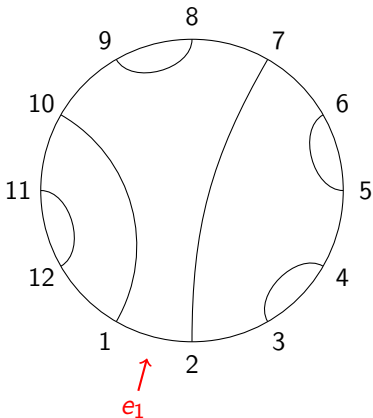


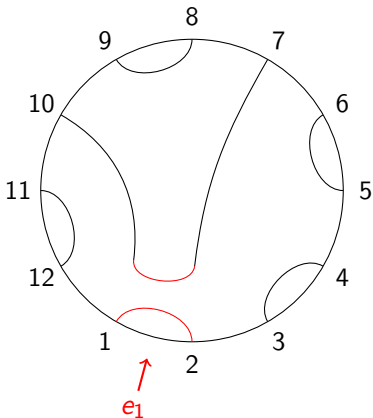
# Opérateur $e_1$

Un couplage de taille 6 (six cordes — *links*) :



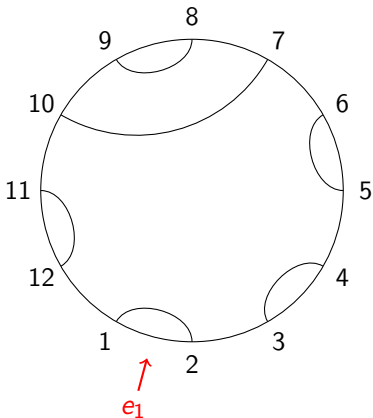
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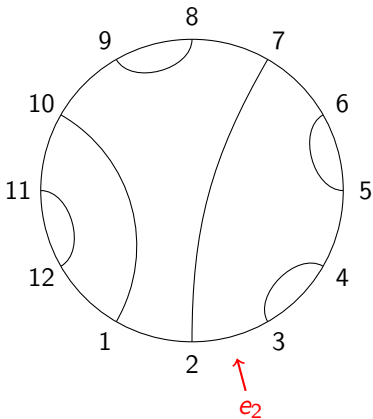
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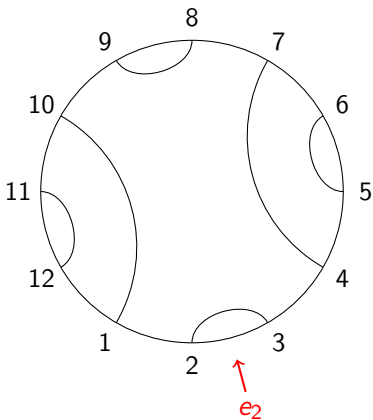
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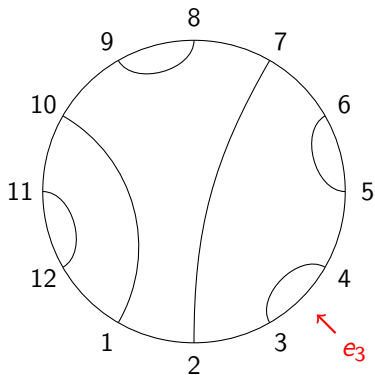
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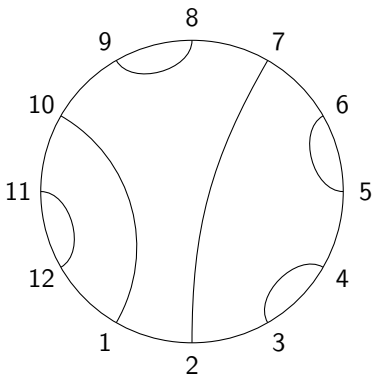
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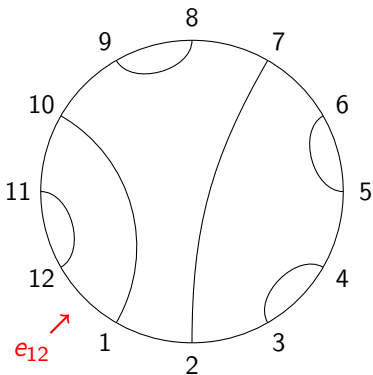
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etc.

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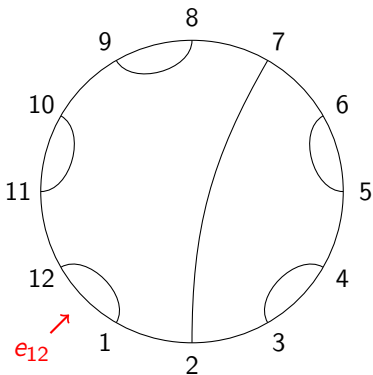


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# Relations

$2n$  opérateurs  $e_i$  sur un couplage de taille  $n$  :

