to the range of $m$, are called unmatched events. The product of two CMSCs $M_1 M_2$ is defined as for MSC, but in addition the $k$-th unmatched send of $M_1$ is matched with the $k$-th unmatched receive of $M_2$ (if they exist) in such a way that the FIFO property is satisfied by matched events. Hence, the product of CMSCs is only partially defined. Moreover, it is not associative, hence we define a product $M_1 M_2 \cdots M_k$ as parenthesized from left to right.

It is not very difficult to see that any CFM can be transformed into an equivalent CMSG of exponential size. The rough idea is that nodes correspond to pairs $(\text{state}, \text{event})$, where state is a global state of the CFM and event is an event enabled in state. There is a transition from $(\text{state}_{1}, \text{event}_{1})$ to $(\text{state}_{2}, \text{event}_{2})$ if state2 is obtained from state1 by an event1-transition (that modifies state1 according to the local transition relation). It is easy to check that any CMSG execution of the CFM with no unmatched receive is an execution of the CMSG, and vice-versa. For instance, the CFM generating the alternating bit protocol from example 1 can be transformed into the CMSG in figure 3. (For CMSGs we draw unmatched events by the solid end of a half-dotted message arrow, that suggests the type of the matching event.)

**Theorem 1.** Any CFM can be transformed into an equivalent CMSG of exponential size.

4 Validating MSC Specifications: Model-Checking and Implementation

MSG specifications are used very early in the design process. Revealing design errors before implementing is of primary importance. This has motivated the de-
sign of algorithms that check specific properties of MSGs such as race conditions [4, 28] and detecting non-local choice [7, 19, 18]. Model-checking MSG specifications has been considered w.r.t. properties expressed as MSG [27], automata [6] and partial-order logics [29, 24]. Another test that may reveal the incompleteness of an MSG specification is the one for implementability. Here, we want to know whether the specification can be transformed into a state-based, distributed model as CFM. As discussed in section 4.2, the definition of implementability is not canonical, and the results strongly depend on the variant we consider.

4.1 Model-Checking

In the common model-checking approach (see for recent textbooks [8, 10]) we usually describe bad execution sequences using the same formalism as for specifying the system (e.g., finite automata over infinite words). Then we need to check the emptiness of the intersection between the bad sequences and the system, and counter-examples can be obtained if the intersection is non-empty. In the MSC setting we cannot use complementation as with finite automata. First, the complement of an MSG is not finitely generated, thus it can never be represented by an MSG. Secondly, even if we take the complement w.r.t. the MSCs generated by the same set of atoms, the complement cannot be represented by an MSG in general. This is similar to the fact that the complement of a rational trace language is not rational, in general [11]. Therefore, we consider two variants of model-checking, positive and negative model-checking. In both cases we specify the property $P$ we want to check, as well as the system $S$ itself, by MSGs. For negative model-checking we view $P$ as a set of bad MSC executions and we ask whether $\mathcal{L}(P) \cap \mathcal{L}(S) = \emptyset$. For positive model-checking we view $P$ as a set of good MSC executions and we ask whether $\mathcal{L}(S) \subseteq \mathcal{L}(P)$.

In the general setting of MSG specifications, both model-checking variants are undecidable [6, 27]. This holds even if the property $P$ is given by a finite-state automaton or an LTL formula [6]:

**Theorem 1** Given a finite-state automaton $P$ and an MSG graph $G$, it is undecidable whether $\mathcal{L}(P) \cap \mathcal{L}(G) = \emptyset$.
The proof for theorem 1 is a straightforward reduction from Post’s correspondence problem (PCP). Recall that an instance of PCP consists of pairs of words \((x_i, y_i)\) \(1 \leq i \leq k\) over the alphabet \(\{0, 1\}\). Then we ask for a non-empty sequence of indices \(i_1, \ldots, i_n\) such that \(x_{i_1} \cdot \cdot \cdot x_{i_n} = y_{i_1} \cdot \cdot \cdot y_{i_n}\).

