Chapter 5

Logic Design with MSI Components and Programmable Logic Devices

The complexity of a chip

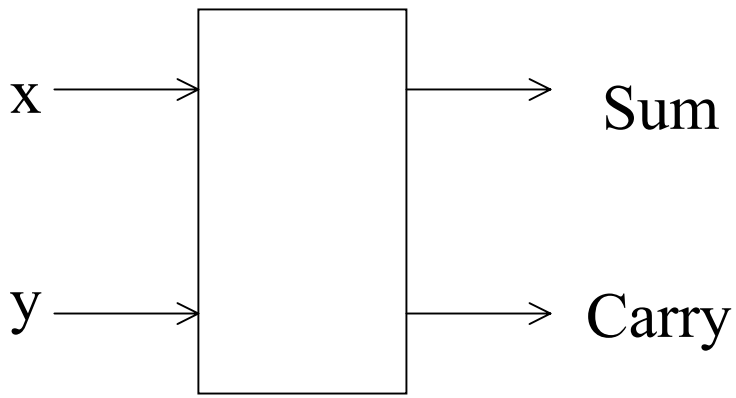
Scale of integration:

- SSI 1 - 10 gates
- MSI 10 - 100 gates
- LSI 100 - 1000 gates
- VLSI > 1000 gates

Specialized MSI components

- adders
- comparators
- encoders/decoders
- multiplexers/demultiplexers

Half Adder

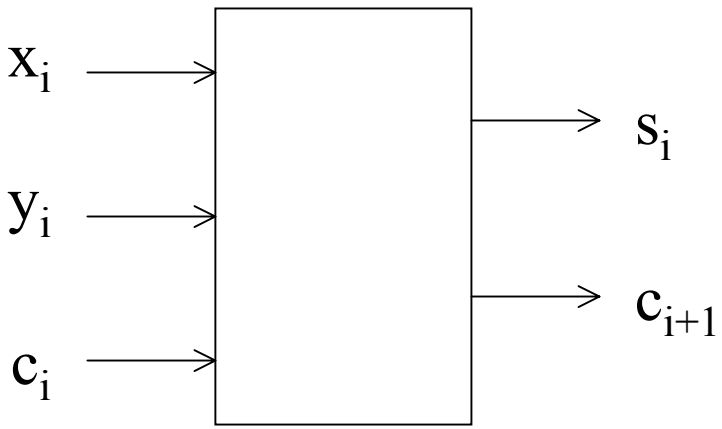


x	y	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

sum = x'y + xy'

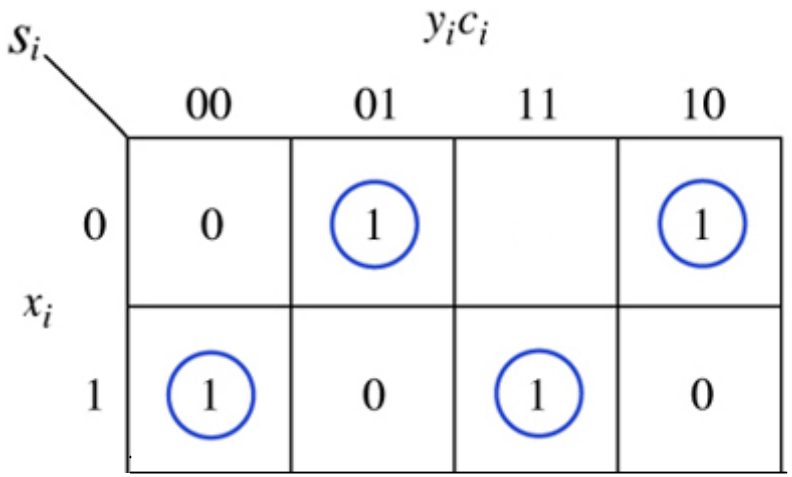
carry = xy

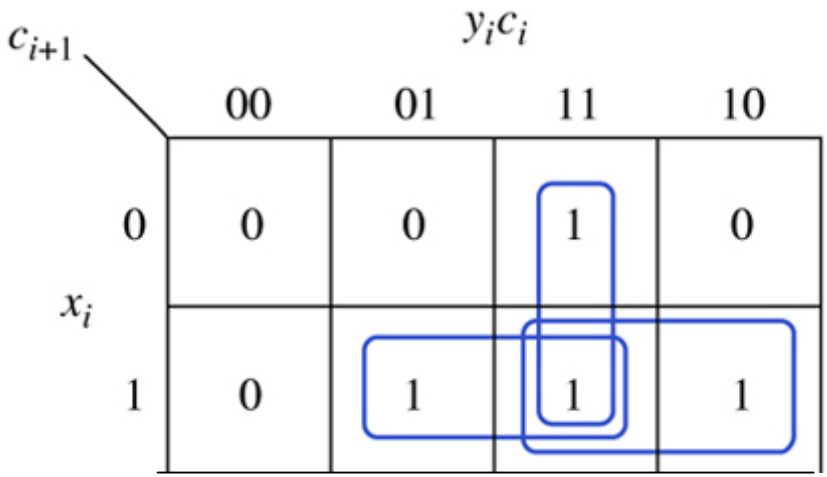
Full Adder



x_i	y_i	c_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

The Karnaugh maps for a full adder



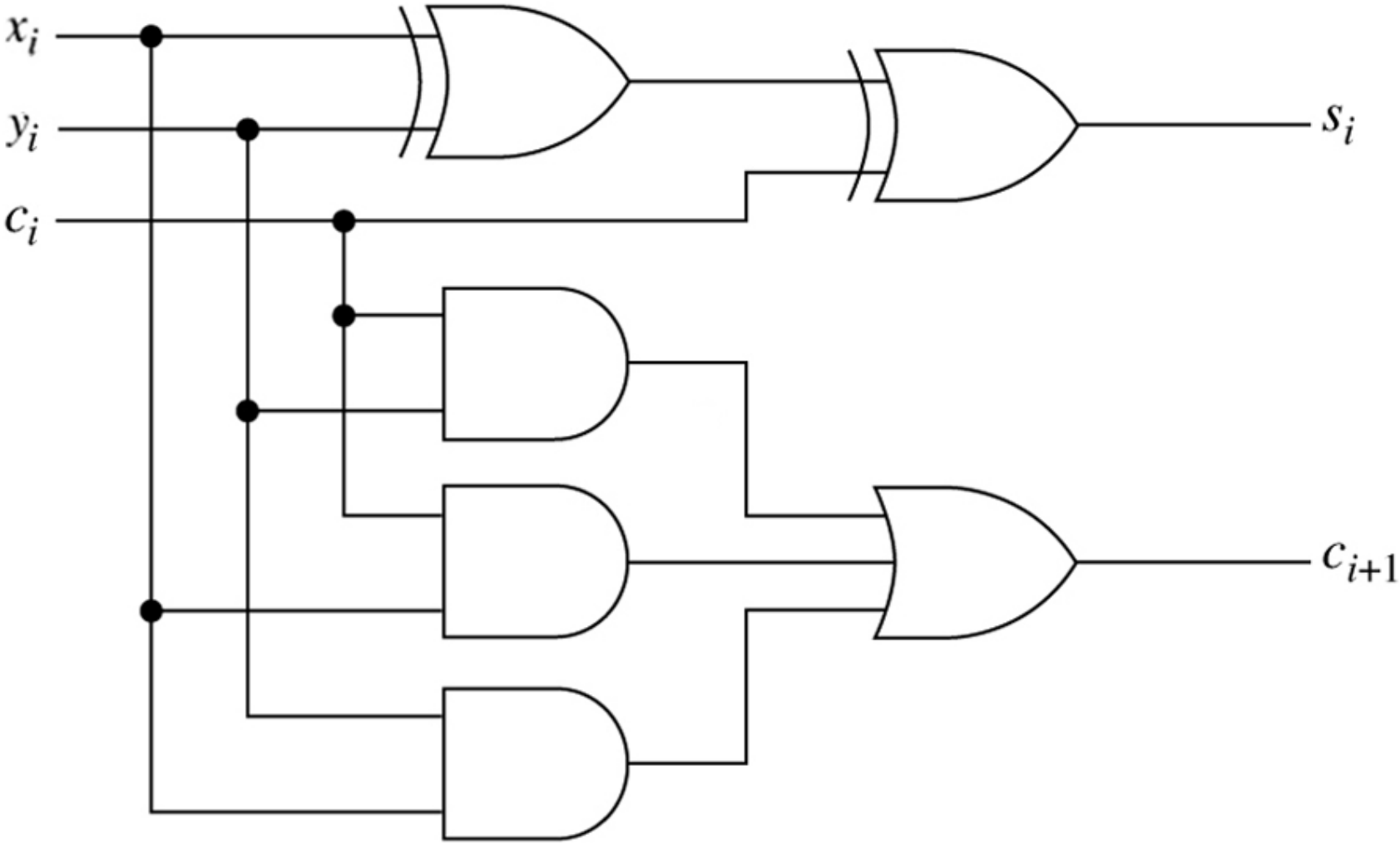


SUM and CARRY functions

SUM = x'y'c + x'yc' + xy'c' + xyc

CARRY = xy + yc + cx

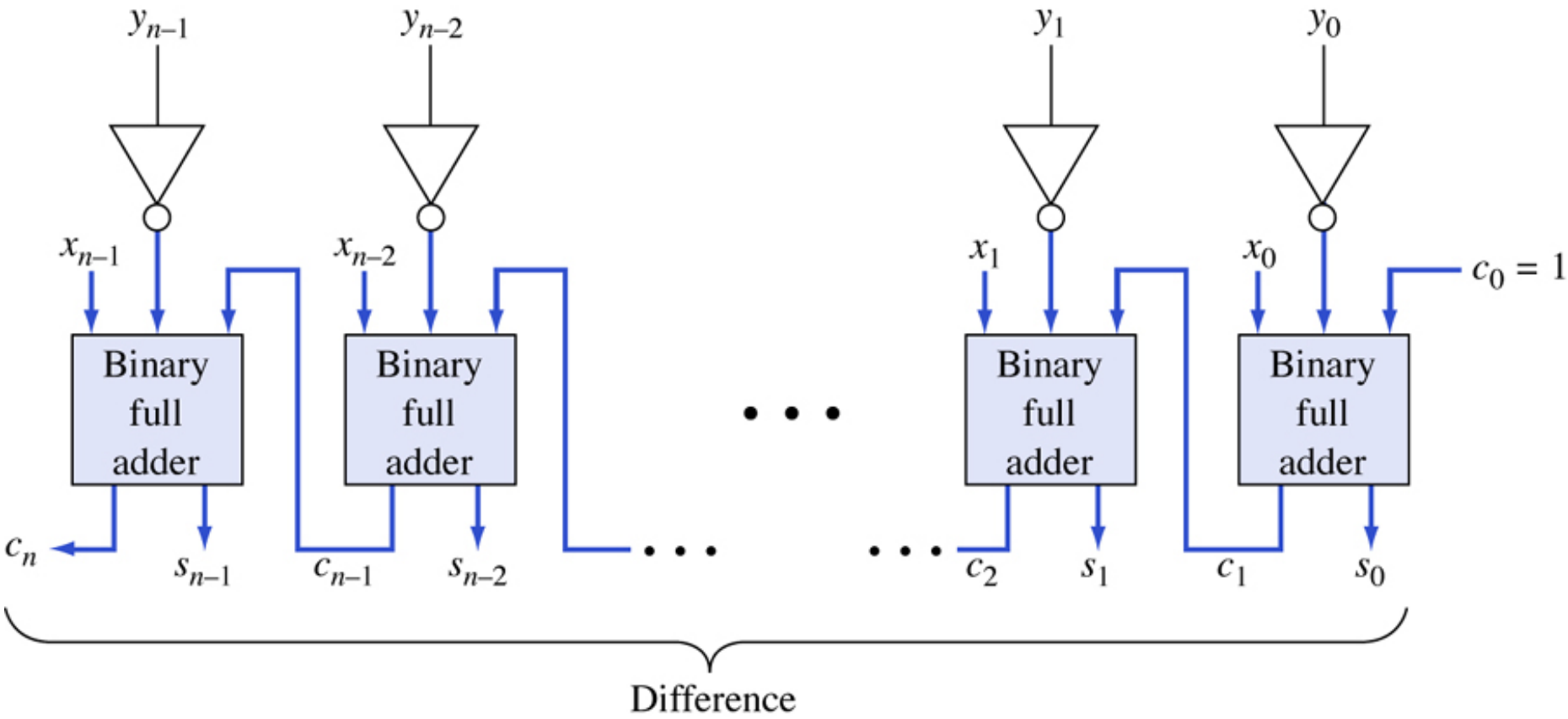
A realization of the binary full adder



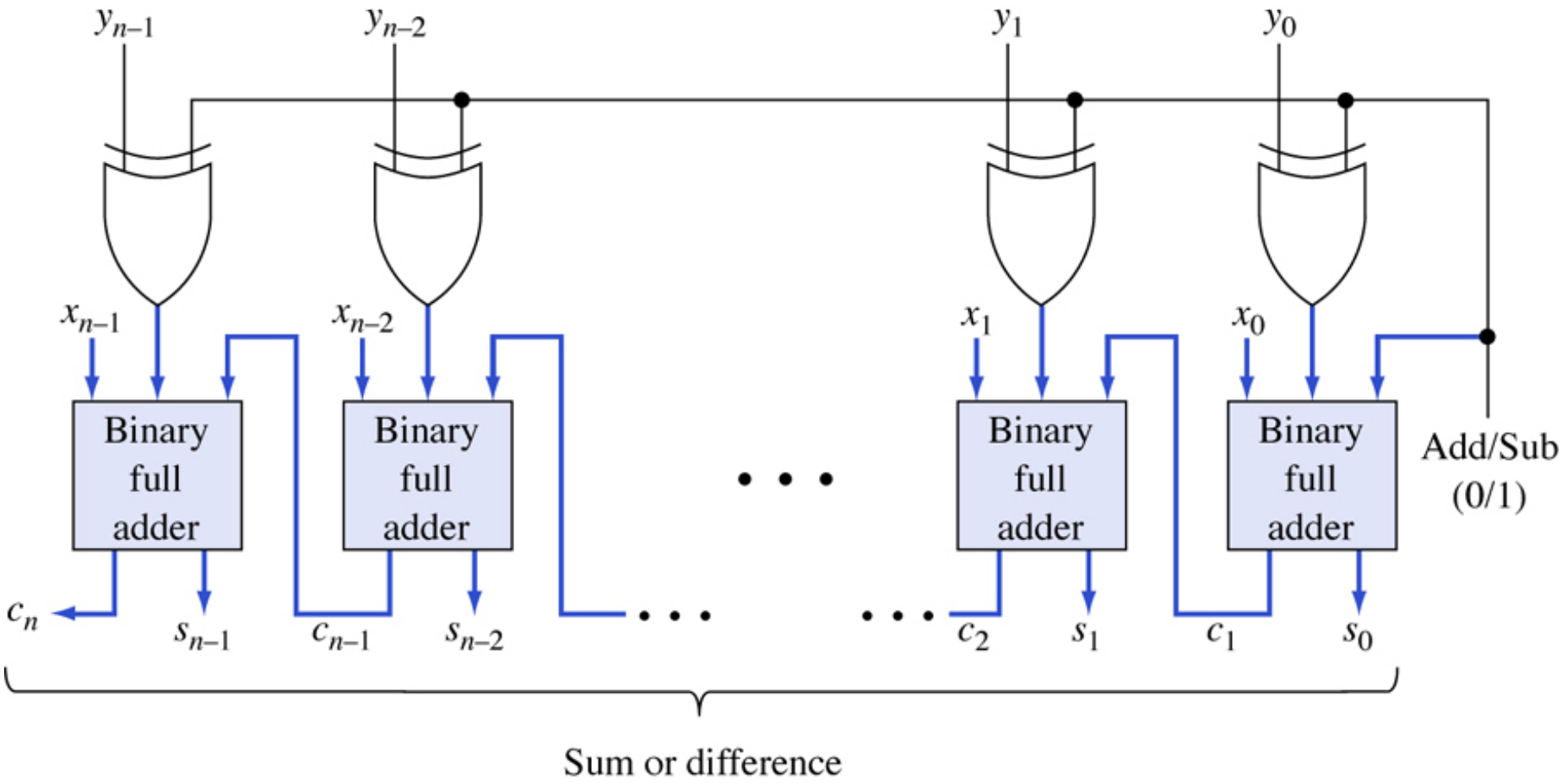
Parallel (ripple) binary adder

Designed to add two binary numbers bit by bit

Parallel binary subtracter constructed by using a parallel binary adder



Parallel binary adder/subtracter



Carry-look-ahead adder

- Problem: the time required to do addition is proportional to the number of bits involved.
- Solution: compute the carry for each stage independently by using a carry-look-ahead network.

Carry-Look-ahead Adder

Recall that

$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i = x_i y_i + (x_i + y_i) c_i$$

Let $g_i = x_i y_i$ be the *carry-generate* function, and

$p_i = x_i + y_i$ be the *carry-propagate* function.

Then we can write $c_{i+1} = g_i + p_i c_i$ and

$$c_1 = g_0 + p_0 c_0$$

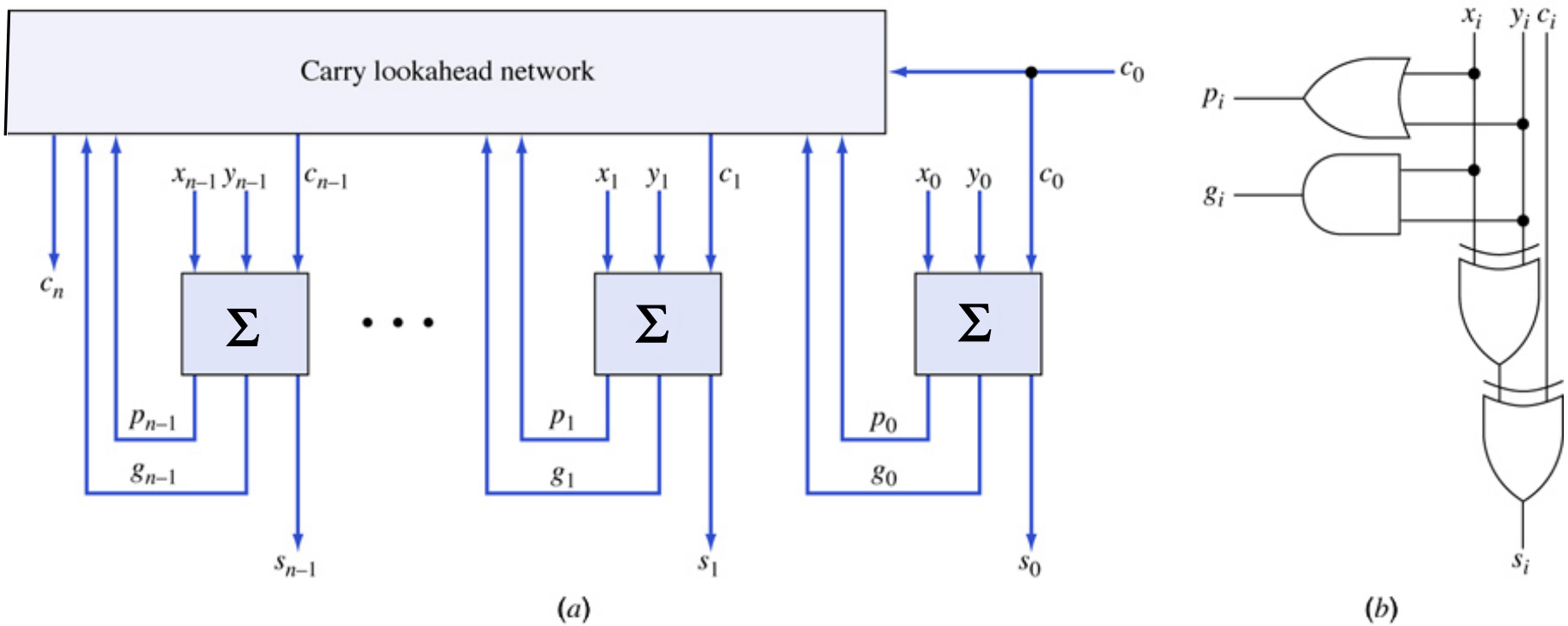
$$c_2 = g_1 + p_1 g_0 + p_1 p_0 c_0$$

$$c_3 = g_2 + p_2 g_1 + p_2 p_1 g_0 + p_2 p_1 p_0 c_0$$

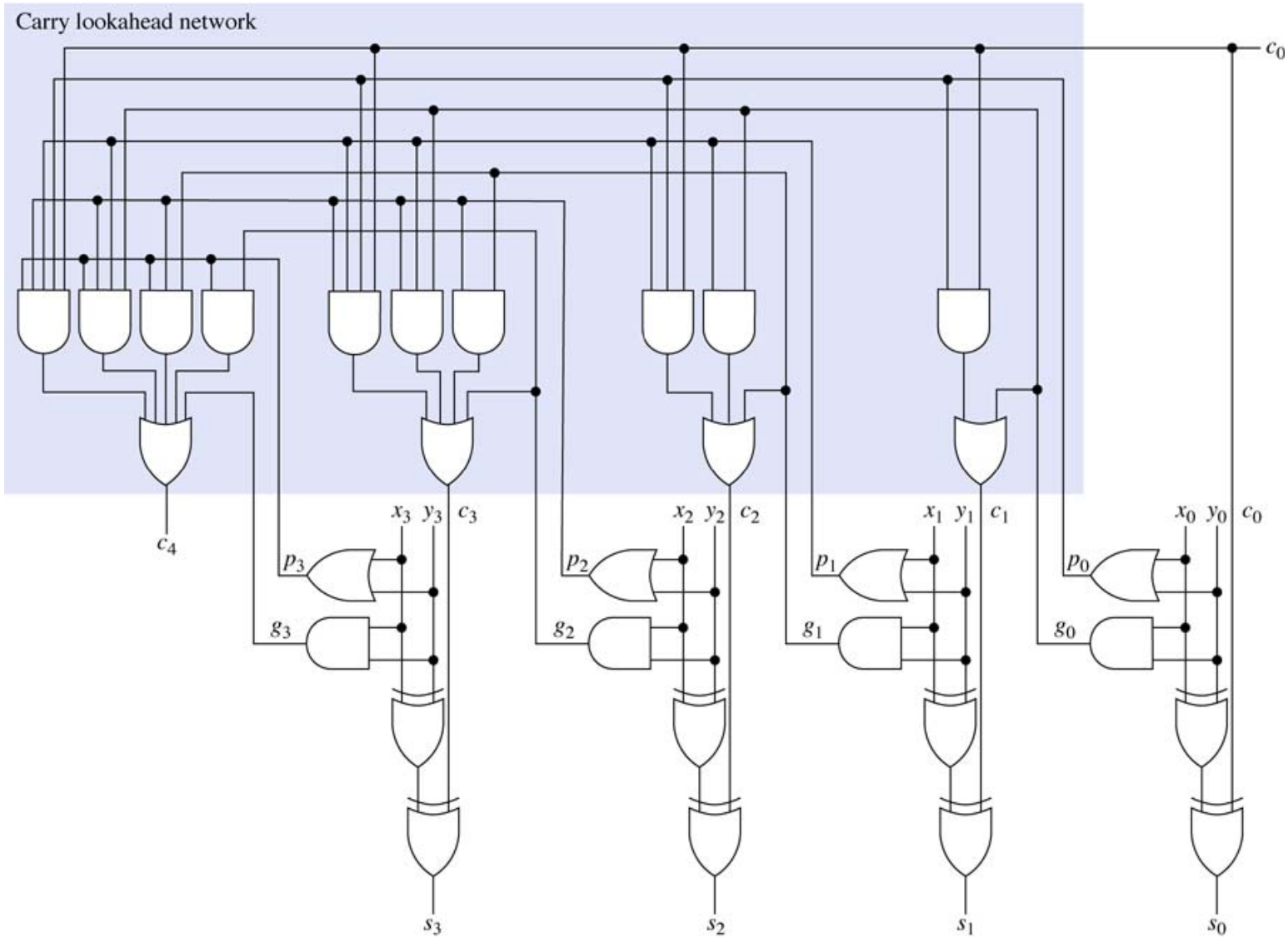
…

We see that all carry signal c_i can be computed by a two level logic circuit.