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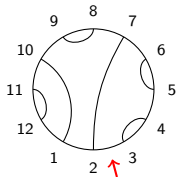
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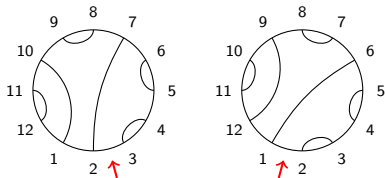
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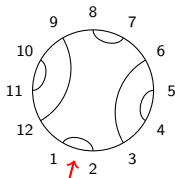
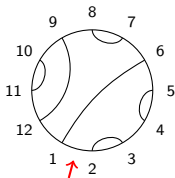
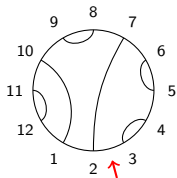
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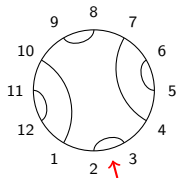
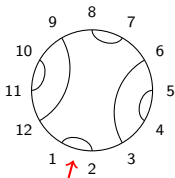
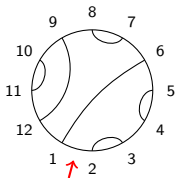
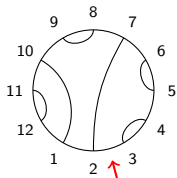
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# Hamiltonien

$$H = \sum_{i=1}^{2n} e_i$$

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Exemple de calcul :

$$H \left| \begin{array}{c} \text{circle with 4 dots} \end{array} \right\rangle = 4 \left| \begin{array}{c} \text{circle with 4 dots} \end{array} \right\rangle + \left| \begin{array}{c} \text{circle with 4 dots and 2 diagonal lines} \end{array} \right\rangle + \left| \begin{array}{c} \text{circle with 4 dots and 2 diagonal lines} \end{array} \right\rangle + \left| \begin{array}{c} \text{circle with 4 dots and 2 diagonal lines} \end{array} \right\rangle + \left| \begin{array}{c} \text{circle with 4 dots and 2 diagonal lines} \end{array} \right\rangle + \left| \begin{array}{c} \text{circle with 4 dots and 2 diagonal lines} \end{array} \right\rangle$$

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Mathématicien ringard :  $H(\pi)$

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$\lambda$ -Informaticien :  $H\pi$

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et bien sûr par rotation :

$$H \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots} \end{array} \right\rangle = 4 \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots} \end{array} \right\rangle + \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots and 1 line} \end{array} \right\rangle + \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots and 1 line} \end{array} \right\rangle + \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots and 1 line} \end{array} \right\rangle + \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots and 1 line} \end{array} \right\rangle + \left| \begin{array}{c} \text{Sphere} \\ \text{with 4 dots and 1 line} \end{array} \right\rangle$$

## Exemple de calcul (suite)

En numérotant les couplages :

$$H|\pi_1\rangle = 4|\pi_1\rangle + |\pi_4\rangle + |\pi_6\rangle + |\pi_8\rangle + |\pi_{10}\rangle$$

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En posant :

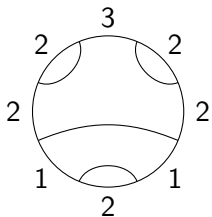
$$|\Pi_1\rangle = |\pi_1\rangle + |\pi_2\rangle, \quad |\Pi_2\rangle = |\pi_3\rangle + \dots + |\pi_{10}\rangle$$

on obtient simplement :

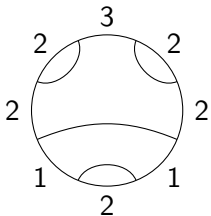
$$H|\Pi_1\rangle = 4|\Pi_1\rangle + |\Pi_2\rangle$$



## Exemple de calcul (fin)

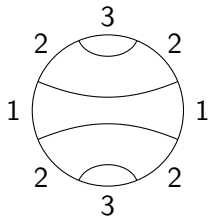
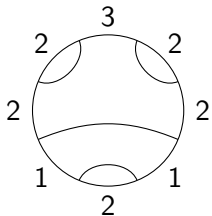


## Exemple de calcul (fin)



$$\begin{aligned} H|\Pi_2\rangle &= \frac{8 \times 2}{2} |\Pi_1\rangle + \frac{8 \times 5}{8} |\Pi_2\rangle + \frac{8 \times 1}{4} |\Pi_3\rangle \\ &= 8|\Pi_1\rangle + 5|\Pi_2\rangle + 2|\Pi_3\rangle \end{aligned}$$

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## Les FPL attaquent

Soit  $\Psi(\pi)$  le nombre de FPL de couplage  $\pi$  :

$$|s\rangle = \sum_{|\pi|=n} \Psi(\pi) |\pi\rangle$$

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Ce vecteur est la *projection* (par couplage) de :

$$||s\rangle\rangle = \sum_{|\Phi|=n} ||\Phi\rangle\rangle$$

où  $\Phi$  désigne un FPL.

# Razumov Stroganov

Enoncé :

$$H|s\rangle = 2n|s\rangle$$

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Formule condensée pour  $n = 4$  :

$$\begin{bmatrix} 4 & 8 & 4 \\ 1 & 5 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix} = 8 \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$



# RS : Rotations et Symétrisation

Définition :

$$S = 1 + R + R^2 + \dots + R^{2n-1}$$

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On peut bien sûr remplacer  $e_1$  par n'importe quel  $e_i$  pour  $i$  fixé.

## RS : exemple

Comparer :

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avec :

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d'où en ajoutant :

$$H|\Pi_1\rangle = S e_1|\Pi_1\rangle$$

et ainsi de suite.

# Action de $e_1$ sur les couplages de taille 4

$$\pi_1 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \circlearrowright \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_2 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \circlearrowright \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array}$$

---

$$\pi_3 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_4 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_5 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_6 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array}$$

$$\pi_7 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_8 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_9 = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_{10} = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array}$$

---

$$\pi_{11} = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_{12} = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_{13} = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array} \quad \pi_{14} = \begin{array}{c} 6 \quad 5 \\ \circlearrowleft \\ 7 \quad 4 \\ \text{---} \\ 8 \quad 3 \\ \circlearrowleft \\ 1 \quad 2 \end{array}$$