The MSG \(G\) consists of \((k + 2)\) nodes \(v^0, v_1, \ldots, v_k, v^l\). Node \(v^0\) (\(v^l\), resp.) is initial (final, resp.), and labeled by the empty MSC. Node \(v_i\) is labeled by a sequence of messages from \(p_1\) to \(p_2\) labeled 0 or 1 such that the sequence of labels equals \(x_i\), and a message from \(p_3\) to \(p_4\) labeled by \(i\). There is a transition from \(v^0\) to each of \(v_i\), from each \(v_i\) to \(v^l\), and one from \(v^l\) to \(v_0\). The automaton \(P\) accepts precisely the set \((X_1 + \cdots + X_k)^+\), where each \(X_i\) is a finite word defined as follows: Let \(y_i = a_1 \cdots a_m\), then \(X_i = p_1 p_2 (a_1) p_2^? p_1 (a_1) \cdots p_1 p_2 (a_m) p_2^? p_1 (a_m) p_3 (i) p_4 (i) p_4 (i)\).

Clearly, since there is no synchronization between the process pairs \(\{p_1, p_2\}\) and \(\{p_3, p_4\}\), both the MSG and the automaton describe MSCs with two parallel threads, one over the PCP words (\(x\) for \(G\), \(y\) for \(P\)) and the other over the corresponding indices. The non-empty intersection between \(G\) and \(P\) reveals then a PCP solution.

**Remark.** Note that the undecidability proof above does not rely on the unboundedness of channels, since \(G\) is existentially-bounded. Actually the construction can be slightly modified such that \(G\) becomes universally-bounded, by adding an acknowledgment after each message. The true reason for undecidability is concurrency, since \(G\) and \(P\) use different linearizations of the same partial orders of MSC.

Several decidable variants of model-checking have been considered in subsequent papers. Some of them are obtained by restricting the properties we want to check, others are obtained by restricting the system specification. However, several variants are based on a similar idea. Suppose for instance that the property \(P\) is given by a *linearization-closed* finite-state automaton \(A\). That is, for every word \(w \in L(A)\), the automaton \(A\) also accepts every linearization \(v \in \text{Lin}(M)\) of the MSC \(M\) defined by \(w\). In this case it suffices to consider *representative linearizations* of the system MSG \(G\): We choose for every node \(v\) of \(G\) some linearization of the MSC labeling \(v\), say \(l_v\). Then we define a finite-state automaton \(A(G)\) from \(G\) by replacing the label of \(v\) by \(l_v\). Thus, states of \(A(G)\) are labeled by words. It is easy to see now that \(L(G) \cap L(A) \neq \emptyset\) if and only if \(L(A(G)) \cap L(A) \neq \emptyset\), and \(L(G) \subseteq L(A)\) if and only if \(L(A(G)) \subseteq L(A)\).

Among the model-checking variants that led to algorithmic solutions we refer to the following ones:

- **Model-checking with gaps** [28]: The property \(P\) is given by an MSG, but its semantics differs from the semantics of the system \(G\). An execution \(M\) of \(P\) is matched *with gaps* by an execution \(M'\) of \(G\) if there is an embedding \(\phi\) of the events of \(M\) in the set of events of \(M'\) such that the visual order is preserved: whenever \(e < f\) in \(M\), we have \(\phi(e) < \phi(f)\) in \(M'\). This problem has been shown to be NP-complete (even if \(P\) is an acyclic MSG).
The main reason for decidability of model-checking with gaps is that gaps lead to very restricted languages, for which we can compute a sort of linearization-closure.

- **Using partial-order specifications** [29, 24]: Here, the property $P$ is given by a partial-order logic, which makes it linearization-closed. In [29] a logic derived from a fragment of TLC [5] is proposed for MSGs. Basically, this logic corresponds to CTL interpreted over partial-order graphs of MSCs, where the edge relation is the immediate successor relation (on each process, resp. for send/receive pairs). It is shown in [29] how to construct an exponential-size automaton from the specification, hence model-checking is PSPACE w.r.t. the specification (and only linear in the size of the system). In [24] the specification formalism is MSO, interpreted over partial-order graphs of MSCs. Here, the complexity is non-elementary, as it is already in the word case.

A further approach leading to a decidable model-checking problem is to syntactically restrict the MSGs, see sections 5 and 6 for details.