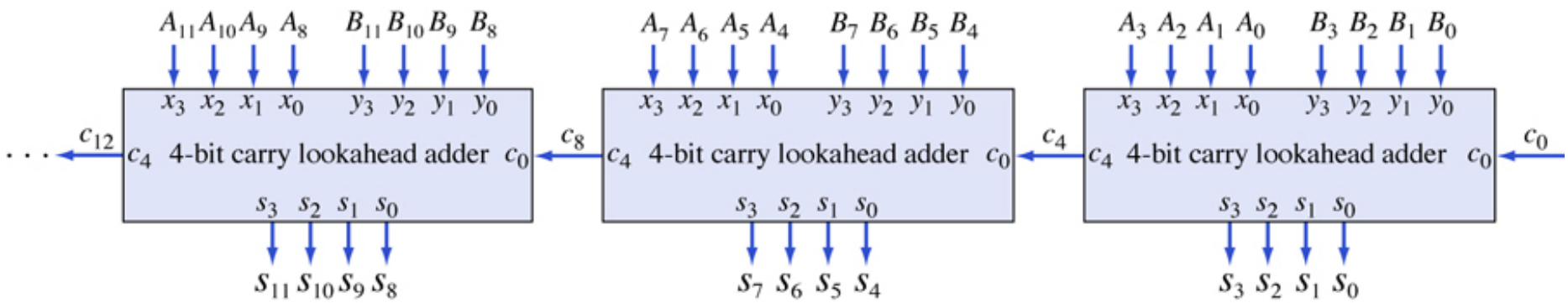
A carry lookahead adder. (*a*) General organization. (*b*) Sigma block



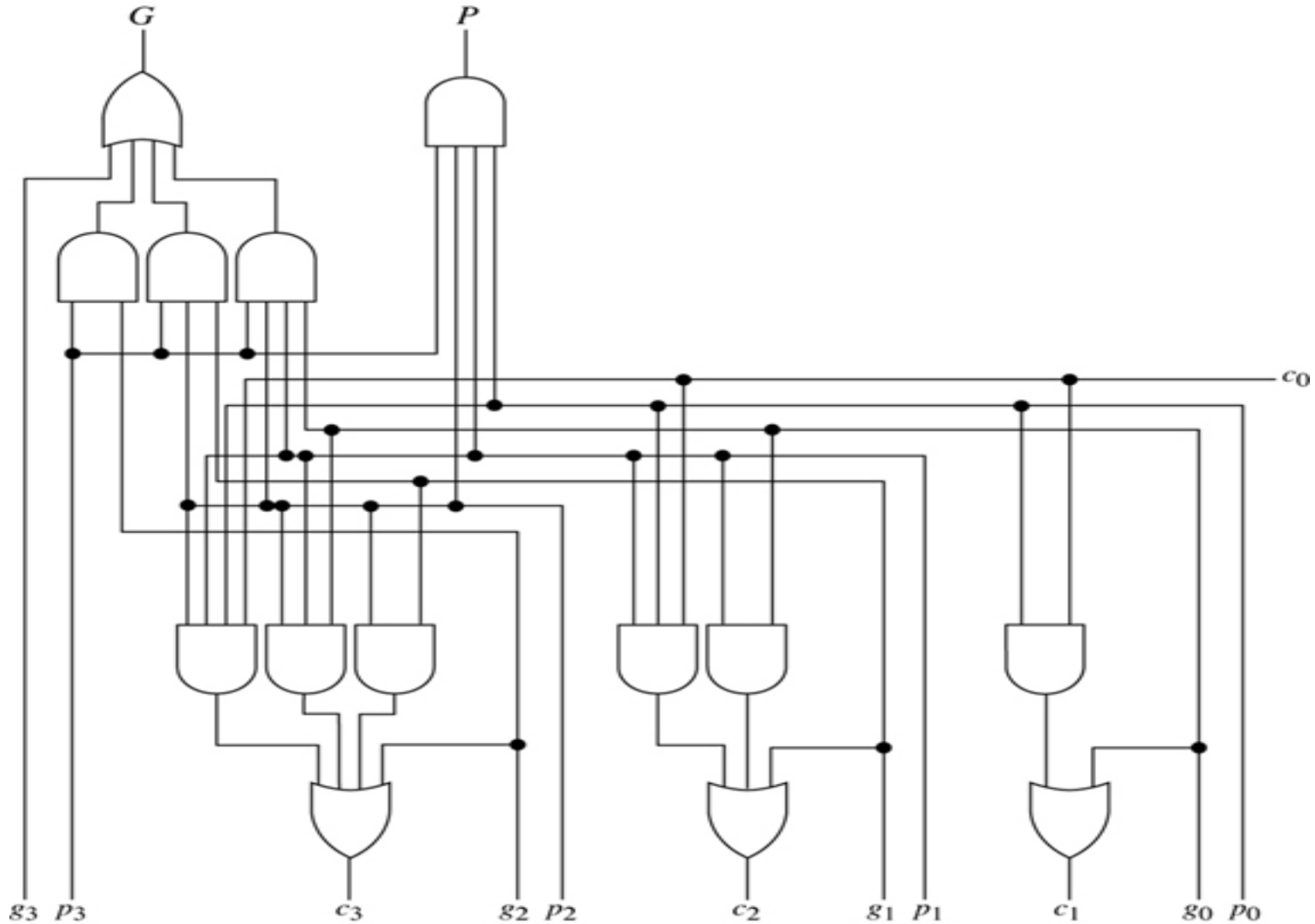
A 4-bit carry lookahead adder



Cascade connection of 4-bit carry lookahead adders

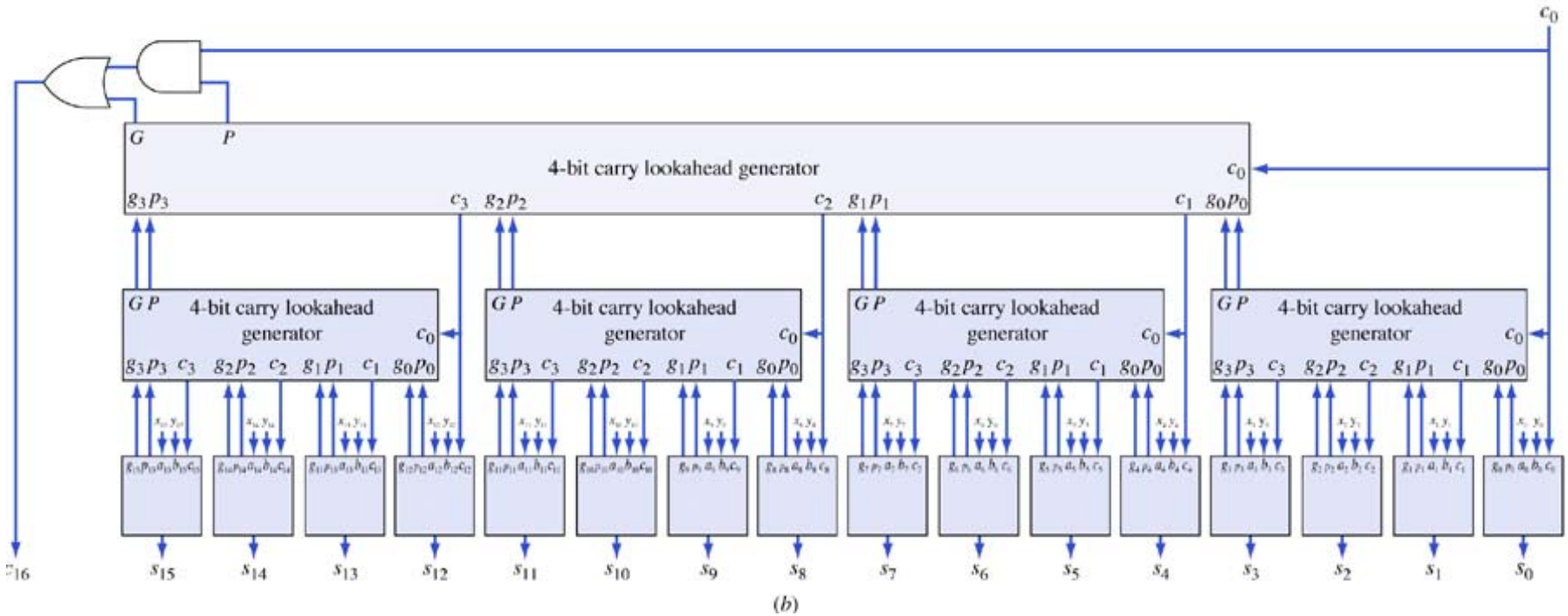


A 4-bit carry look-ahead generator



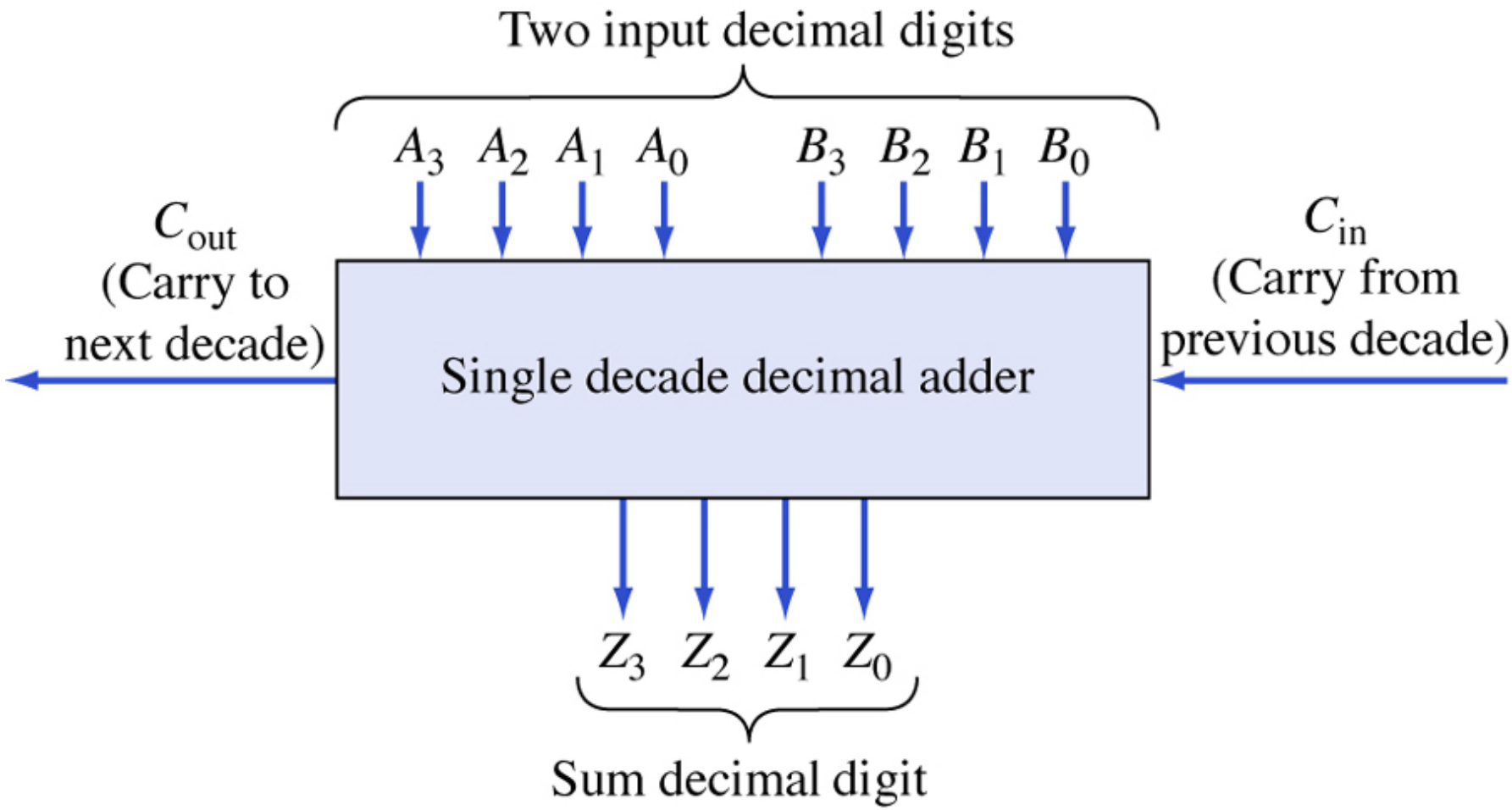
(a)

A 16-bit high-speed adder

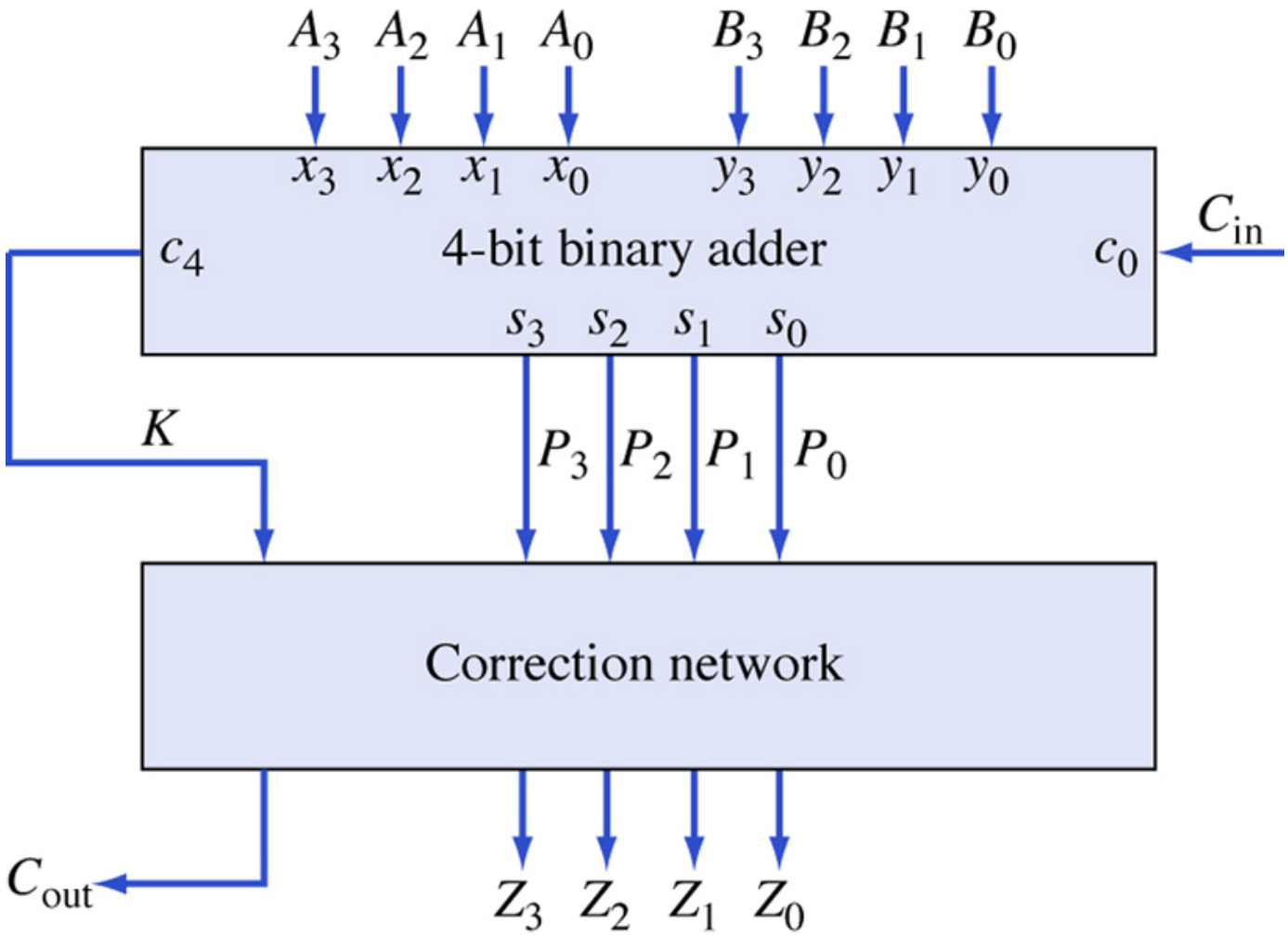


(b)

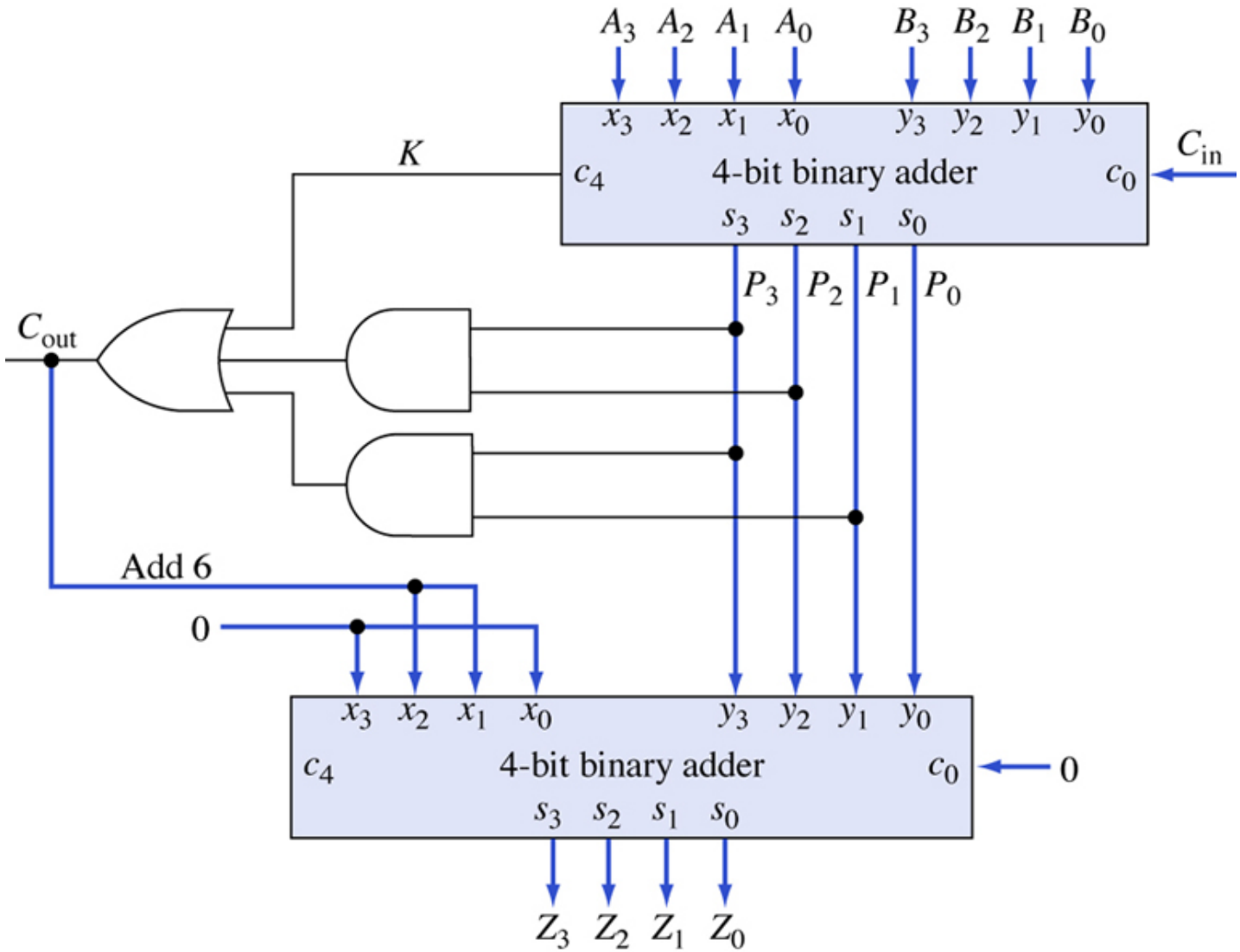
Organization of a single-decade decimal adder