### 4.2 Implementability

As previously mentioned, the MSG formalism is useful as a specification notation, but it does not provide directly a protocol model. Such a model is usually state-based and distributed, whereas MSGs provide an implicit global control over the behavior of the processes. This allows for specifications that are not implementable because of global choices (see for instance figure 4 which is discussed below). Being able to generate an implementation for an MSC specification also allows to perform tests on the level of requirements, hence it is not longer required to generate code before testing.

The protocol model generally used is CFM over the same set of processes as the MSG specification. But we still have some choice for the semantics of the implementation. For instance, we could allow for an implementation with more (or less) behavior than the MSG. The most natural notion is that the implementation is equivalent to the MSG: An MSG $G$ is implementable, if some CFM $\mathcal{A}$ exists such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(G)$. Furthermore, we allow the implementation to contain additional data in messages. That is, the message contents of the CFM come from a finite set $C' = C \times D$, where $C$ is the set of message contents of the MSG and $D$ is some finite set. Then, the equality $\mathcal{L}(\mathcal{A}) = \mathcal{L}(G)$ is required up to the additional data $D$. A further, even more relaxed notion of implementability, would also allow for additional messages. Notice that this would make every MSG implementable, since the additional messages can be used for synchronizing all processes after each node. We do not allow additional messages, since in many applications they are neither desired nor possible (e.g., applications where acknowledgments cannot be provided).

The first notion of implementation, which we denote as standard implementation, has been proposed in [2, 3]. The standard implementation of the MSG $G = (V, R, V^D, V_f, \lambda)$ over the process set $\mathcal{P}$ does not add any data and it is fully
determined by the MSG, being defined process by process: The automaton \( \mathcal{A}_p \) for process \( p \) generates the projection of \( \mathcal{L}(G) \) on the events of process \( p \).

We call an MSG standard-implementable if it is implementable w.r.t. the standard version of implementability. Notice that this notion is actually too weak, since it captures just a small subset of implementable specifications. The simplest counter-example (see figure 4) is a set of two MSCs over the processes \( p_1, p_2 \) where the first MSC \( M_1 \) has a message from \( p_1 \) to \( p_2 \), followed by one from \( p_2 \) to \( p_1 \). In the second MSC \( M_2 \) we have first a message from \( p_2 \) to \( p_1 \), then one from \( p_1 \) to \( p_2 \). These two MSCs are not standard-implementable since we can combine the projection of \( M_1 \) on \( p_1 \) with the projection of \( M_2 \) on \( p_2 \) and we obtain the MSC \( M_3 \). This set is not implementable even with additional data. Changing slightly this example we obtain one which is not standard-implementable, but is implementable with additional data. For this, we just add at the beginning of both \( M_1, M_2 \) a first unlabeled message from \( p_1 \) to \( p_2 \), see figure 5. Then the non-implementability argument given previously still works. However, with additional data we use the initial message for letting \( p_1 \) decide on the outcome \( M_1 \) or \( M_2 \), and inform \( p_2 \).

**Fig. 4.** The set \( \{M_1, M_2\} \) is not implementable (it does not contain the implied MSC \( M_3 \)).

Two striking weaknesses of the standard notion is that not even simple MSGs are standard-implementable, as seen from the example above. Furthermore, for the restricted class of regular MSGs defined in section 5, the question of standard-implementability is undecidable. However, regular MSGs are implementable with additional data, see section 5 for more details.

Nevertheless, the results of [2, 3] show that standard implementability becomes decidable at least for regular MSGs if one looks for deadlock-free implementations, only (called safe realizability in [2, 3]), albeit with high algorithmic complexity. A CFM is called a deadlock-free implementation of an MSG \( G \) if \( \mathcal{L}(\mathcal{A}) = \mathcal{L}(G) \) and every configuration of \( \mathcal{A} \) that has no successor, is such that all processes have reached a final state and all channels are empty. Deadlock-freeness is of course required in practice, since real-life protocols should not be aborted in some unclean state. We will recall the various results on the implementability problem in sections 5 and 6.
Fig. 5. CFM implementing the set \( \{ M_1, M_2 \} \) with additional data.
5 Regular MSC Specifications

Regular MSGs have been proposed in the context of model-checking, as a subclass for which both variants of model-checking are decidable [6, 27]. It is a syntactic restriction that ensures that the set of all linearizations, i.e., the set Lin(G), is regular. Regular MSGs provided to be a theoretically robust class, in terms of logical and automata-theoretic characterizations. In particular, regular MSGs can be implemented with additional data by CFM with universally-bounded channels. However, the CFM implementation is not deadlock-free, in general.