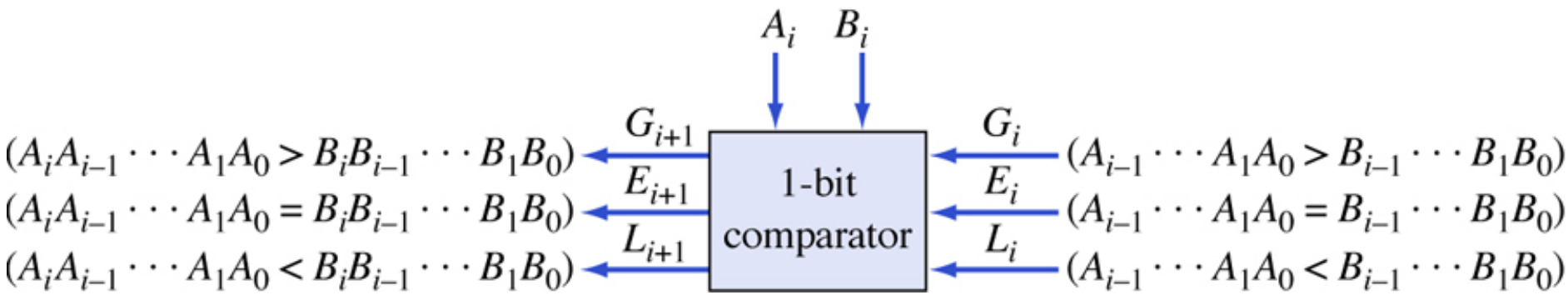
Organization of a single-decade BCD adder



A single-decade BCD adder



Organization of a 1-bit comparator



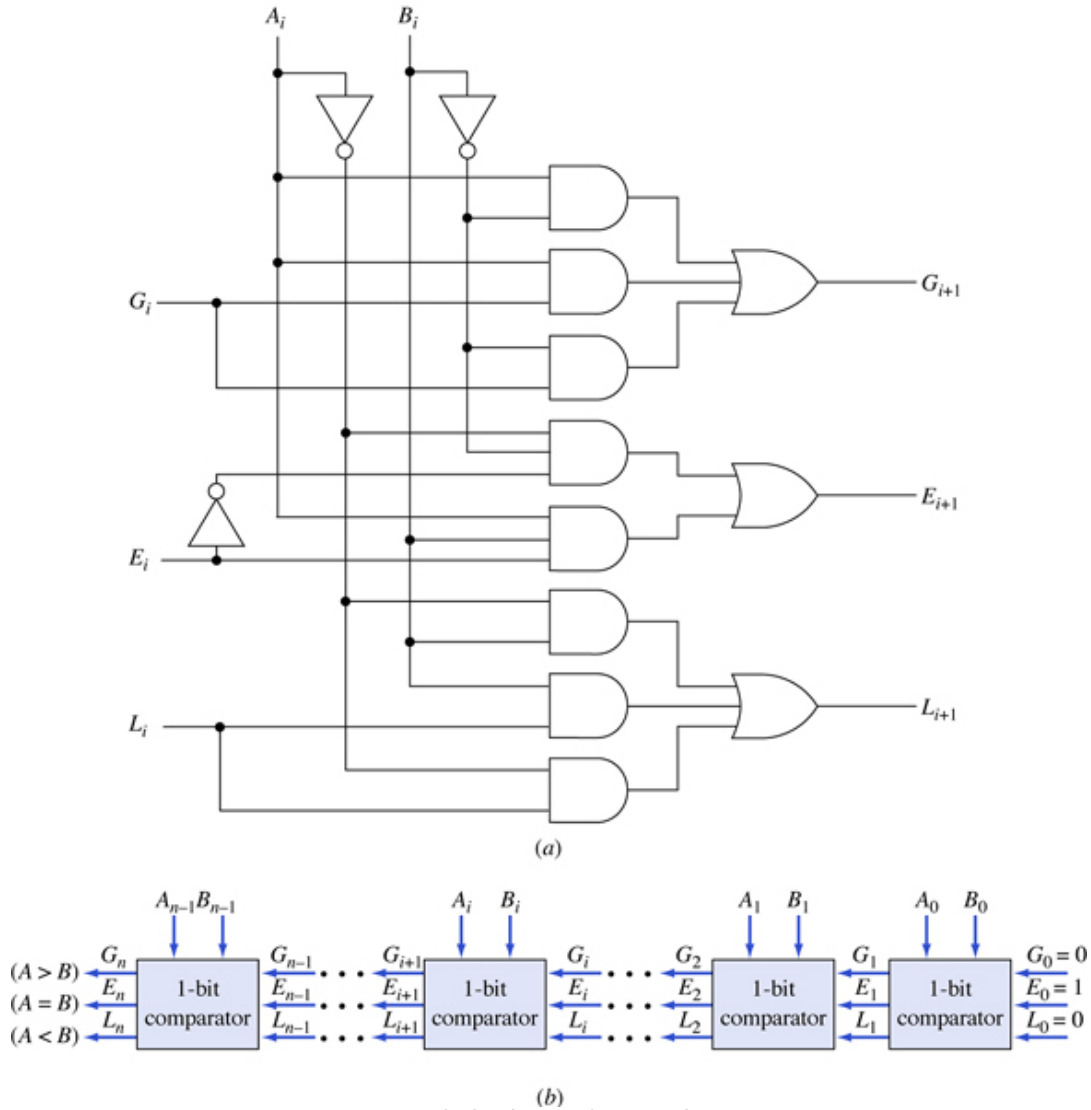
From the truth table (Table 5.4) on page 247 we obtained

$$G_{i+1} = A_i B'_i + A_i G_i + B'_i G_i$$

$$E_{i+1} = A'_i B'_i E_i + A_i B_i E_i$$

$$L_{i+1} = A'_i B_i + B_i L_i + A'_i L_i$$

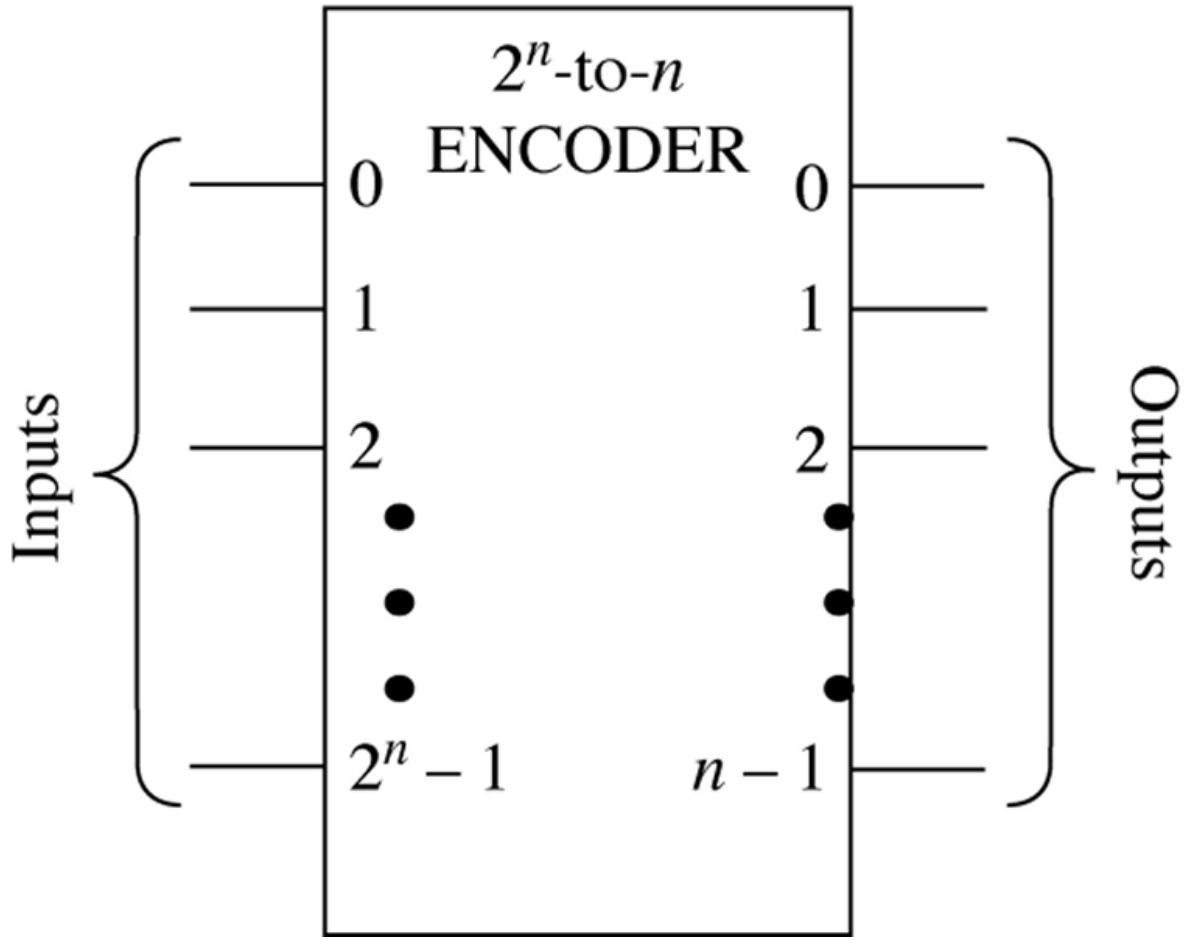
Comparing two binary numbers A and B. (a) 1-bit comparator network. (b) Cascade connection of 1-bit comparators.



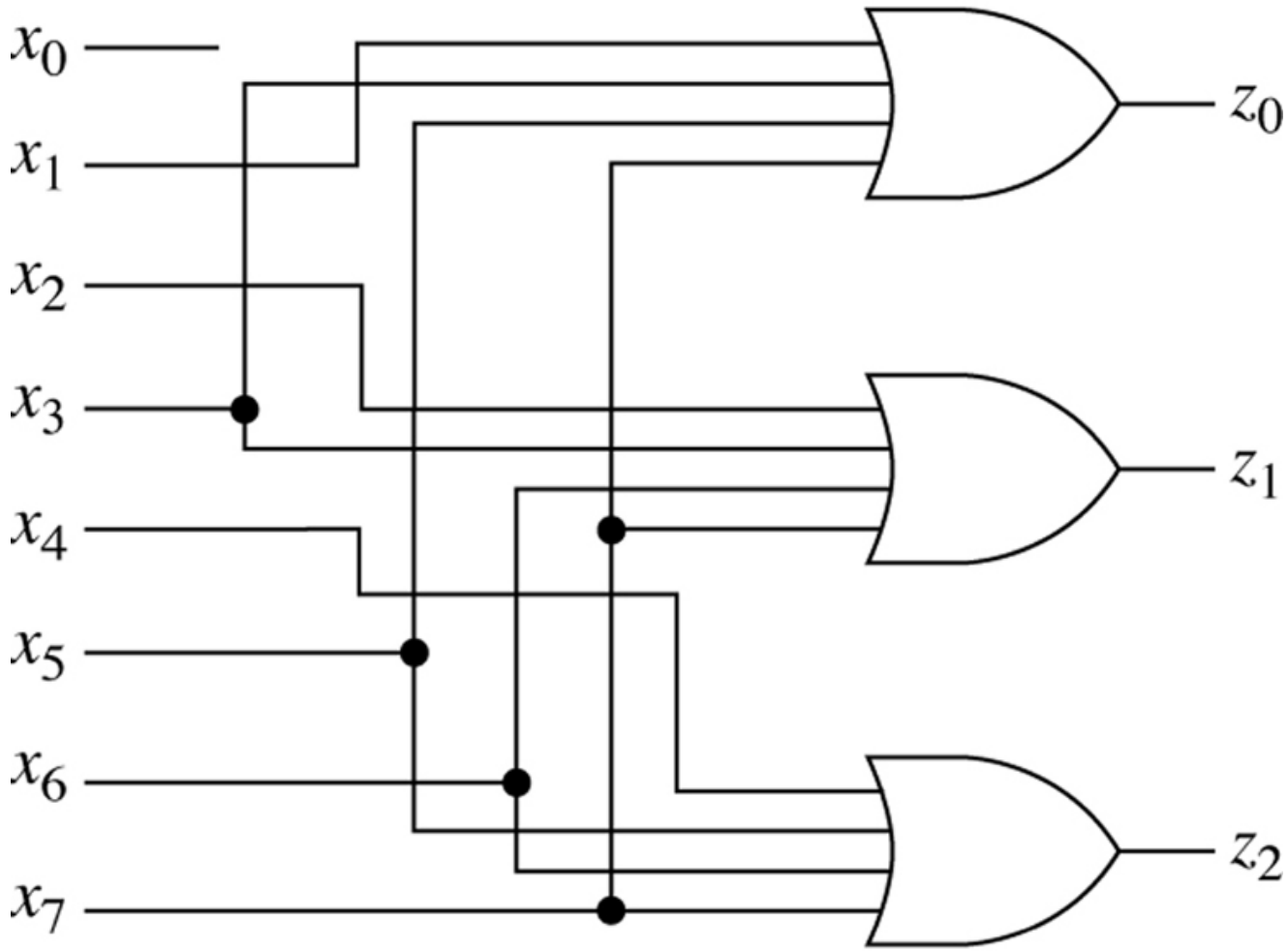
Main function of encoder and decoder

The purpose is to reduce the number of wires required for interconnection.

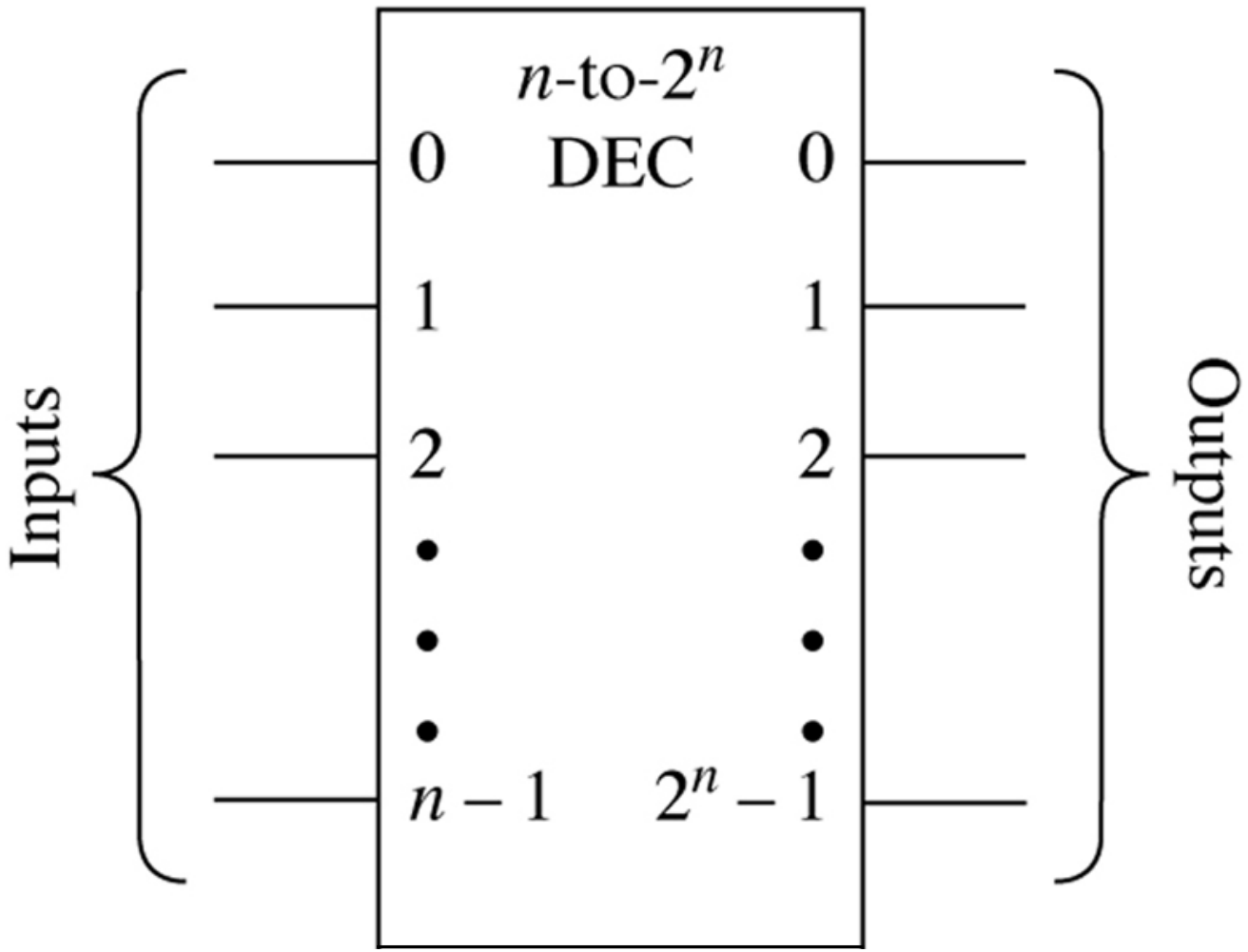
A 2^n-to-n-line encoder symbol



An 8-to-3-line encoder



Symbol for an n-to-2^n-line decoder

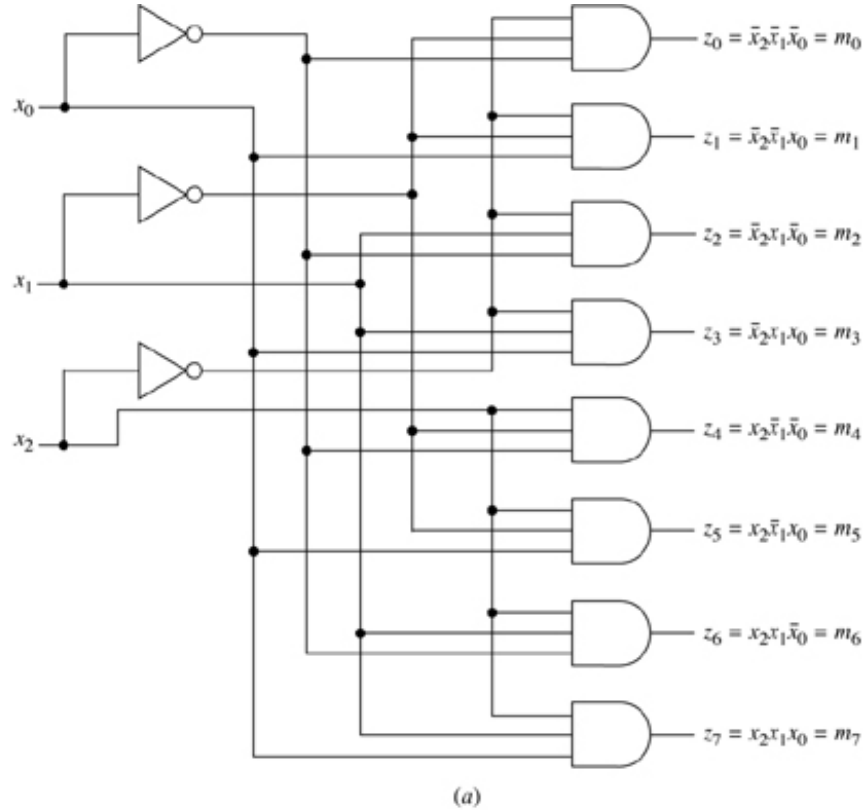


A 3-to-8-line decoder

(a) Logic diagram.

(b) Truth table.