A set X of finite MSCs is called regular if Lin(X) is a regular string language over the alphabet T of event types [20]. Moreover, there is a syntactic condition ensuring that an MSG G generates a regular set L(G) of MSCs. This condition roughly means that communication in a loop must be acknowledged to all active processes. Formally, we need to define the communication graph of an MSC M: it is a directed graph over the set of communicating processes in M with an edge from process p to process q whenever M contains a message from p to q. An MSG G is called a regular MSG (locally-synchronized in [27], bounded in [6]) if any MSC labeling a loop of G has a strongly connected communication graph. This condition is co-NP complete [27].

For an example, consider the MSG in figure 2. It is a regular MSG, since every loop involves only A, C, and the communication graph of AC is strongly connected (the firewall is connected with both user and server by bidirectional arcs).

Putting together the results from [6, 27, 20] we have the following relationship between regular sets of MSCs and regular MSGs:

**Theorem 2** 1. For every regular MSG G the set L(G) of generated MSCs is regular [6, 27].
2. For every regular and finitely generated set X of MSCs there exists a regular MSG G with X = L(G) [20].

The main interest in regular MSGs was to obtain a subclass of MSC specifications with a decidable model-checking problem:

**Theorem 3** [6, 27] The negative model-checking problem L(G) ∩ L(H) ≠ ∅ where G is a regular MSG, is PSPACE-complete. The positive model-checking problem L(G) ⊆ L(H) where H is a regular MSG, is EXPSPACE-complete.

The theorem above shows that model-checking MSGs is rather expensive, which is actually not very surprising when we deal with concurrent models. The reason is MSGs are more compact than finite-state automata. The upper bounds in the theorem above are based on the fact that if G is a regular MSG then we can compute a finite automaton of exponential size generating Lin(G).

Regular MSC languages also have nice characterizations in the logical and communicating automata framework. The logic used in [21, 24] is MSO with atomic propositions ℓ(e) = t ∈ T, e ≤ f and e ∈ E that have the usual interpretation, as type labeling, partial order of the MSC and membership in a second order variable E.
Theorem 4 \cite{21, 22} Let $X$ be a universally-bounded set of MSCs. The following assertions are equivalent:

1. $X$ can be implemented by a (deterministic) CFM with additional data.
2. There exists an MSO formula $\phi$ such that $X$ is the set of bounded MSCs satisfying $\phi$.

In the first part of the theorem above the implementation is not deadlock-free, since the constructed CFM uses global final states for accepting $X$. On the other hand, as we mentioned in section 5, the standard implementation is not really helpful when applied to regular MSGs (the upper bound is due to [3], and the lower bound to [23]):

Theorem 2. \cite{3, 23} It is undecidable to know whether a regular MSG is standard-implementable. It is EXPSPACE-complete to know whether a regular MSG is standard-implementable without deadlocks.

Remark. The undecidability result in theorem 2 heavily depends on the fact that channels are FIFO. Without FIFO, standard implementability for regular MSGs becomes decidable. 

\[\square\]

6 Globally-Cooperative MSGs

As seen in section 5, model-checking for regular MSGs is decidable and of tractable complexity (PSPACE for the basic variant). However, the situation is far from being ideal. Notice first that some trivial protocols cannot be represented by regular MSGs. For instance, the protocol where process $p_1$ can send any number of messages to process $p_2$. The reason is that regular MSGs have universally bounded channels, which restricts severely their expressive power. Second, for real life communication protocols one can usually find a (sufficiently large) bound $b$ so that any run of the protocol can be executed with channels each bounded by $b$. Similarly to MSGs, we call such protocols existentially $b$-bounded. Whereas an algorithm exists to check whether a CFM or a finite-state machine is existentially $b$-bounded for a given $b$, its complexity depends severely (exponentially) on $b$. Hence, in practice we cannot hope to be able to fix a sufficiently large bound $b$ that takes care of all executions. The last problem is that we cannot obtain in general an automaton generating all linearizations of executions for models that are strictly more expressive than regular MSGs. Instead, we can try to use representative linearizations rather than all linearizations, requiring that the set of representative linearizations is $b$-bounded, with $b$ as small as possible.

Definition 1 An MSG $G$ is called globally-cooperative (gc-MSG for short) if every loop of $G$ has a weakly connected communication graph.

Thus, an MSG $G$ is a gc-MSG if any MSC $M$ labeling a loop cannot be written as $M = M_1 || M_2$ with $M_1, M_2$ non-empty MSCs with no common process. It is
co-NP complete to know whether an MSG is a gc-MSG. For an example of a gc-MSG, see figure 6, or suppose that we add a self-loop on node $A$ in figure 2. The MSG thus obtained is not regular anymore, but it is a gc-MSG. Clearly, every regular MSG is also a gc-MSG. Moreover, it can be noted that regular MSGs correspond exactly to gc-MSGs with universally-bounded channels.

The representative linearizations that we use for model-checking are the linearizations that execute atoms one by one. More precisely, for any $M \in \mathcal{L}(G)$ we consider only linearizations in Lin($M$) of the form $w = w_1 \cdots w_n$, where $M = A_1 \cdots A_n$ is some decomposition of $M$ into atoms $A_i$ and $w_i \in \text{Lin}(A_i)$ for all $i$. Let us denote by Lin$^a(G) \subseteq \text{Lin}(G)$ the set of such linearizations of MSCs of $\mathcal{L}(G)$. For an example, let $G$ be the graph consisting of a single node with a self-loop, labeled by a message from $p_1$ to $p_2$. Let $s = p_1!p_2$ and $r = p_2?p_1$, then Lin$^a(G) = (sr)^\ast$. Of course, Lin$^a(G)$ is not regular, it corresponds to the Dvck language over one pair of brackets.

![Fig. 6. Globally-cooperative MSG.](image)

We can use representative linearizations for model-checking as follows. Let $G, H$ be two MSGs. Then it is easy to see that $\mathcal{L}(G) \cap \mathcal{L}(H) = \emptyset$ if and only if Lin$^a(G) \cap \text{Lin}^a(H) = \emptyset$ (respectively, $\mathcal{L}(G) \subseteq \mathcal{L}(H)$ if and only if Lin$^a(G) \subseteq \text{Lin}^a(H)$). Recall that for any MSG $G$, atoms of $\mathcal{L}(G)$ are finite and finitely many. Hence the set of representative linearizations Lin$^a(G)$ is $b$-bounded, where $b$ is such that $G$ is existentially $b$-bounded. For getting a regular set of representative linearizations Lin$^a(G)$ we impose a theoretically well-known restriction, that of loop-connectedness.

We can change slightly the graph of a gc-MSG $G$ by replacing each node $v$ labeled by some non-empty MSC $M$ by a path of new nodes $v_1, \ldots, v_k$ where $v_1$ is labeled by $A_1$ and $M = A_1 \cdots A_k$ is some decomposition of $M$ into atoms $A_i$. The new graph $G'$ can be seen as an automaton with states labeled over the alphabet of atoms $\text{At}(G)$. The property of $G$ being a gc-MSG translates to $G'$ being loop-connected, which is a well-known property from the theory of Mazurkiewicz traces. It means that every loop of $G'$ is labeled by a subalphabet of $\text{At} = \text{At}(G)$ that is connected w.r.t. the symmetric dependence $D = (\text{At} \times \text{At}) \setminus I$, that is $\text{At} \cdot \text{At}'$ if $A$ and $A'$ share at least one process. With this restriction it is well-known that the closure under commutation $I$ of the regular set generated by $G'$ is regular, and an automaton generating the closure
can be effectively computed \([11, 27]\). From this automaton we obtain \(\text{Lin}^a(G)\) and an automaton generating it simply by replacing every atom \(A \in \text{At}(G)\) by some linearization of \(A\). Since the size of the automaton generating \(\text{Lin}^a(G)\) is exponential in the size of \(G\) we obtain:

**Theorem 5** \([15]\) Given a ge-MSG \(G\) and an arbitrary MSG \(H\), it is PSPACE-complete to decide whether \(\mathcal{L}(G) \cap \mathcal{L}(H) = \emptyset\). The positive model-checking problem \(\mathcal{L}(G) \subseteq \mathcal{L}(H)\) where \(H\) is a ge-MSG, is EXPSPACE-complete.