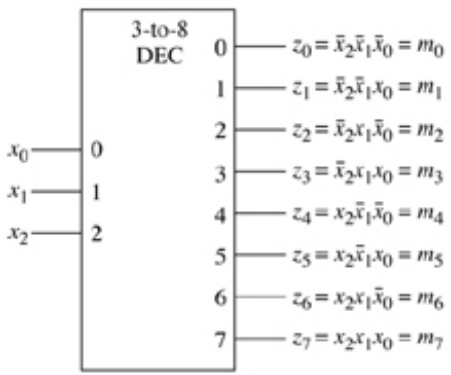
(*c*) Symbol



(*a*)

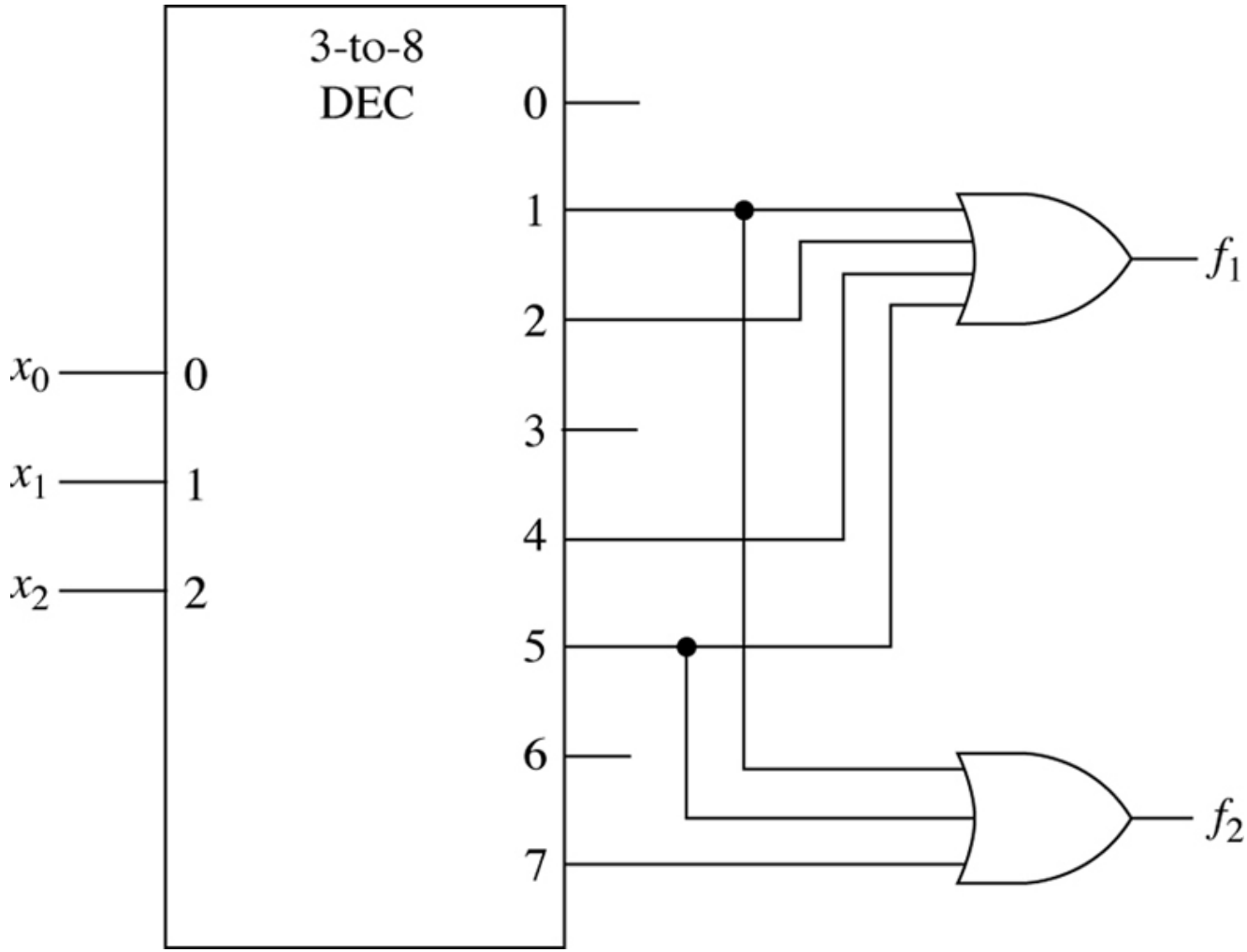
Inputs			Outputs							
x_2	x_1	x_0	z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

(*b*)

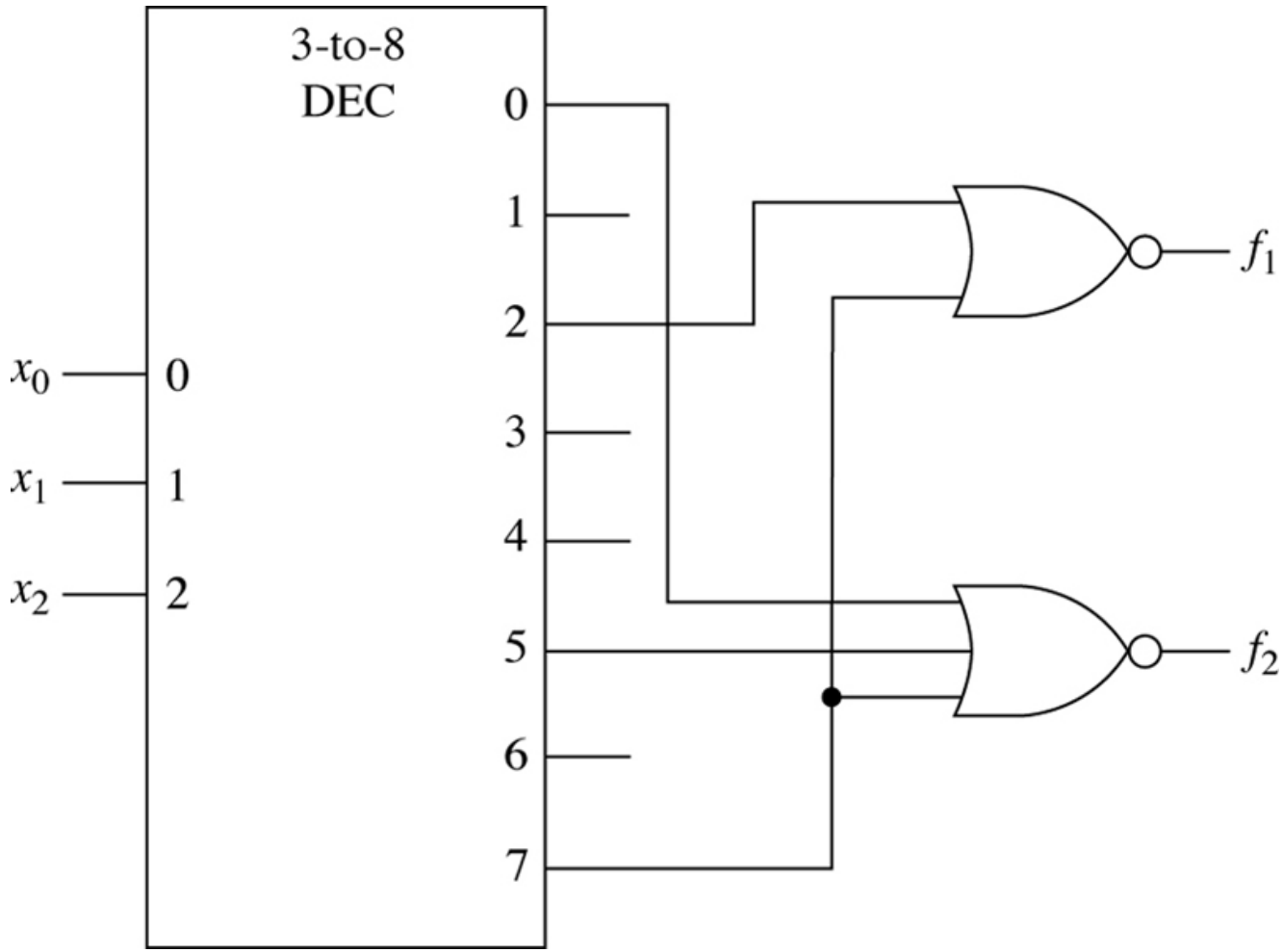


(*c*)

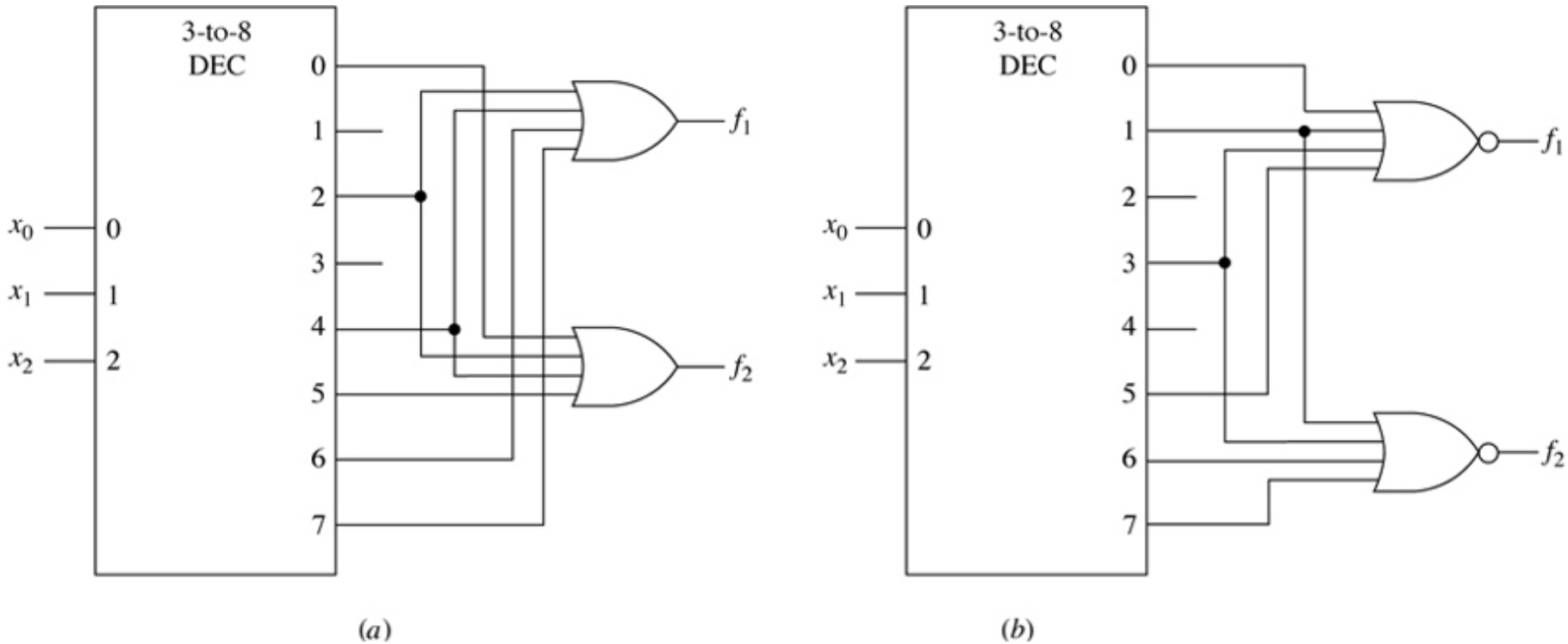
Decoder realization of $f_1(x_2,x_1,x_0) = \Sigma m(1,2,4,5)$ and $f_2(x_2,x_1,x_0) = \Sigma m(1,5,7)$



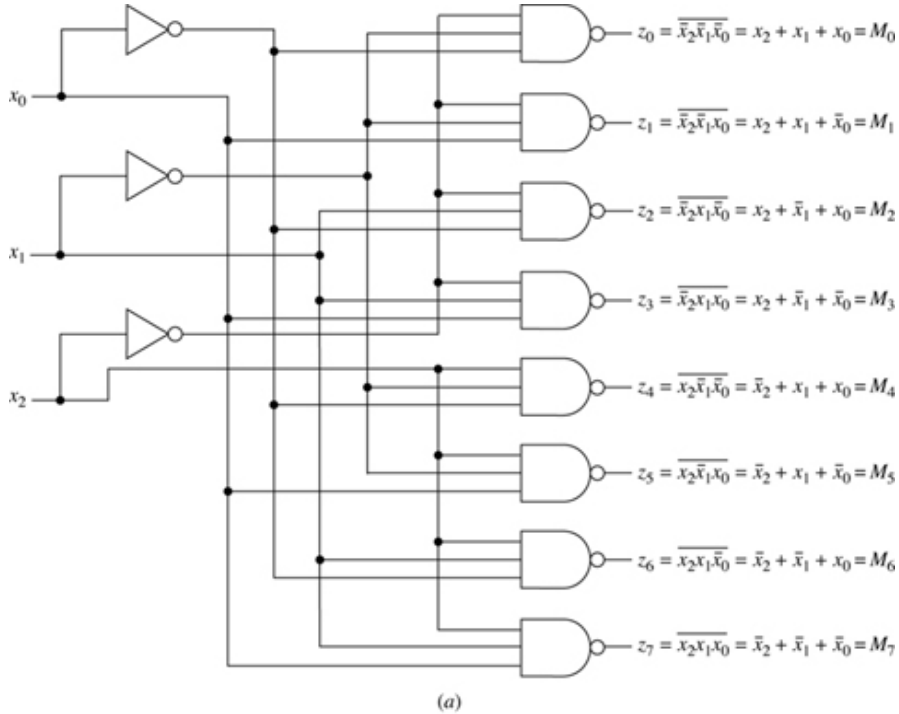
Decoder realization of Boolean functions $f_1 = \Pi M(2, 7)$ and $f_2 = \Pi M(0, 5, 7)$



A decoder realization of $f_1(x_2,x_1,x_0) = \Pi M(0,1,3,5)$ and $f_2(x_2,x_1,x_0) = \Pi M(1,3,6,7)$ (*a*) Using output or-gates. (*b*) Using output nor-gates.



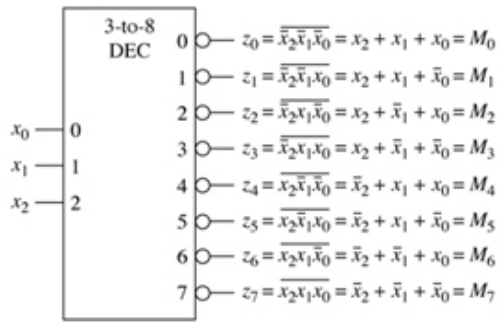
A 3-to-8-line decoder using nand-gates



(a)

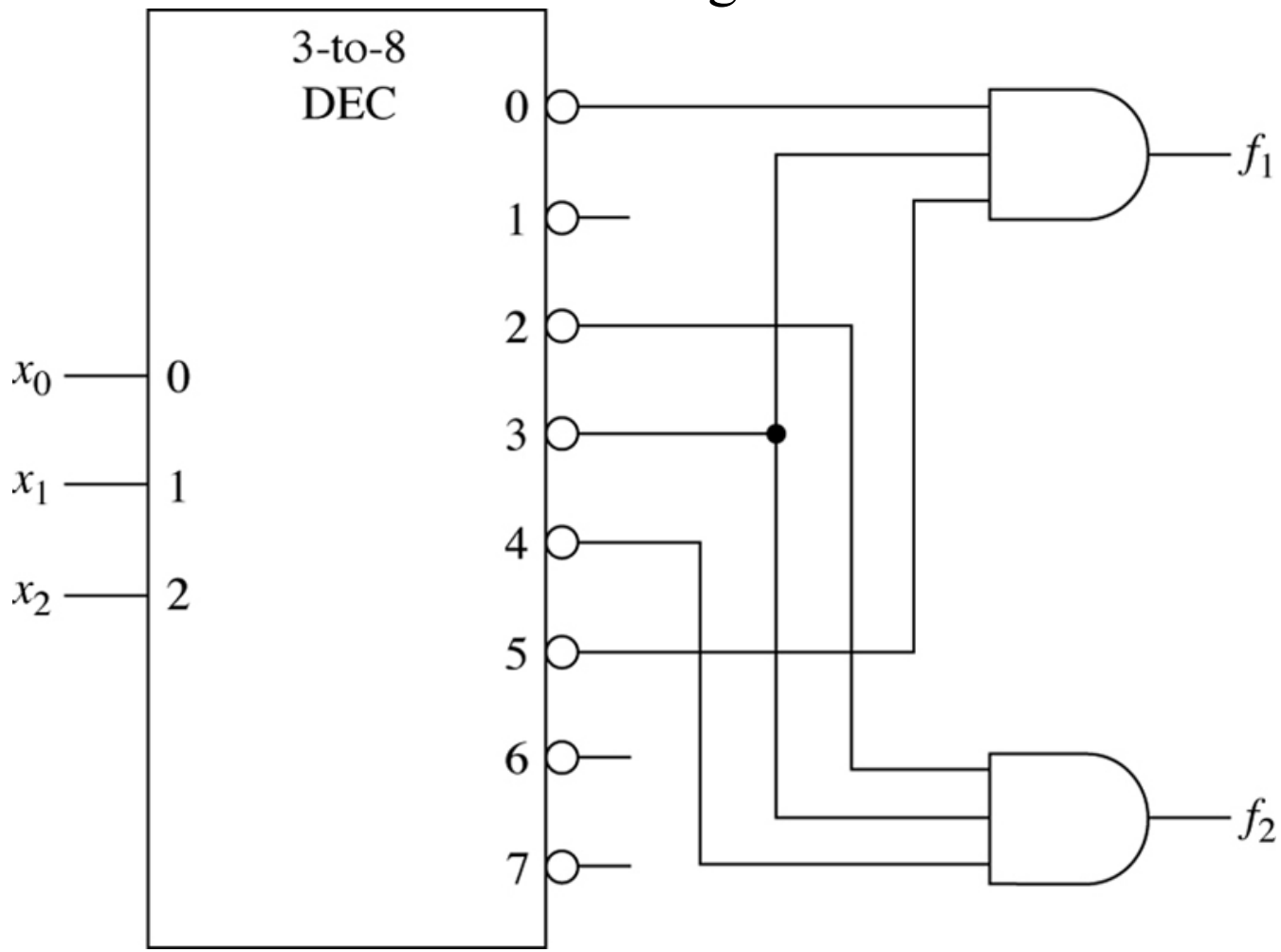
Inputs			Outputs							
x_2	x_1	x_0	z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

(b)

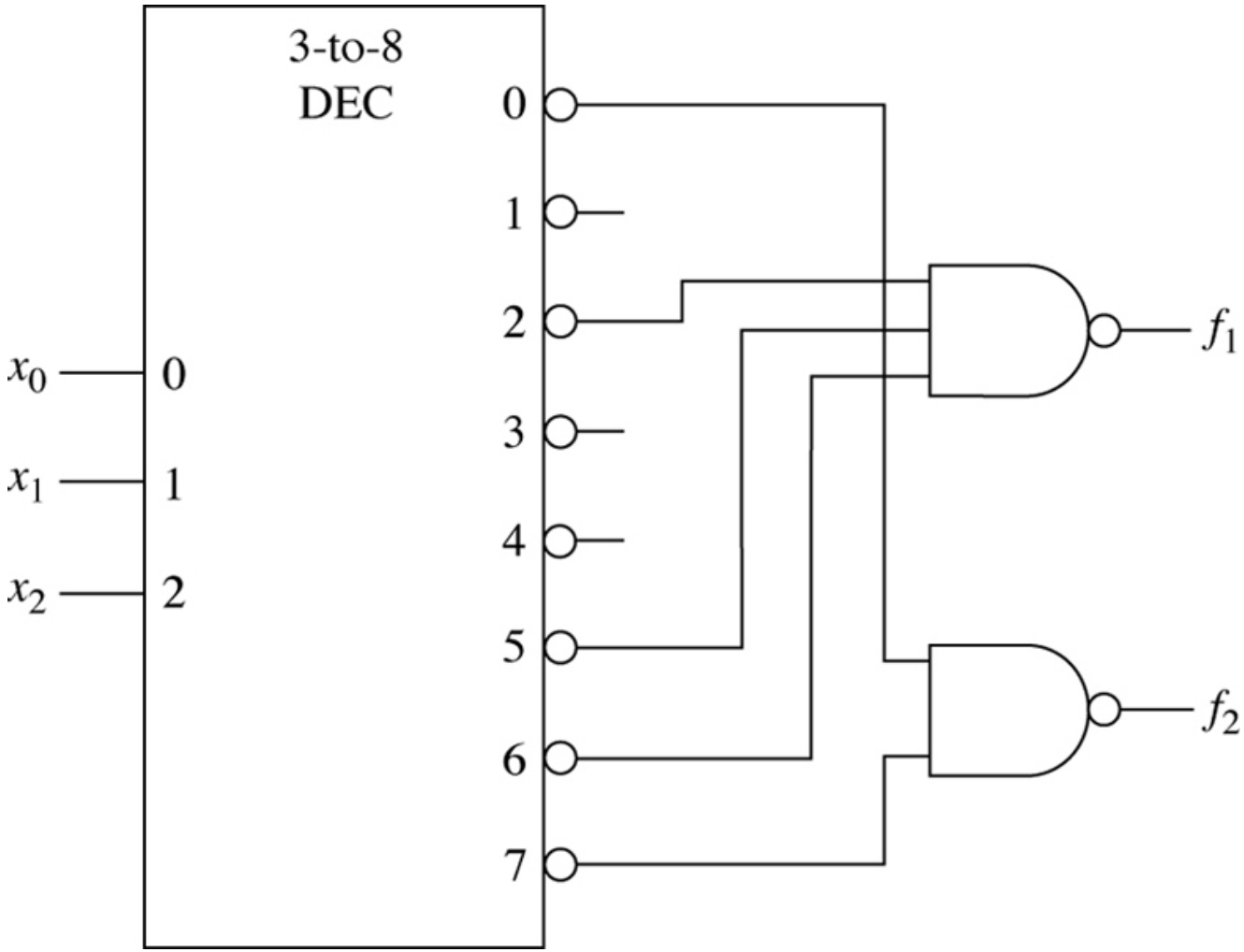


(c)

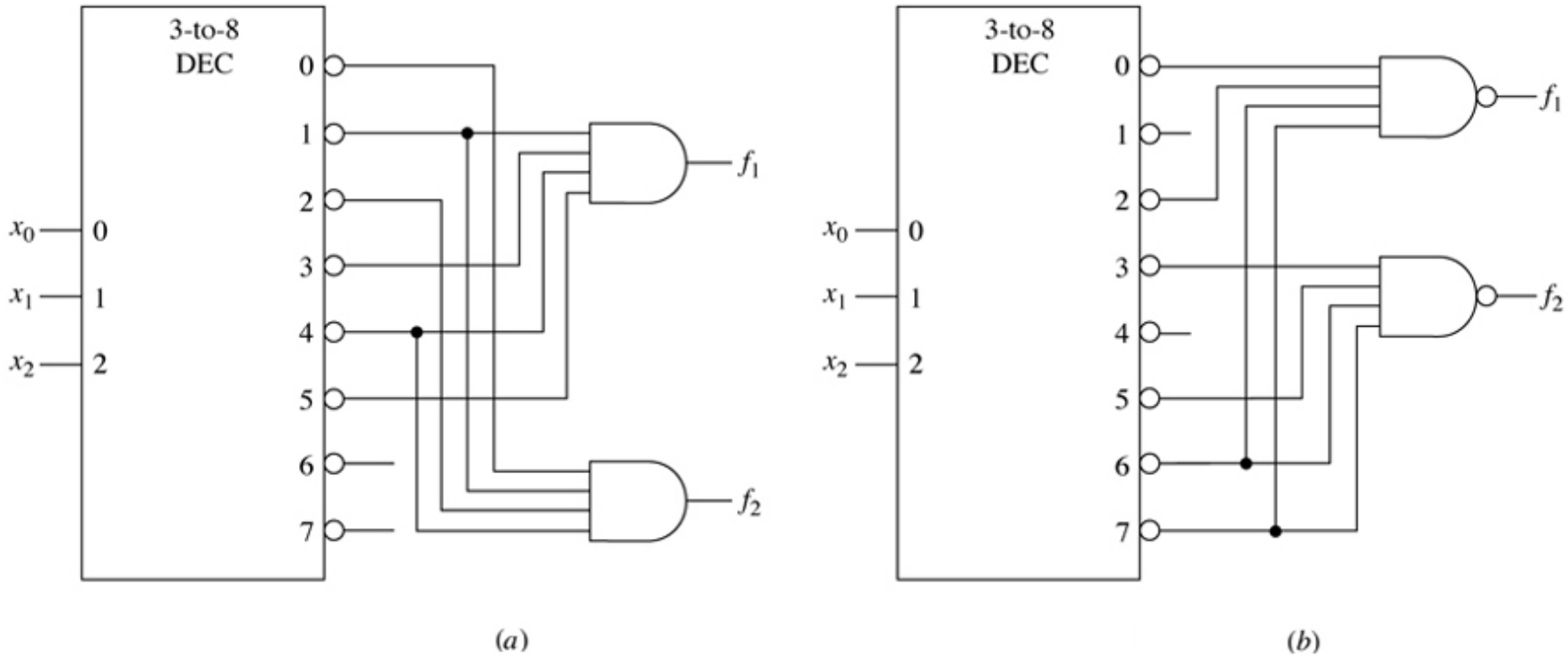
Realization of the pair of maxterm canonical expressions $f_1(x_2,x_1,x_0) = \Pi M(0,3,5)$ and $f_2(x_2,x_1,x_0) = \Pi M(2,3,4)$ with a 3-to-8-line decoder and two and-gates.