Notice that the complexity of model-checking ge-MSGs is not higher than for regular MSGs. Moreover, the situation for ge-MSGs is better, since we do not have to compute all linearizations, but a smaller subset that has the additional property of being \(b\)-bounded for a small \(b\), yielding an algorithm that is faster in practice than the one given for regular MSGs. A regular MSG \(G\) is universally \(B\)-bounded with a \(B\) that can be exponential in the size of \(G\).

We turn now to the implementation problem. The situation here enforces the idea that that universal channel bounds are not needed. For the safe variant of the standard implementability problem, i.e., where the implementation is not allowed any additional data but must be deadlock free, the complexity is the same for regular MSGs and for ge-MSGs:

**Theorem 6** \([\beta, 23]\) Given a ge-MSG \(G\), it is EXPSPACE-complete to decide whether there exists a deadlock-free CFM \(A\) with \(\mathcal{L}(G) = \mathcal{L}(A)\).

Again, this result is not really practical, given the high complexity. Moreover, in practice it might be the case that the standard implementation does not work for some ge-MSG \(G\), but that \(G\) is still implementable with a little more data. For an example see figure 8 in section 7.

In the case where one allows data to be added to messages but deadlocks are not allowed, there are ge-MSGs that cannot be implemented. For instance, consider the ge-MSG \(G\) in figure 6 with two nodes with self-loops, and two edges between them. Both nodes are labeled by MSCs with two messages, one from \(p_1\) to \(p_2\) and one from \(p_1\) to \(p_2\). The first node has its messages carrying the data \(a\), while the second node carries the data \(b\). Both nodes are initial and final. In any CFM implementation processes \(p_1\) and \(p_2\) should decide to send either both \(a\) or both \(b\), but this is impossible with no additional synchronization (messages). Hence, this protocol cannot be implemented without deadlocks. It remains open whether every ge-MSG can be implemented with additional data and allowing deadlocks. The conjecture in \([15]\) is that this is always possible.

### 7 Choice and Implementability

Deadlock-free implementability being a key feature required for communicating protocols, tractable algorithms that help implementing an MSG with additional data are needed. One reasonable way of doing this is first to exhibit a non-trivial subclass of MSGs that is always implementable with additional data and no
deadlocks. Then we want to test whether an MSG can be represented inside our subclass, preserving the MSC language.

As mentioned before, the reason for non-implementability of an MSG is the global control, whereas the choice in a CFM must be done locally. The idea is then to define MSGs that have only local choices, that is any node is controlled by a single process [7, 19].

**Definition 2** An MSG \( G = (V, R, v^0, V_f, \lambda) \) is called local-choice (lc-MSG for short) if each MSC labeling any node \( v \) of \( G \) is a triangle, that is it has a single minimal event \( \min(v) \) in the partial order \( \preceq \). Moreover, \( \min(w) \) belongs to the process set of node \( v \), whenever \( (v, w) \in R \).

Figure 7 shows an lc-MSG \( G \). Note that \( G \) is equivalent to the MSG in figure 2, which is not an lc-MSG. Checking that an MSG is local-choice can be done in polynomial time.

![Fig. 7. Local-choice MSG.](image-url)

It is not very hard to translate a lc-MSG into a deadlock-free CFM, using linear additional data. The idea is to use a leader process and to let the current leader choose the current node to be executed and the next leader. The node is chosen among the nodes that follow the node being executed, and that begin with a minimal event belonging to the leader. The next leader should be chosen among the minimal processes of nodes that follow the chosen node.

In the procedure polling-state below process \( p \) waits for a message informing it about the next node to execute and the next leader:
void polling_state()
{ while (true) {
    if p receives a message \((a,v,q)\) then
        \{ current_node=v; next_leader=q; return; \} \}

Initially, the current node is initialized by letting next_leader = \(p_r(\min((t^0)))\).
Before executing its event from current_node, process \(p\) goes to a polling state, unless it is the leader process. Here is the algorithm for process \(p\):

initialization();
while (true)
{ if \((p \neq \text{next\_leader})\) polling_state();
 else \{current_node=guess(current_node);
        next_leader=guessp(current_node);\}
        execute_path(current_node); \}

The algorithm execute\_path(current_node) above makes that process \(p\) executes its events from current_node, if any. In this case each message sent by \(p\) contains the additional data \((current\_node, next\_leader)\).

**Theorem 7** [15] Every \(le\)-MSG \(G\) is implementable by a deadlock-free CFM with additional data which is of size linear in \(|G|\).

Note that in a triangle (see definition 2), every process but the minimal process begins by a receive. A process that is chosen to be the leader is always informed, since it occurs in the node where it is chosen.

![Fig. 8. Le-MSG not implementable without additional data.](image)

It is important to see that if additional data is forbidden, then there are \(le\)-MSG that are not implementable, even when allowing deadlocks. Consider
for example an lc-MSG with three nodes 1, 2, 3, see figure 8. The initial node consists of a message from process \( p_1 \) to \( p_2 \). Then either node 2 is executed, with process \( p_1 \) sending to process \( p_R \), or node 3 is executed with process \( p_2 \) sending to process \( p_L \). Since processes \( p_1 \) and \( p_2 \) do not know which one will be next (no additional data, same past), both can begin, thus both nodes 2 and 3 can start. The execution must stop, and there is no distributed way to know whether the protocol went fine or not. Hence without additional data this protocol is not implementable at all. [19] proposes a sufficient condition for the standard-implementability of lc-MSGs.

While local-choice is defined syntactically, we show that it corresponds to a semantic property. Moreover, one can test whether an MSG is transformable into a lc-MSG. Triangles are of huge importance here. We first define a generic lc-MSG \( H_n \) over triangles of size bounded by \( n \). The MSG \( H_n \) has for each triangle \( T \) of size at most \( n \), one node \( v_T \) labeled by \( T \). There is an edge \( v_T \rightarrow v_{T'} \) if \( \Pr(\min(T')) \in \Pr(T) \).

**Proposition 1** [14] An MSG \( G \) is equivalent to some lc-MSG iff there exists some \( n \) such that \( \mathcal{L}(G) \subseteq \mathcal{L}(H_n) \). If this is the case, then we can obtain a lc-MSG equivalent to \( G \), of size exponential in \( |G| \) and \( n \).

If the test in proposition 1 on \( G \) answers yes, then an equivalent lc-MSG can be constructed by synchronizing \( G \) and \( H_n \).

While it is PSPACE to test whether \( \mathcal{L}(G) \subseteq \mathcal{L}(H_n) \) by theorem 9, the value of \( n \) is not bounded so far. For testing, we need a bound. We use for this the following three structural properties of lc-MSG \( G \).

1. Every MSC \( M \) in \( \mathcal{L}(G) \) is a triangle.
2. There is a bound \( b \) s.t. for every MSC in \( \mathcal{L}(G) \) containing a factor \( (U||V) \) with \( U, V \) MSCs (that is, \( U, V \) share no process), either \( |U| < b \) or \( |V| < b \). This implies that for an MSG to be equivalent to some lc-MSG, it is necessary to be a gc-MSG.
3. There is a bound \( b \) s.t. for every MSC in \( \mathcal{L}(G) \) of the form \( URV \) with \( R \) an MSC of size at least \( b \), there exists triangle \( T \) that is a suffix of \( RV \), such that \( \min(T) \) belongs to \( R \).

Obviously, an MSG \( G \) that is equivalent to some lc-MSG satisfies these three properties. The important point is that the converse holds, too. It allows us to state:

**Theorem 8** [14] Testing whether an MSG \( G \) is equivalent to some lc-MSG is in PSPACE. Moreover, if the answer is positive, then an equivalent lc-MSG of doubly exponential size can be constructed.

**Proof.** We can check whether \( G \) is a gc-MSG in co-NP [15]. Checking the first property above is in polynomial time. Checking the second property for gc-MSG is in co-NP. If true, the test provides a bound \( b \) that is polynomial in \( |G| \).
Checking the third property for gc-MSGs is in PSPACE. If true, the test provides a bound $b$ that is exponential in $|G|$.