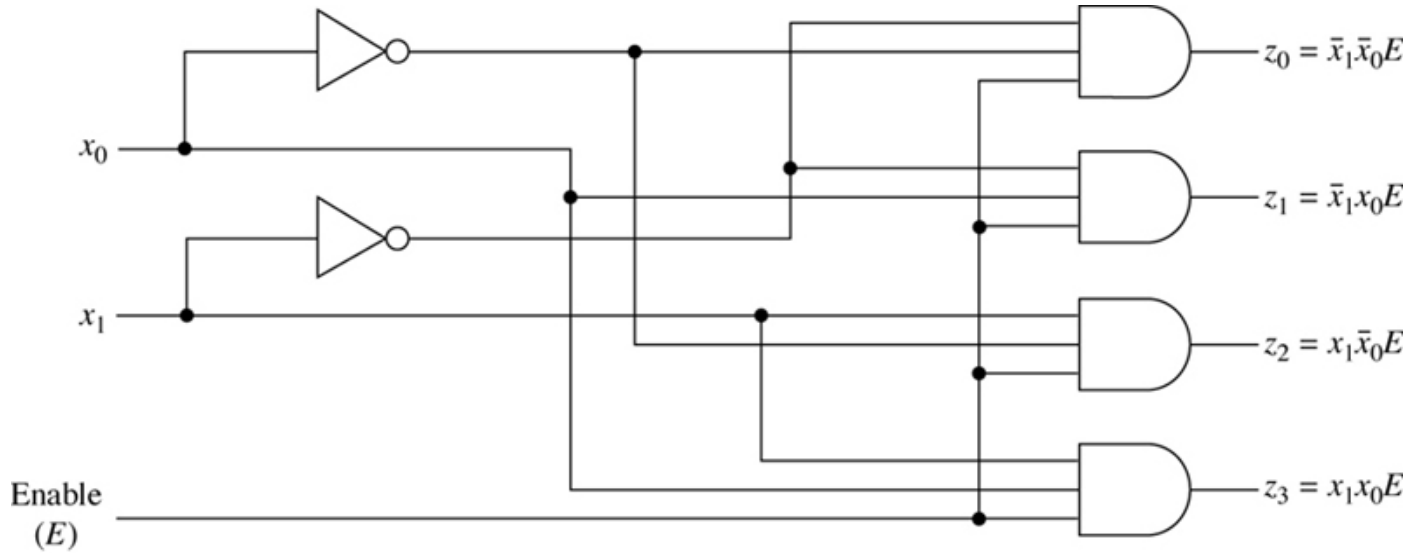
Realization of the Boolean expressions $f_1(x_2,x_1,x_0) = \Pi M(0,1,3,4,7)$ with a 3-to-8-line decoder and two nand-gates.



A decoder realization of $f_1(x_2,x_1,x_0) = \Sigma m(0,2,6,7)$ and $f_2(x_2,x_1,x_0) = \Sigma m(3,5,6,7)$ (*a*) Using output and-gates. (*b*) Using output nand-gates.



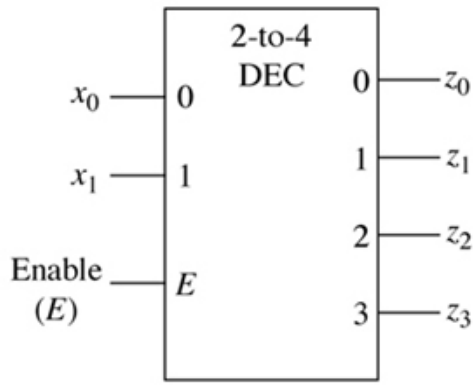
And-gate 2-to-4-line decoder with an enable input. (*a*) Logic diagram. (*b*) Compressed truth table. (*c*) Symbol.



(*a*)

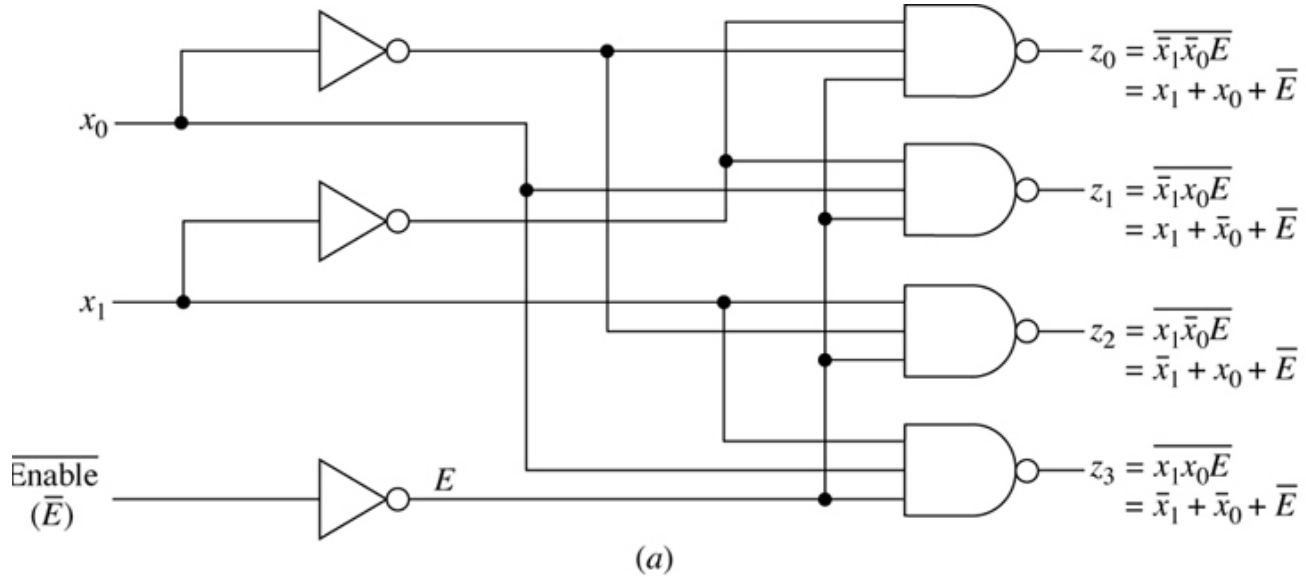
Inputs			Outputs			
E	x_1	x_0	z_0	z_1	z_2	z_3
0	×	×	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

(*b*)



(*c*)

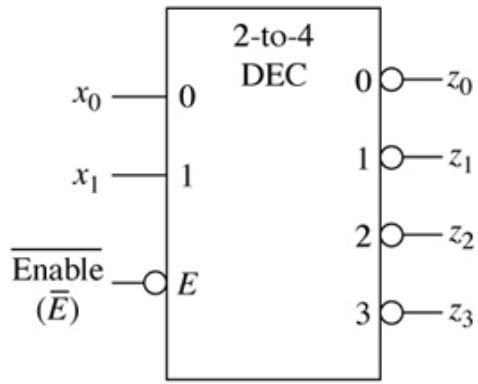
Nand-gate 2-to-4-line decoder with an enable input



(*a*)

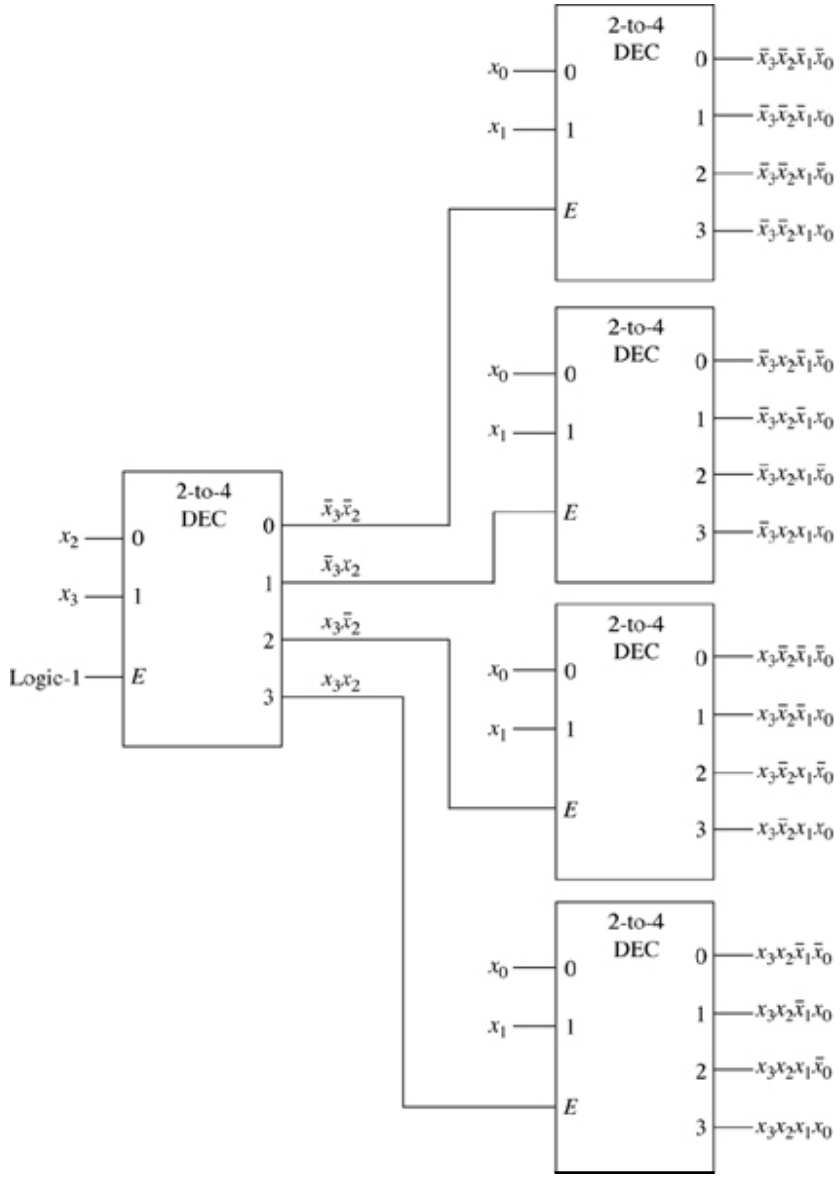
Inputs			Outputs			
$\overline{E}$	x_1	x_0	z_0	z_1	z_2	z_3
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0
1	×	×	1	1	1	1

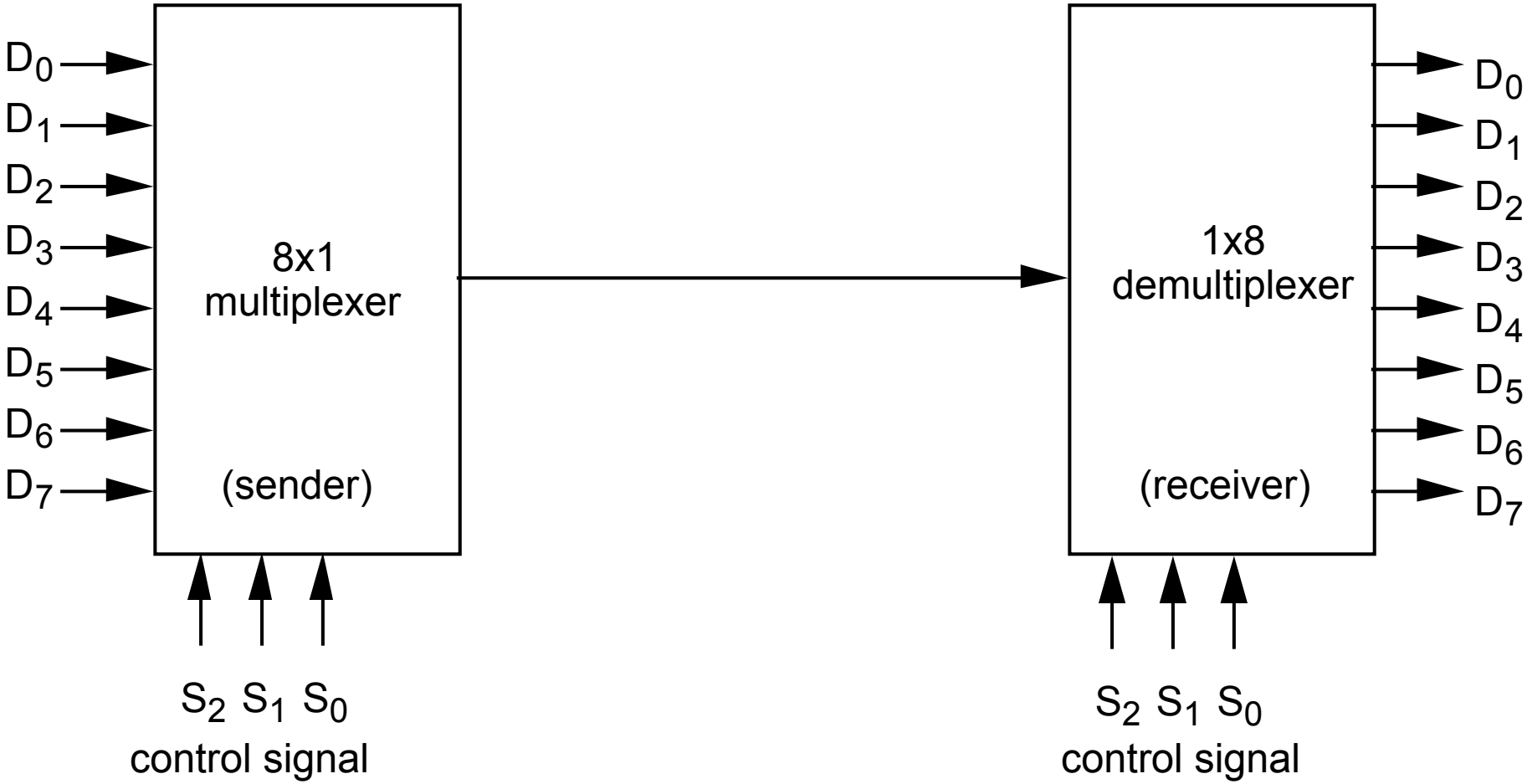
(*b*)



(*c*)

A 4-to-16-line decoder constructed from 2-to-4-line decoder

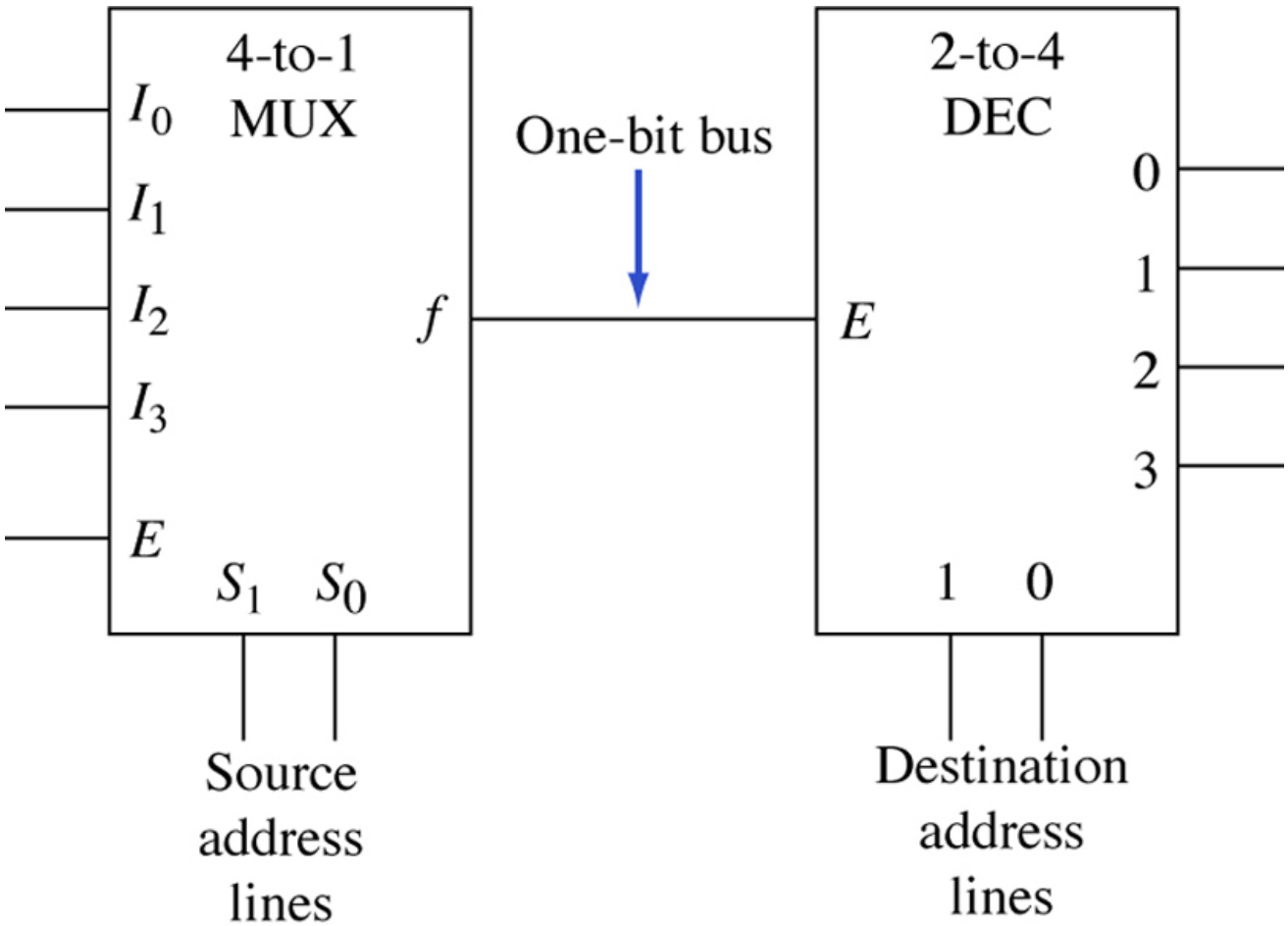




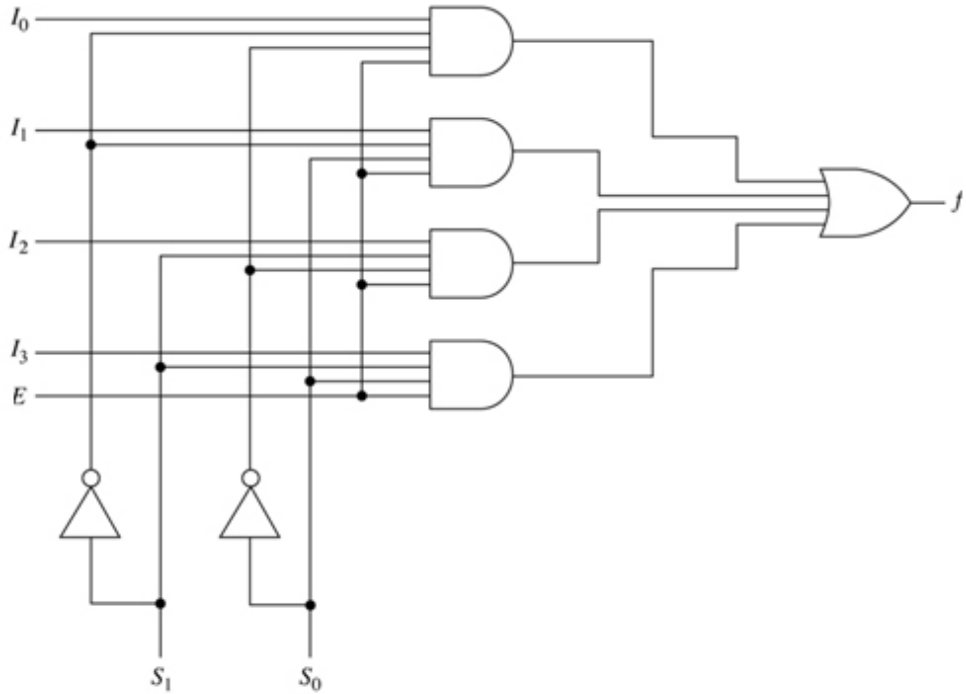
Main function of multiplexer and demultiplexer

The purpose is to reduce the number of wires required for interconnection by making the signals to time-share the link.

A multiplexer/demultiplexer arrangement for information transmission.



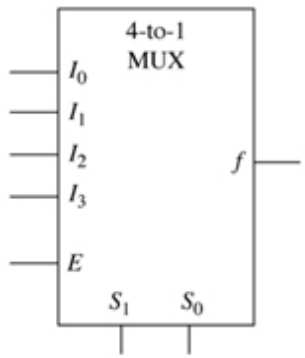
A 4-to-1-line multiplexer



(a)

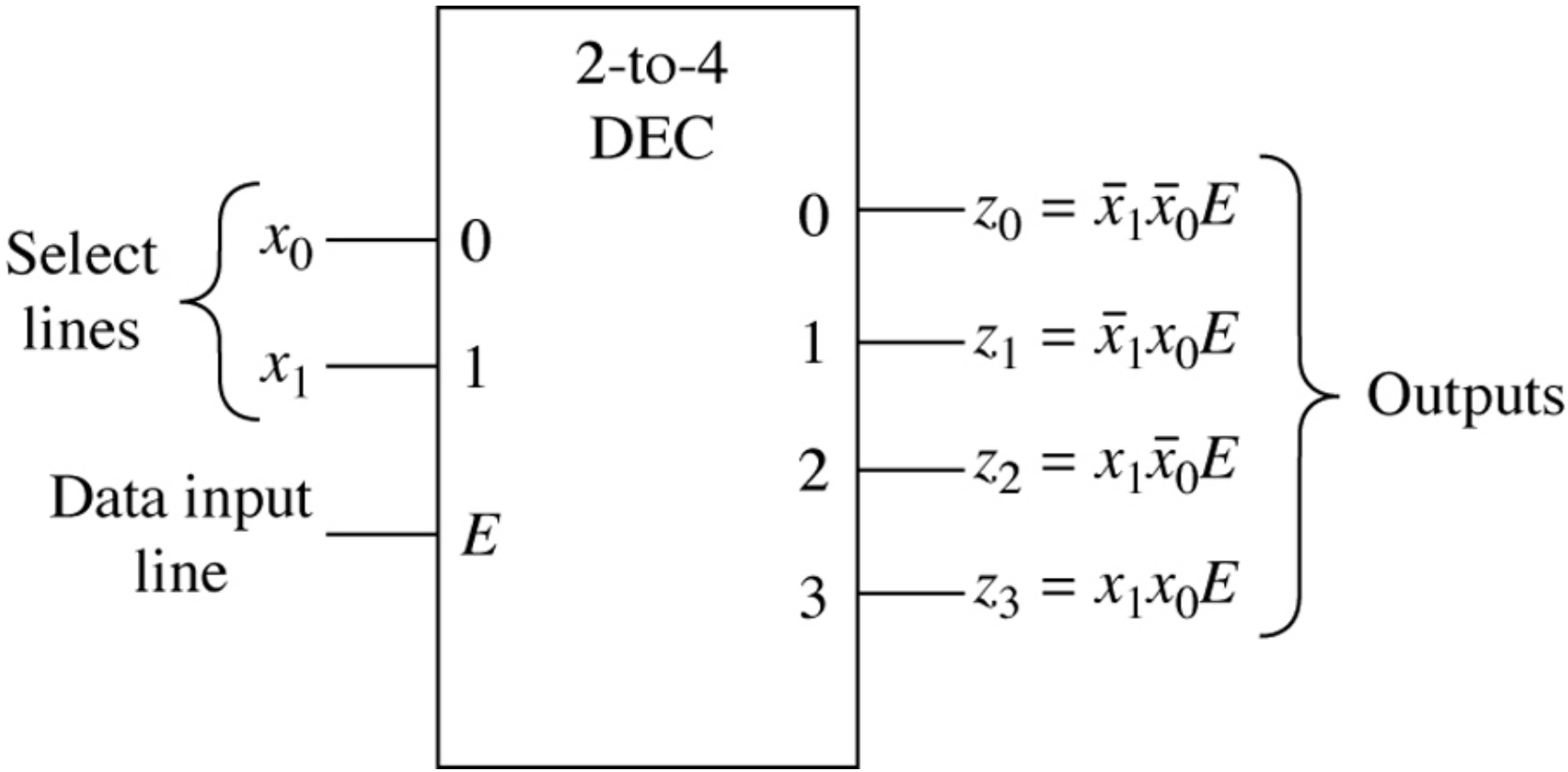
E	S_1	S_0	I_0	I_1	I_2	I_3	f
0	×	×	×	×	×	×	0
1	0	0	0	×	×	×	0
1	0	0	1	×	×	×	1
1	0	1	×	0	×	×	0
1	0	1	×	1	×	×	1
1	1	0	×	×	0	×	0
1	1	0	×	×	1	×	1
1	1	1	×	×	×	0	0
1	1	1	×	×	×	1	1

(b)

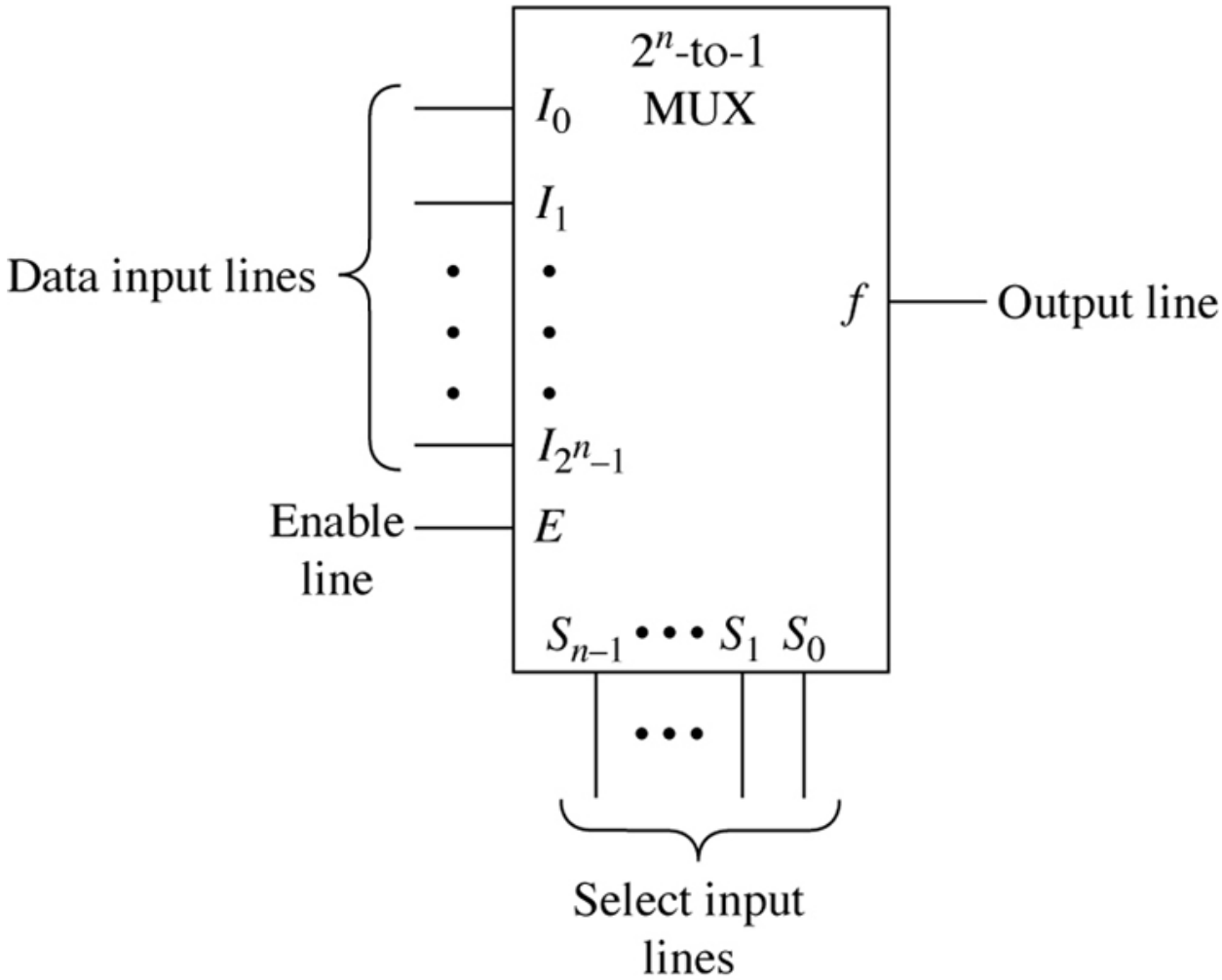


(c)

Demultiplexer



A 2^n-to-1-line multiplexer symbol



MUX implementation of a Boolean function

- Any Boolean function of n variables can be implemented by a multiplexer with n control inputs in a straightforward manner.

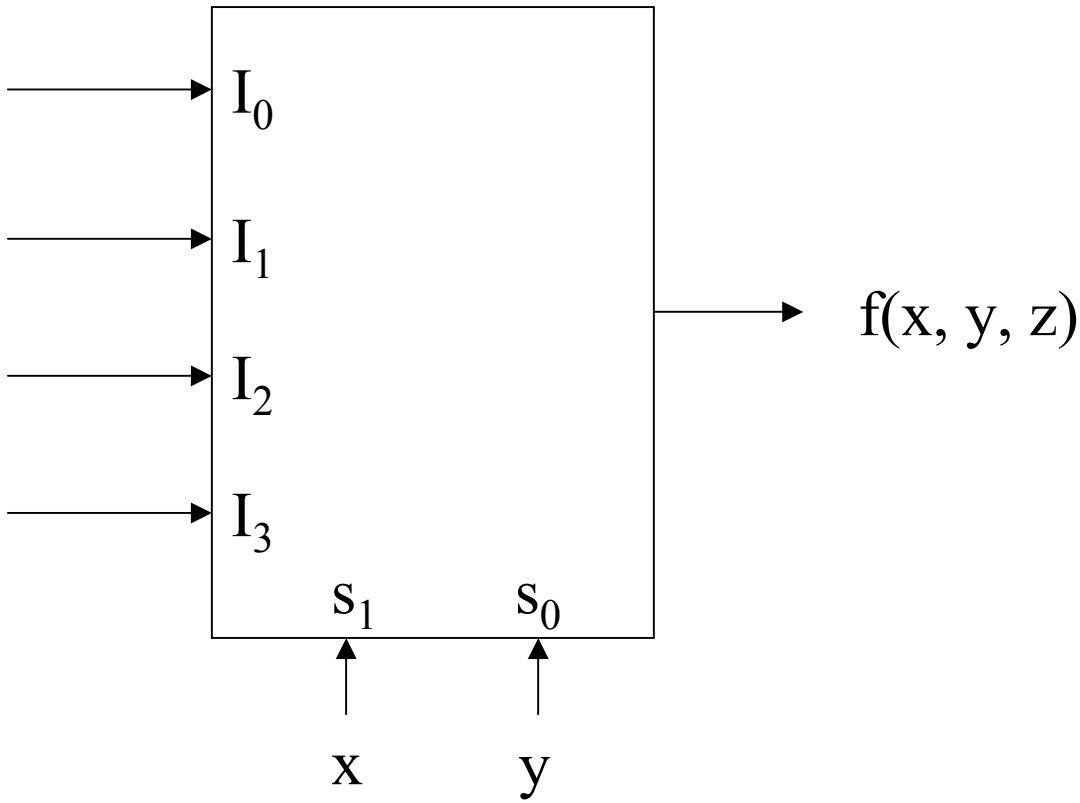
Example: $f(x, y, z) = \Sigma m(2, 5, 6, 7)$

x	y	z	f(x, y, z)	=
0	0	0	f(0, 0, 0)	0
0	0	1	f(0, 0, 1)	0
0	1	0	f(0, 1, 0)	1
0	1	1	f(0, 1, 1)	0
1	0	0	f(1, 0, 0)	0
1	0	1	f(1, 0, 1)	1
1	1	0	f(1, 1, 0)	1
1	1	1	f(1, 1, 1)	1

MUX implementation of a Boolean function

- Even better, any Boolean function of n variables can be implemented by a multiplexer with n-1 control inputs as illustrated in the following.

Implementing a function of 3 variables with a 4x1 MUX: Method 1



Using a multiplexer to implement a Boolean function: Method 1

Note that the output of a 4x1 multiplexer is

$F(x, y, z) = x'y'I_0 + x'yI_1 + xy'I_2 + xyI_3$

Now, given a Boolean function

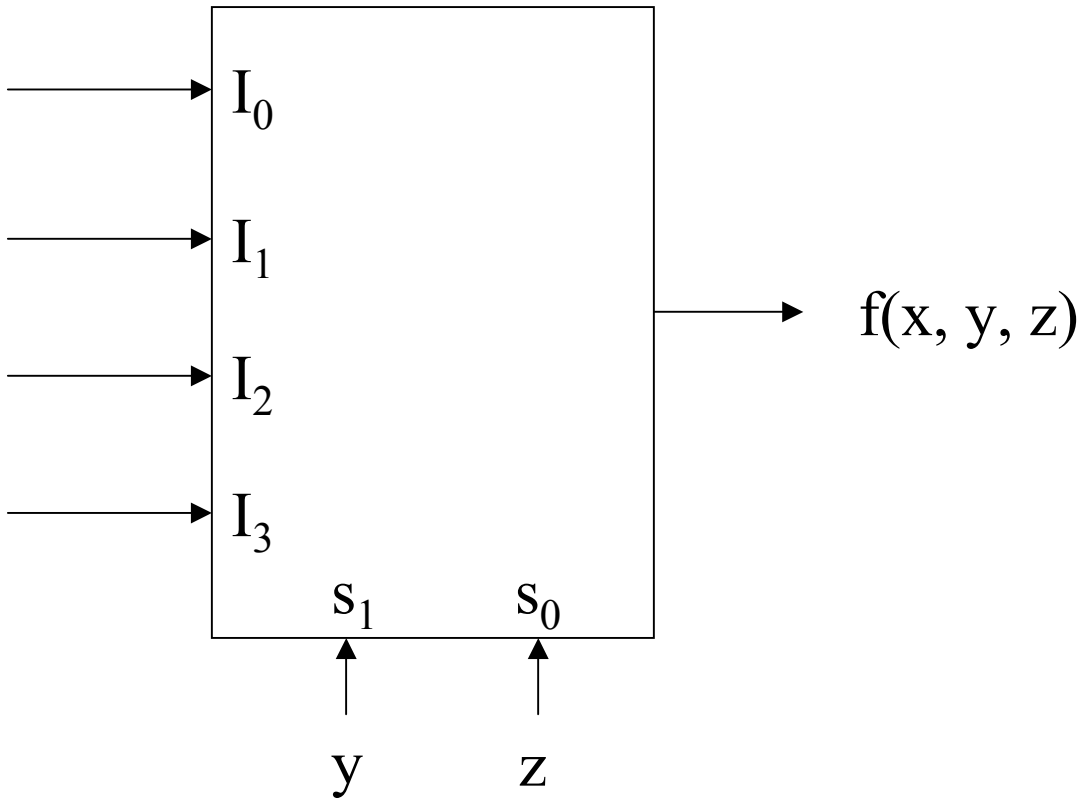
$f(x, y, z) = f(0, 0, 0)x'y'z' + f(0, 1, 0)x'yz' + f(1, 0, 0)xy'z' + f(1, 1, 0)xyz'$
$+f(0, 0, 1)x'y'z + f(0, 1, 1)x'yz + f(1, 0, 1)xy'z + f(1, 1, 1)xyz$

The value for input I_0 is to be determined as follows.

if f(0, 0, 0) =	and f(0, 0, 1) =	then f(0, 0, 0)x'y'z' + f(0, 0, 1)x'y'z =	and thus we should let I_0 =
0	0	0=x'y'0	0
0	1	x'y'z	z
1	0	x'y'z'	z'
1	1	x'y'=x'y'1	1

The value for I_1 , I_2 , and I_3 are to be determined in a similar manner.

Implementing a function of 3 variables with a 4x1 MUX: Method 2



Using a multiplexer to implement a Boolean function: Method 2

Note that the output of a 4x1 multiplexer is

$F(x, y, z) = I_0y'z' + I_1y'z + I_2yz' + I_3yz$

Now, given a Boolean function

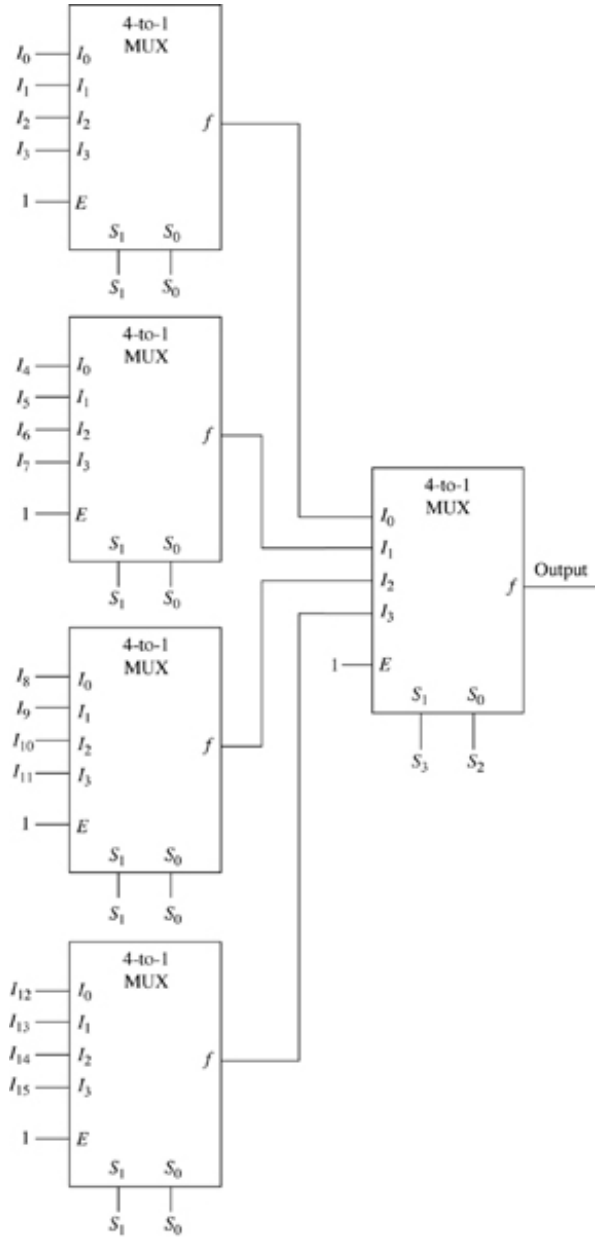
$$f(x, y, z) = f(0, 0, 0)x'y'z' + f(0, 0, 1)x'y'z + f(0, 1, 0)x'yz' + f(0, 1, 1)x'yz + f(1, 0, 0)xy'z' + f(1, 0, 1)xy'z + f(1, 1, 0)xyz' + f(1, 1, 1)xyz$$

The value for input I_0 is to be determined as follows.

if f(0, 0, 0) =	and f(0, 0, 1) =	then f(0, 0, 0)x'y'z' + f(1, 0, 0)xy'z' =	and thus we should let I_0 =
0	0	0=0y'z'	0
0	1	xy'z'	x
1	0	x'y'z'	x'
1	1	y'z'=1y'z'	1

The value for I_1 , I_2 , and I_3 are to be determined in a similar manner.

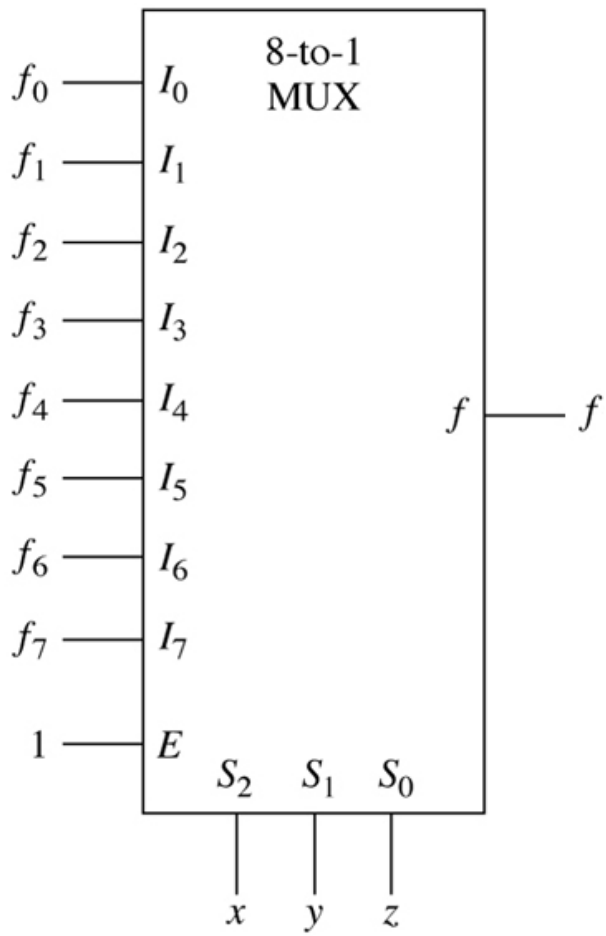
A multiplexer tree to form a 16-to-1-line multiplexer



Realization of a three-variable function using a 8-to-1-line multiplexer.
(a) Three-variable truth table
(*b*) General realization.

x	y	z	f
0	0	0	f_0
0	0	1	f_1
0	1	0	f_2
0	1	1	f_3
1	0	0	f_4
1	0	1	f_5
1	1	0	f_6
1	1	1	f_7

(*a*)

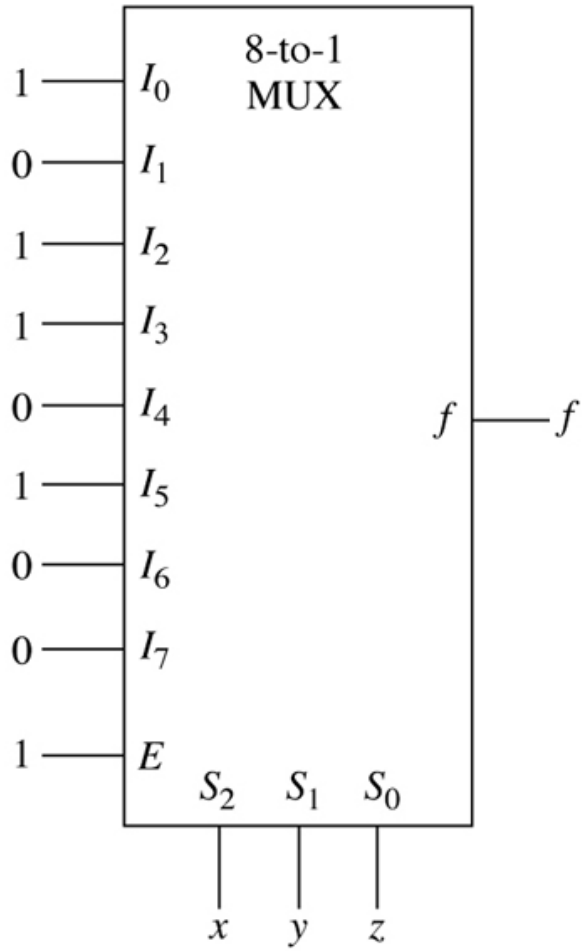


(*b*)