We can then compute an equivalent lc-MSG building the product $H_b \times \text{Lin}^*(G)$. As $b$ is exponential in $|G|$ and $H$ is exponential in $b$, the result is at most doubly exponential in $|G|$. \hfill \Box

One important question is whether local-choice is expressive enough, else the test to know whether an MSG is equivalent to some lc-MSG would almost certainly lead to a negative answer. Comparing lc-MSGs to regular MSGs, lc-MSGs tend to be more useful in practice. In particular, the restriction of universally-bounded channels of regular MSGs is not required for lc-MSGs. Moreover, lc-MSGs can be implemented without deadlock, while this is not the case for regular MSGs. A drawback of lc-MSGs is the fact that they exclude long parallel MSCs, while this is possible with regular MSGs (albeit not in the same loop of the graph). Actually, it would not be difficult to cut a protocol into parallel ones, and implement each one using lc-MSGs.

Since lc-MSGs form a subclass of gc-MSGs, one can hope that they are easier to model-check than gc-MSGs. In order to improve the model-checking algorithm, triangles can be used as generators instead of atoms. For a given lc-MSG each node $v$ labeled by a triangle $T$ can be sliced into two nodes labeled by triangles $R, S$, as long as $T = RS$ satisfies $pr(\text{min}(w)) \in S$ for every $v \rightarrow w$. Notice that by the definition of a triangle, we have that $pr(\text{min}(S)) \in R$. Let $T_1 \cdots T_n, T'_1 \cdots T'_{n'}$ be sequences of triangles labeling two paths $\rho, \rho'$ in lc-MSGs $G, H$ sliced in this way. Then there exist $k, X$ s.t. $T_i = T'_i$ for all $i < k$, and $T_k = XT_{k+1} \cdots T_n, T_k = XT'_k \cdots T'_{n'}$. Hence, $T_{k+1} \cdots T_n$ is smaller than the largest node of $G$. The same applies for $T'_{k+1} \cdots T'_{n'}$. This idea allows to do model-checking very similarly to word automata.

**Theorem 9** [15] Given two lc-MSGs $G, H$, the negative model-checking question $\mathcal{L}(G) \cap \mathcal{L}(H) = \emptyset$ can be answered in quadratic time. The positive model-checking question $\mathcal{L}(G) \subseteq \mathcal{L}(H)$ with $H$ an lc-MSG and $G$ an arbitrary MSG, is PSPACE-complete.

### 8 Conclusions

The MSC/MSG standard is a popular notation for concurrent system specification, in particular for communication protocols. Stemming from its successful use by software engineers, new techniques and tools have been developed for MSC/MSG analysis. The finite states model was designed by researchers. Although this model has many mathematical properties, it is not always easy to transfer its related technology to the software developers. The MSC notation, on the other hand, has gained first popularity with the software developers. Consequently, this notation does not fit directly the main classes of formal languages. This calls for studying the expressiveness of the notation and developing new validation and implementation methods.
It is evident from the collection of results surveyed here that one of the main challenges in studying MSCs/MSGs is how to achieve the appropriate expressiveness, while maintaining decidability with respect to automatic verification. This calls for developing various extensions and restrictions on the allowed class of MSCs/MSGs.

The MSC/MSG standard provides an alternative for the communicating automata model. In particular, the main compositional operator for the former is sequential composition, while the main way to connect communicating automata is using parallel composition. Although sequential composition is often considered simpler than the parallel one, it is evident that this is not the case here. The reason is that the sequential composition is asynchronous, relating partial orders. In particular, the parallel composition of two MSCs (i.e., that share no process) is expressed when we compose them sequentially (as is the case in classical Mazurkiewicz trace theory [1]). This is also manifested by the high complexity results on MSG decision procedures. Note however that subclasses as lc-MSGs have the same complexity as finite-state machines.

The theory of MSCs is related to models of true concurrency, including partial orders and Mazurkiewicz traces. While these theories flourished in the recent decades, their practical use was limited, due to the high complexity they generally possess, when compared to the finite-state machine model. The MSC model provides an important use of these true concurrency models. The intuitive nature of these models is manifested by the use of the MSC as a popular visual notation for concurrency.

References