Example: realization of $f(x,y,z) = \Sigma m(0,2,3,5)$

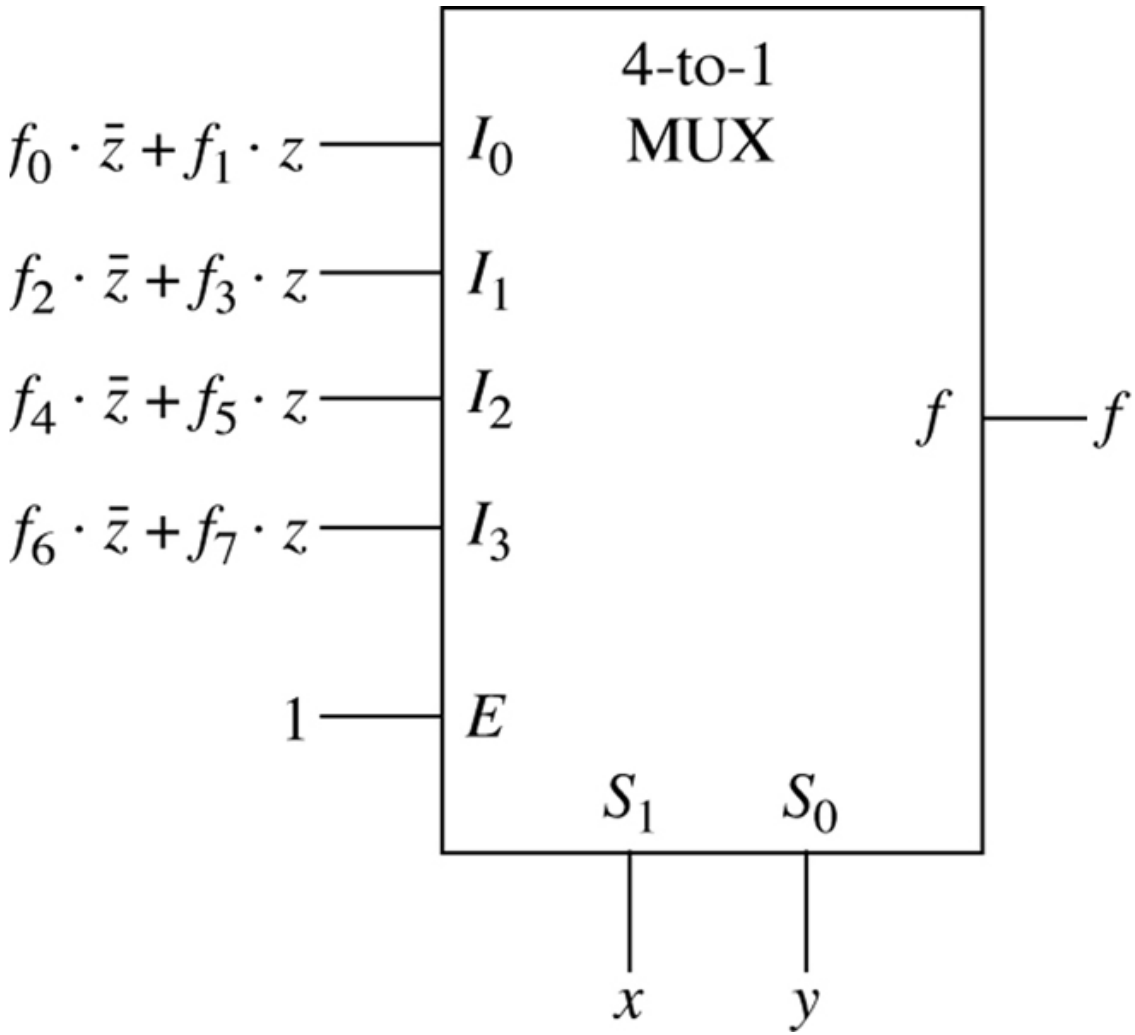
x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

(a)

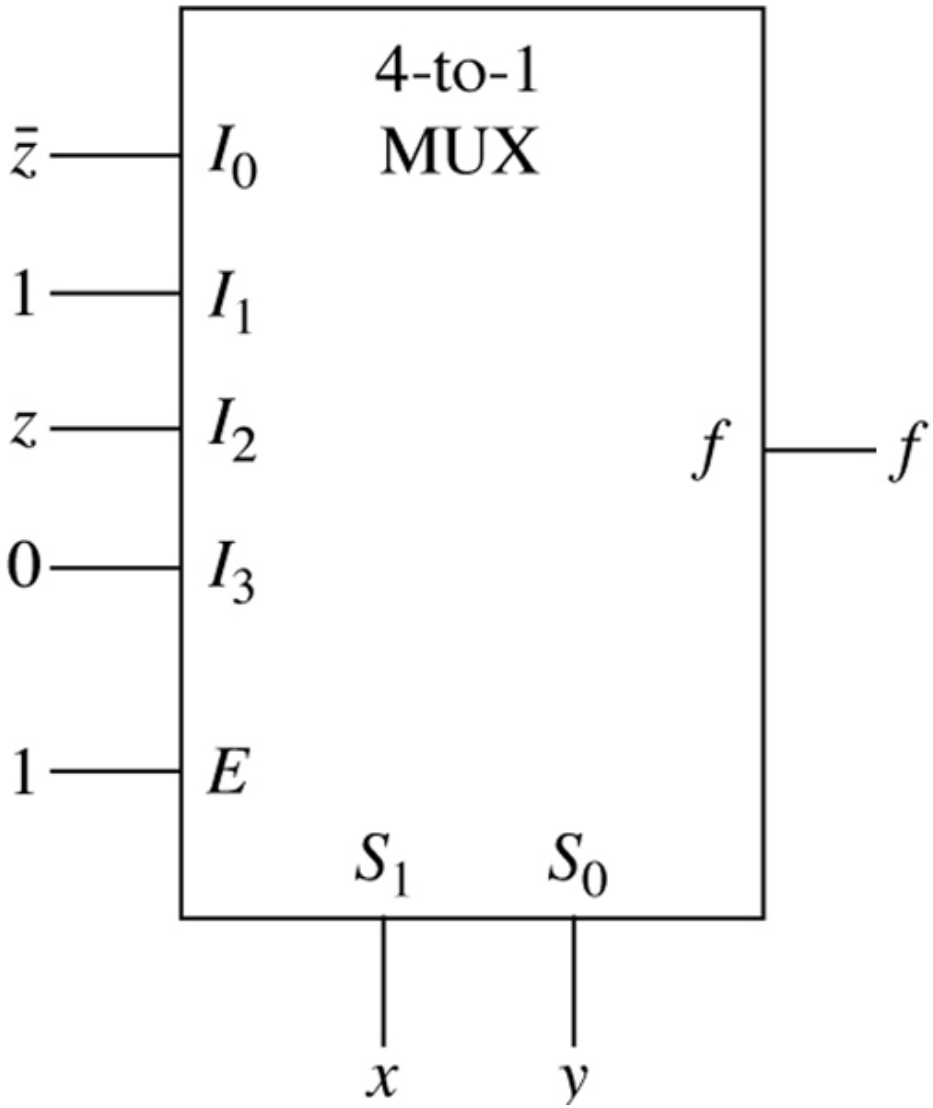


(b)

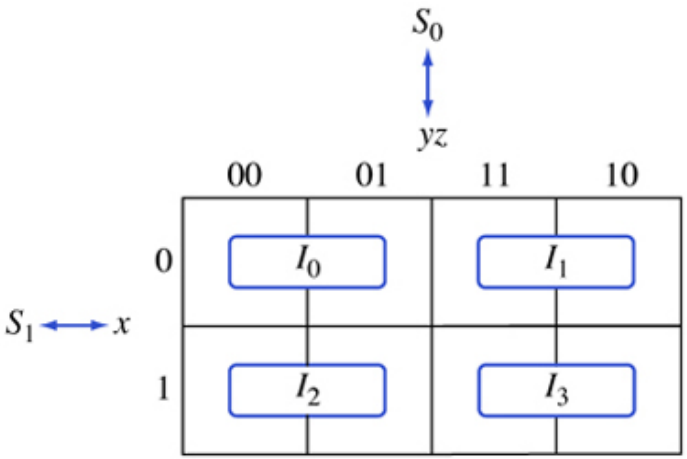
Realizing a 3-variable Boolean function with a 4-to-1 multiplexer



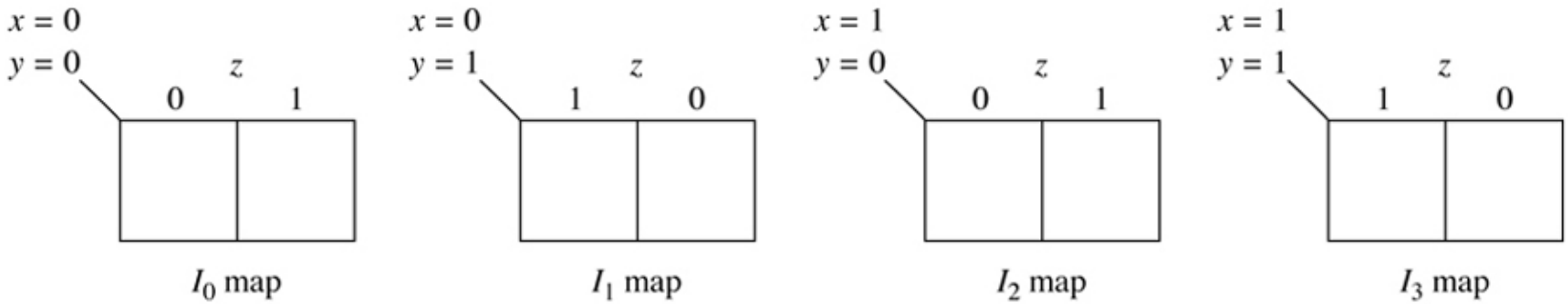
Realization of $f(x,y,z) = \Sigma m(0,2,3,5)$ using a 4-to-1-line multiplexer



Obtaining multiplexer realizations using Karnaugh maps.
(*a*) Cell groupings corresponding to the data line functions.
(*b*) Karnaugh maps for the I_i subfunctions.

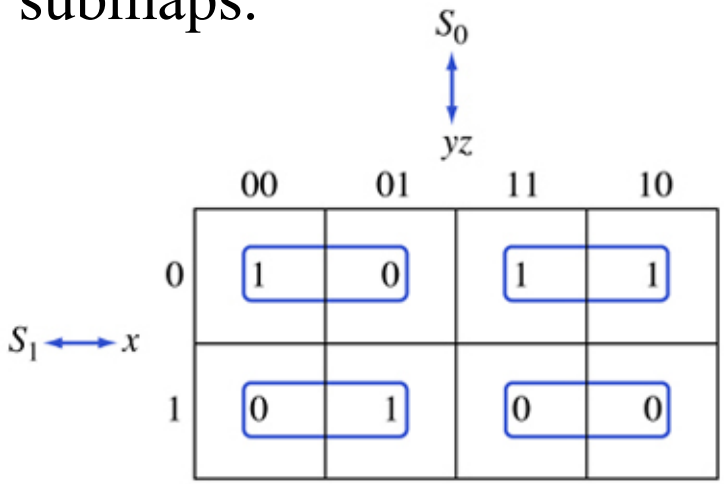


(*a*)

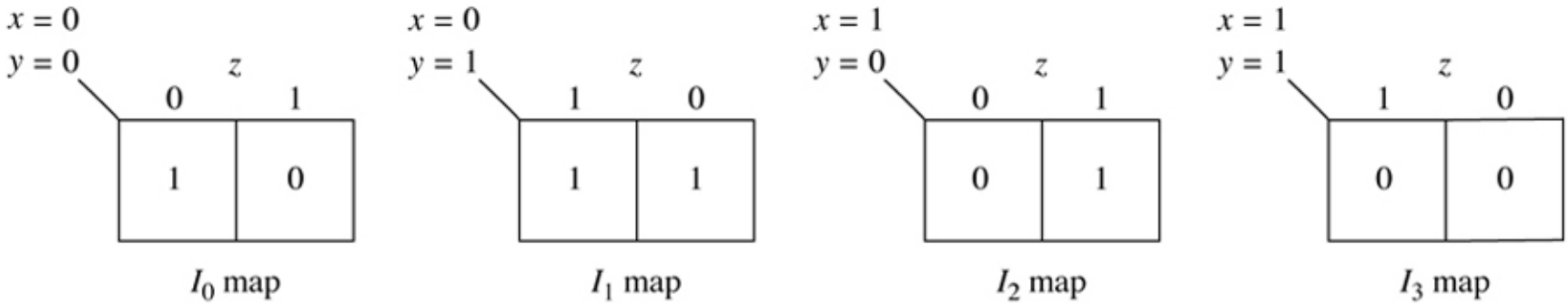


(*b*)

Realization of $f(x,y,z) = \Sigma m(0,2,3,5)$.
(a) Karnaugh map.
(b) I_0, I_1, I_2, and I_3 submaps.

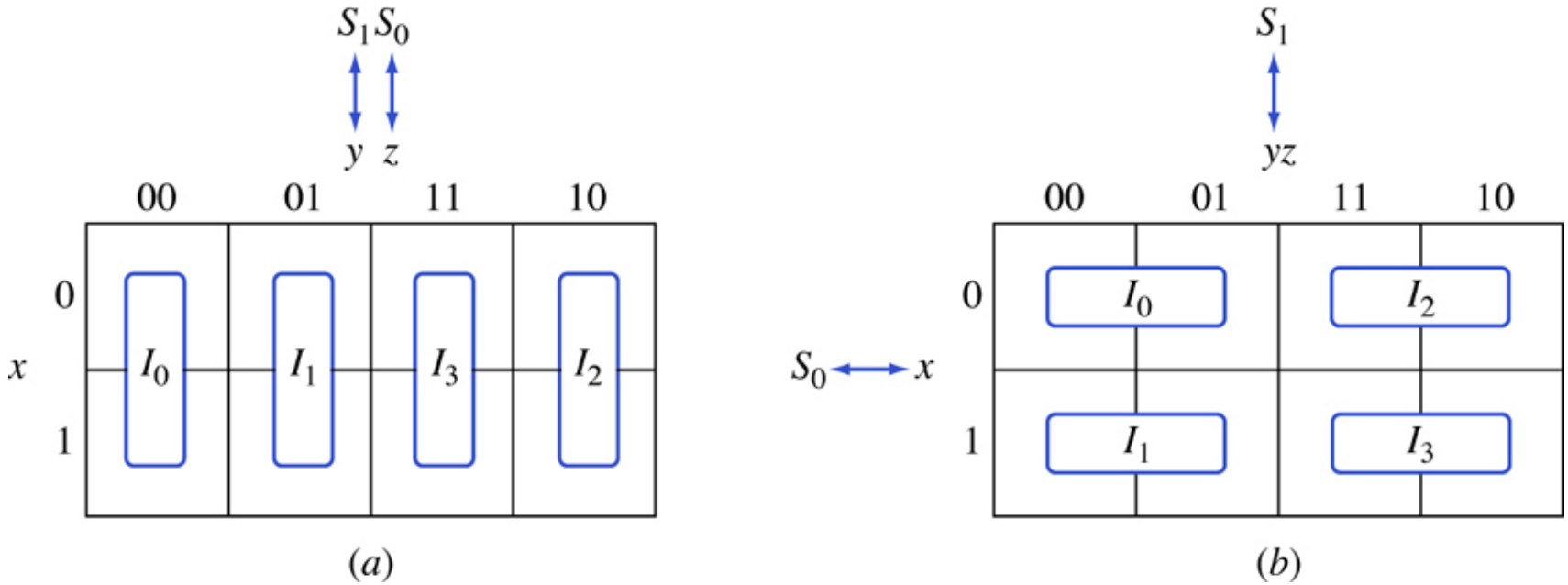


(a)

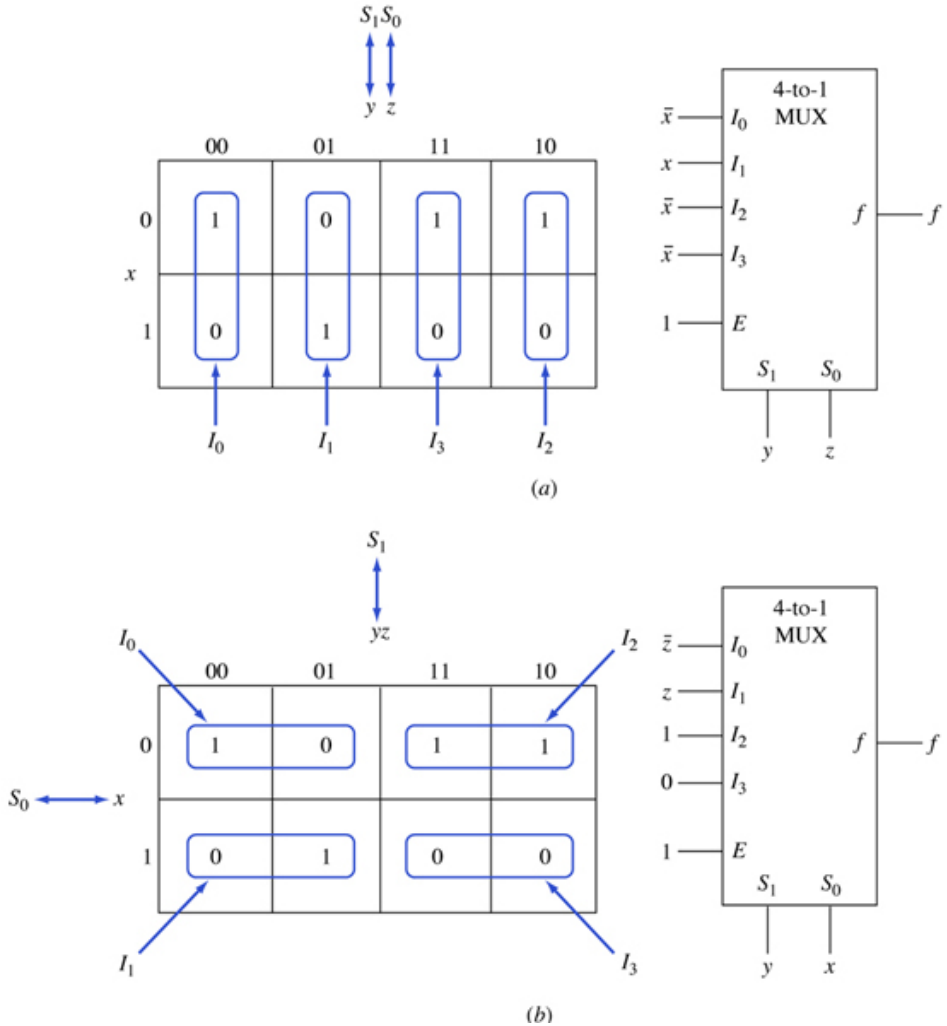


(b)

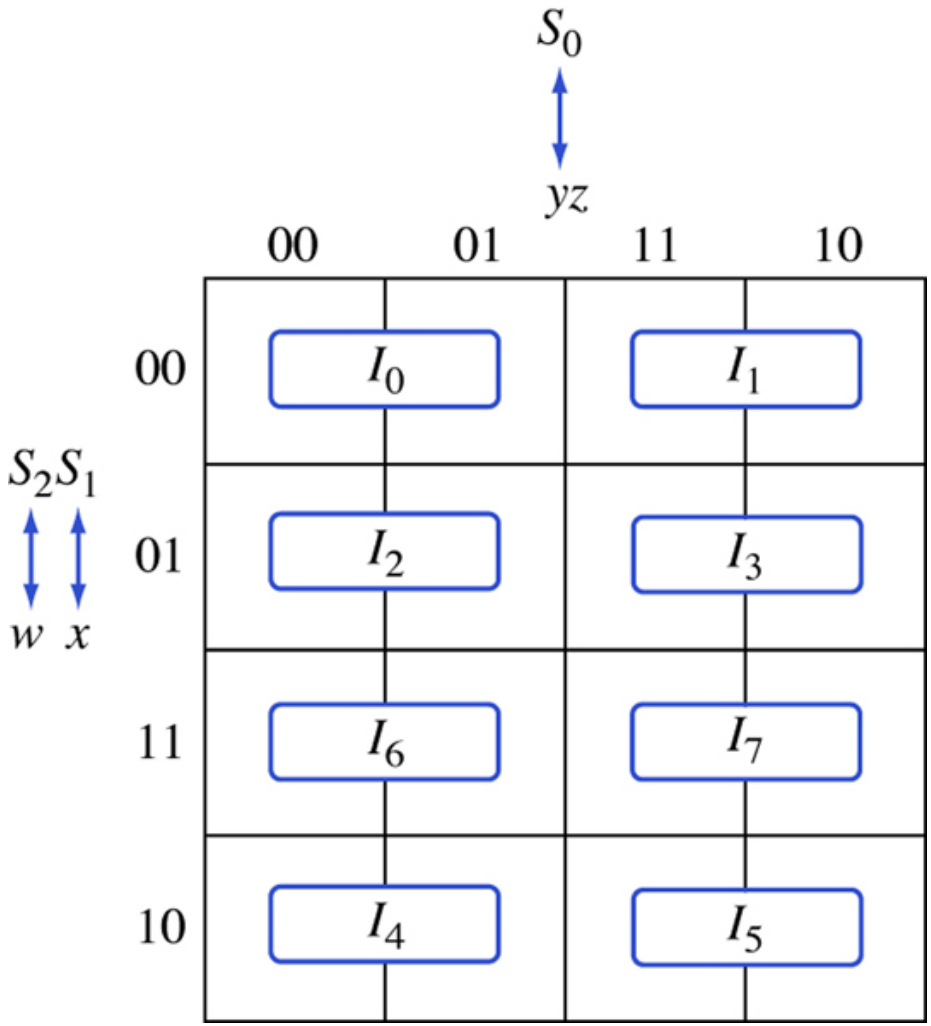
Using Karnaugh maps to obtain multiplexer realizations under various assignments to the select inputs.
(*a*) Applying input variables *y* and *z* to the S_1 and S_0 select lines.
(*b*) Applying input variables *x* and *y* to the S_0 and S_1 select lines.



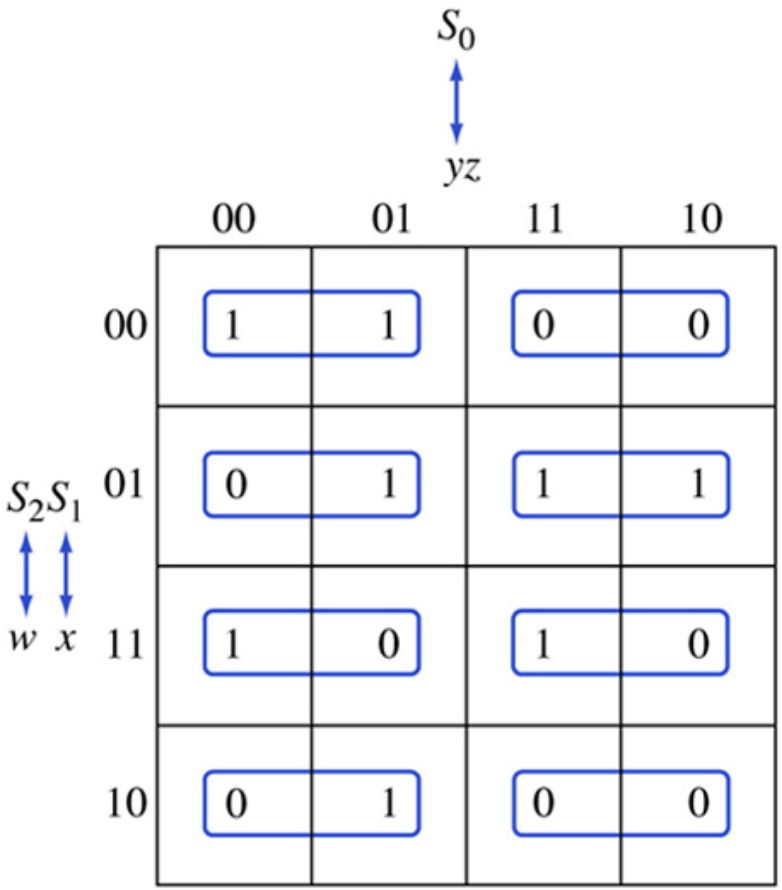
Alternative realizations of $f(x,y,z) = \Sigma m(0,2,3,5)$.



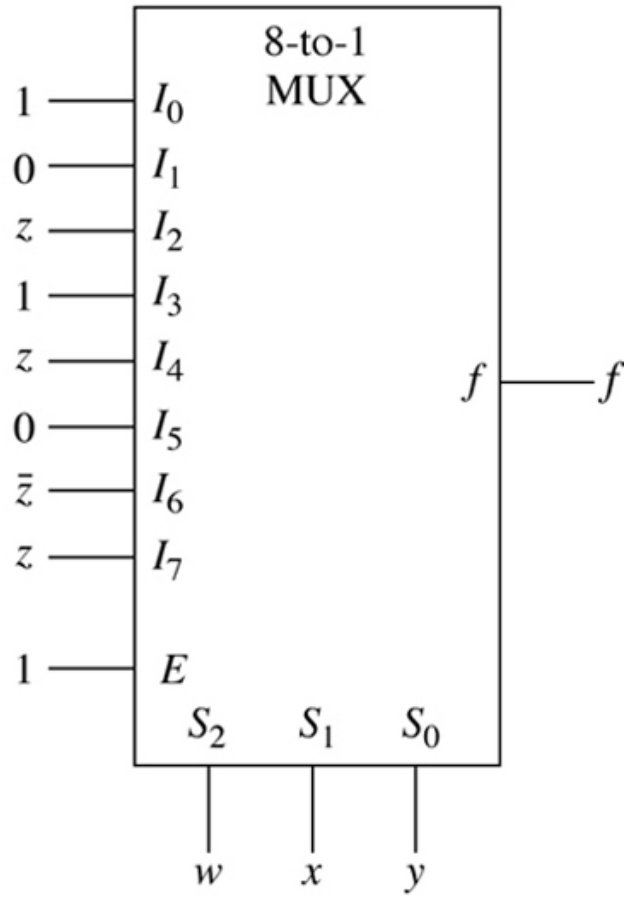
A select line assignment and corresponding data line functions for a multiplexer realization of a four-variable function.



Realizations of $f(w,x,y,z) = \Sigma m(0,1,5,6,7,9,12,15)$.

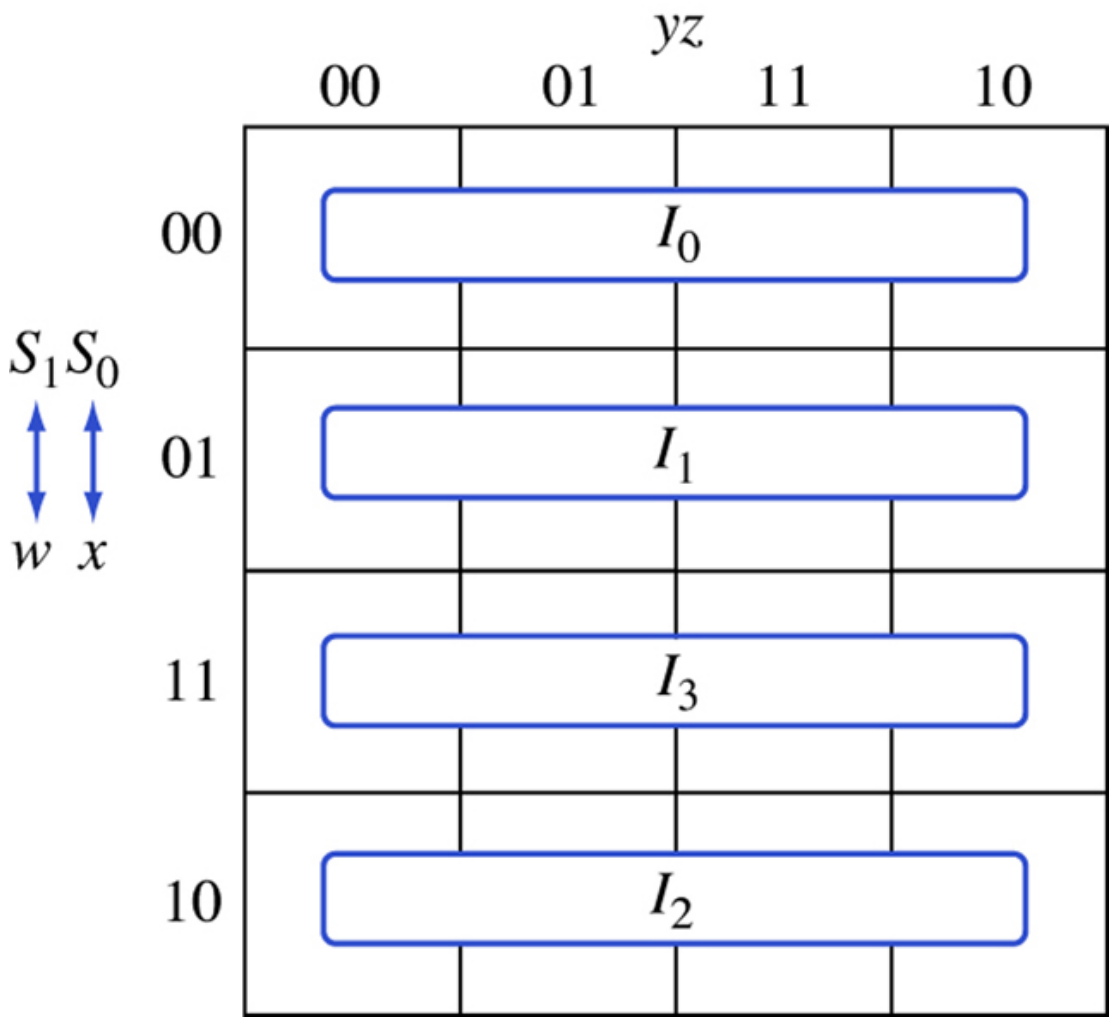


(a)

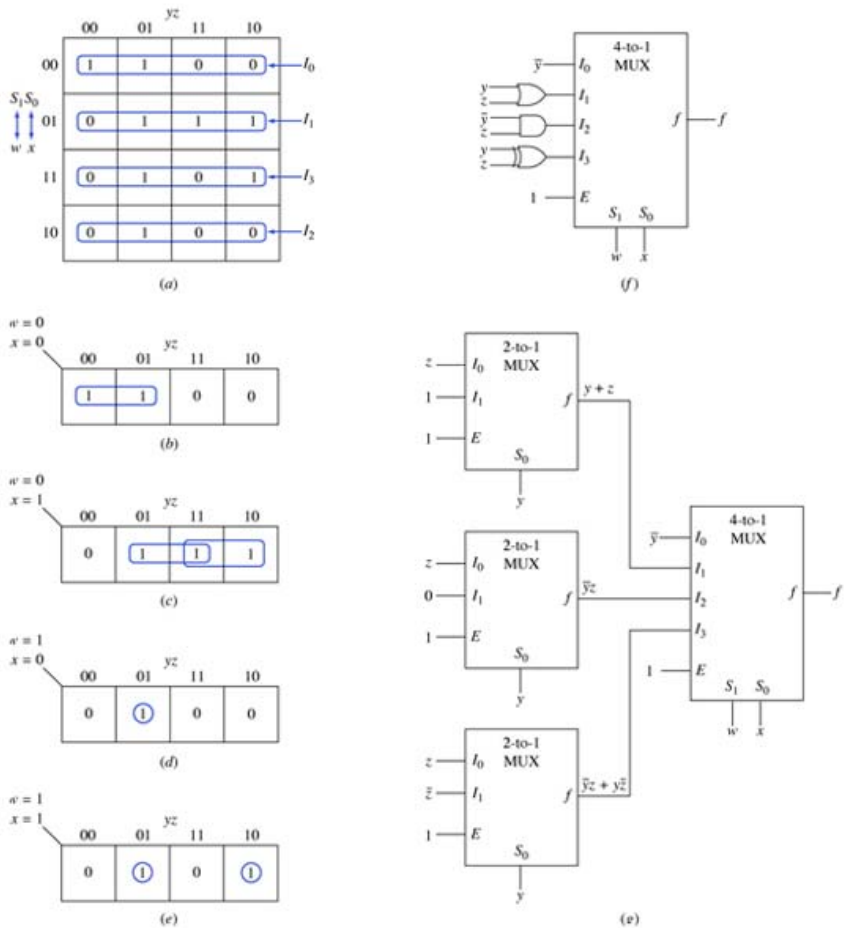


(b)

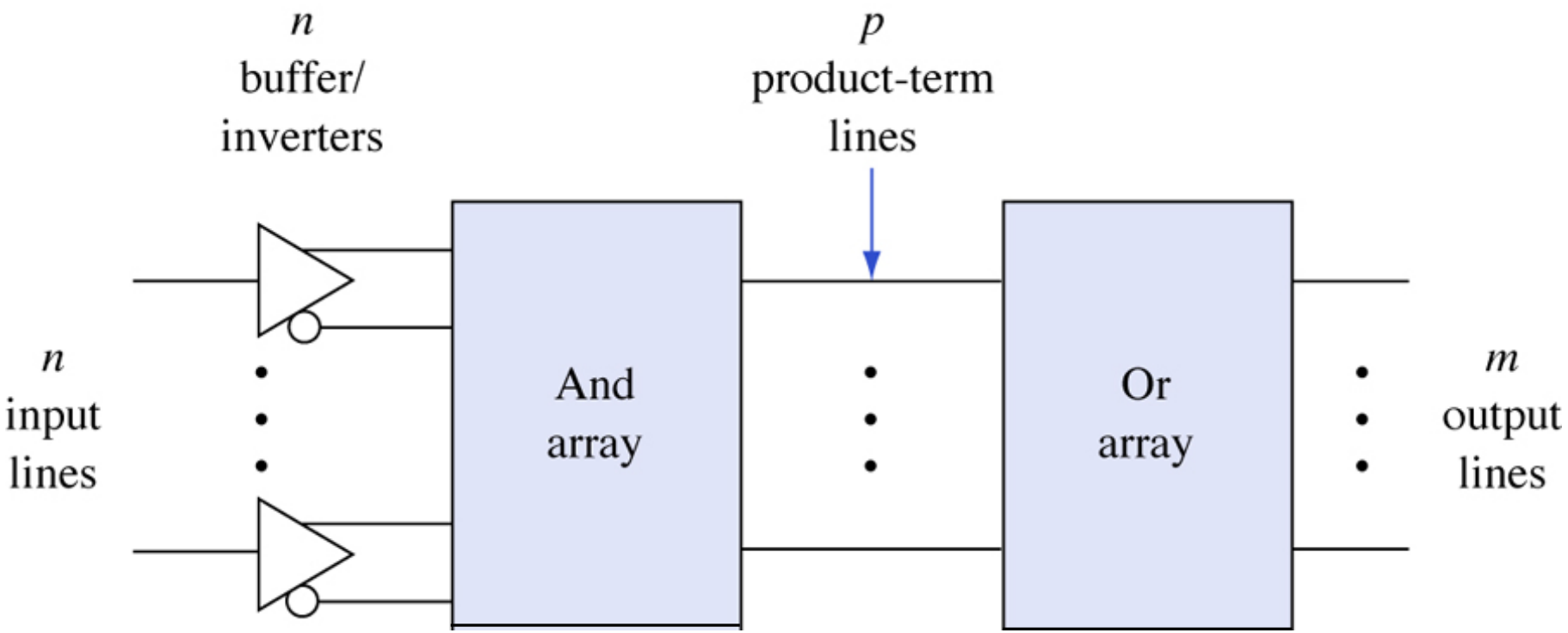
Using a four-variable Karnaugh map to obtain a Boolean function realization with a 4-to-1-line multiplexer.



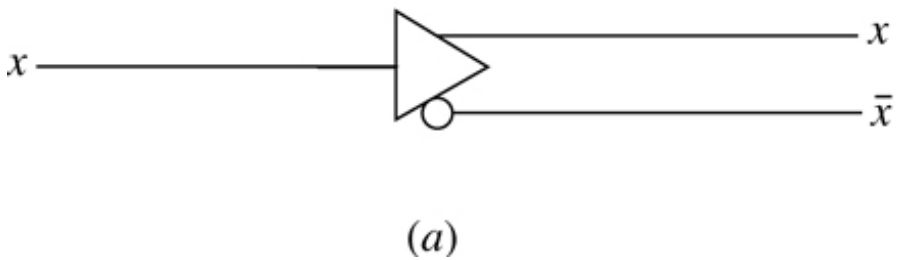
Realizations of the Boolean function $f(w,x,y,z) = \Sigma m(0,1,5,6,7,9,13,14)$.



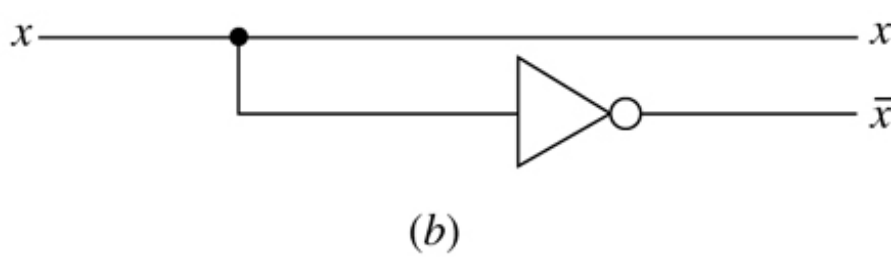
General structure of Programmable Logic Devices (PLDs)



Buffer/inverter. (*a*) Symbol. (*b*) Logic equivalent



(*a*)

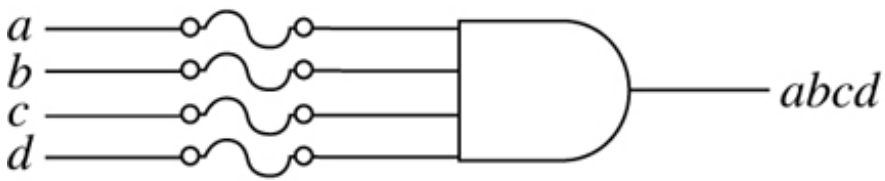


(*b*)

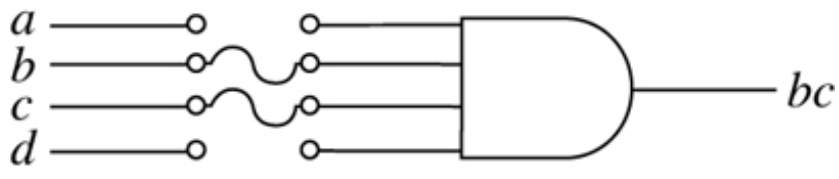
Types of PLDs

Device	AND-array	OR-array
PROM	Fixed	Programmable
PLA	Programmable	Programmable
PAL	Programmable	Fixed

Programming by blowing fuses.
(*a*) Before programming.
(*b*) After programming.

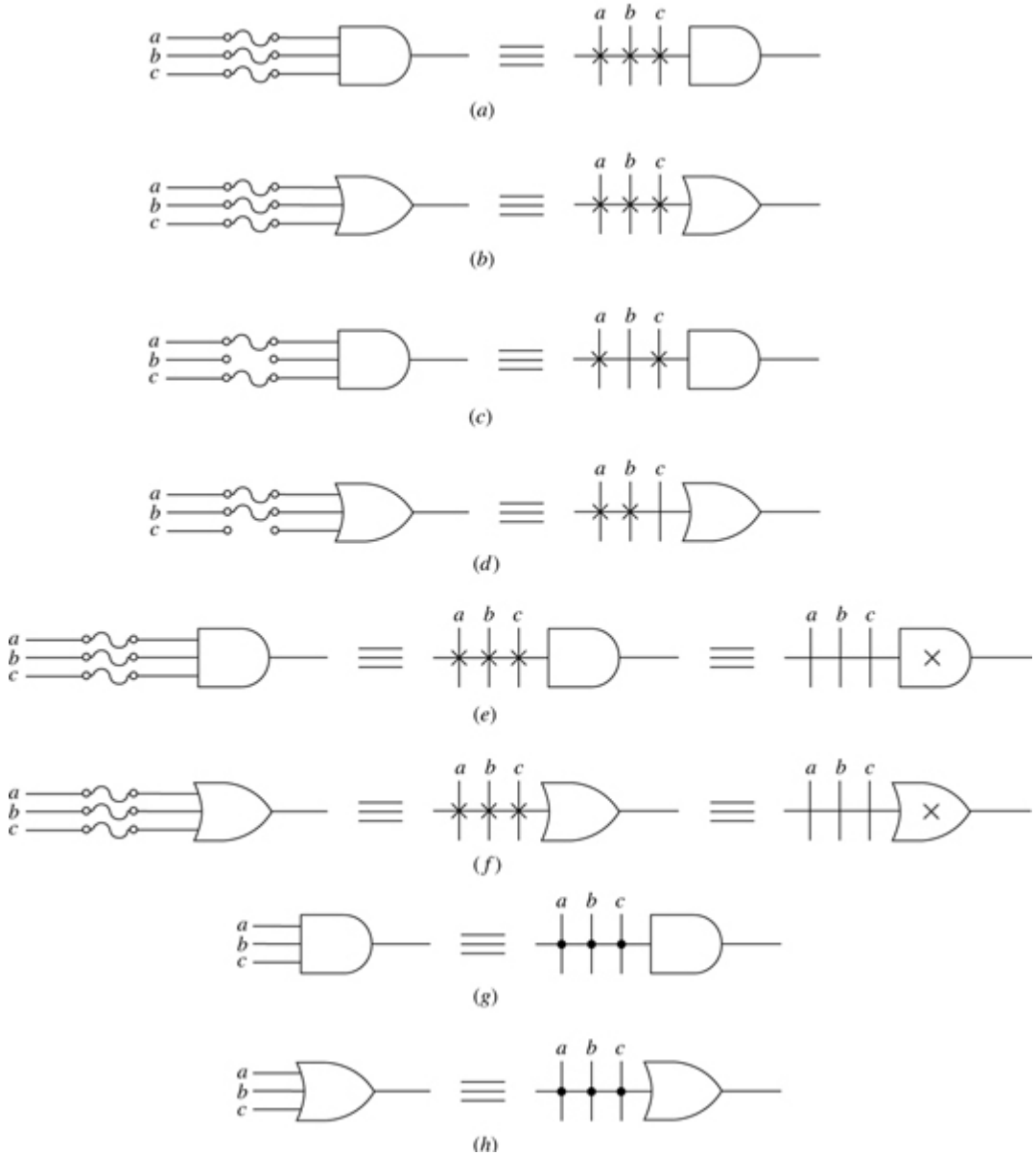


(*a*)

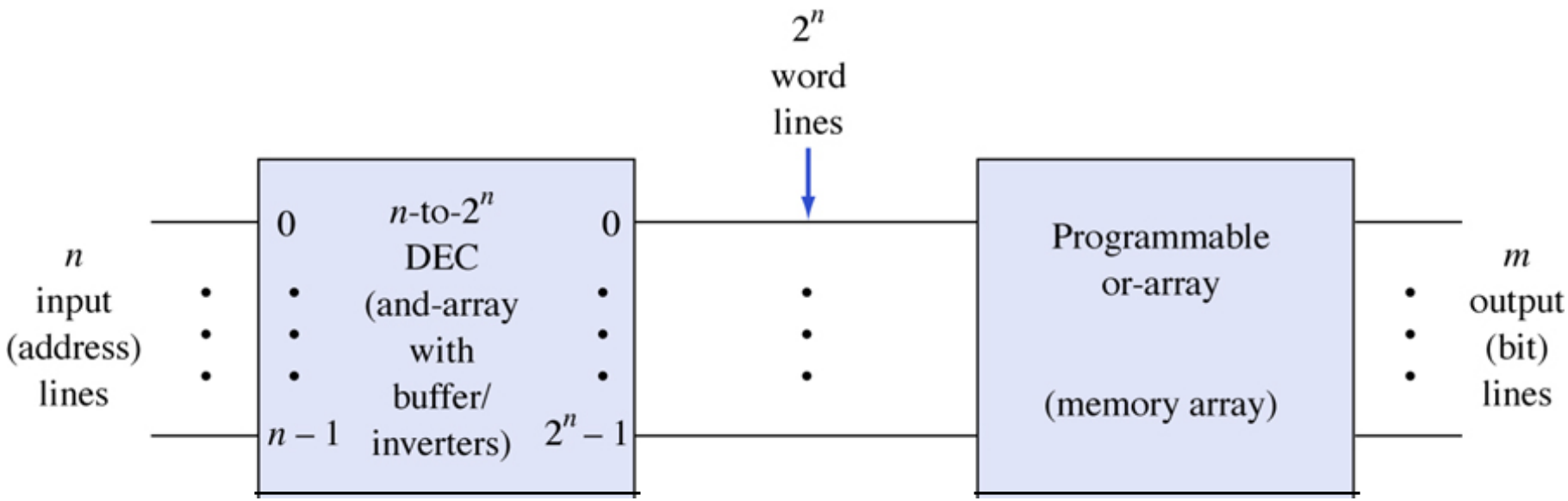


(*b*)

PLD notation



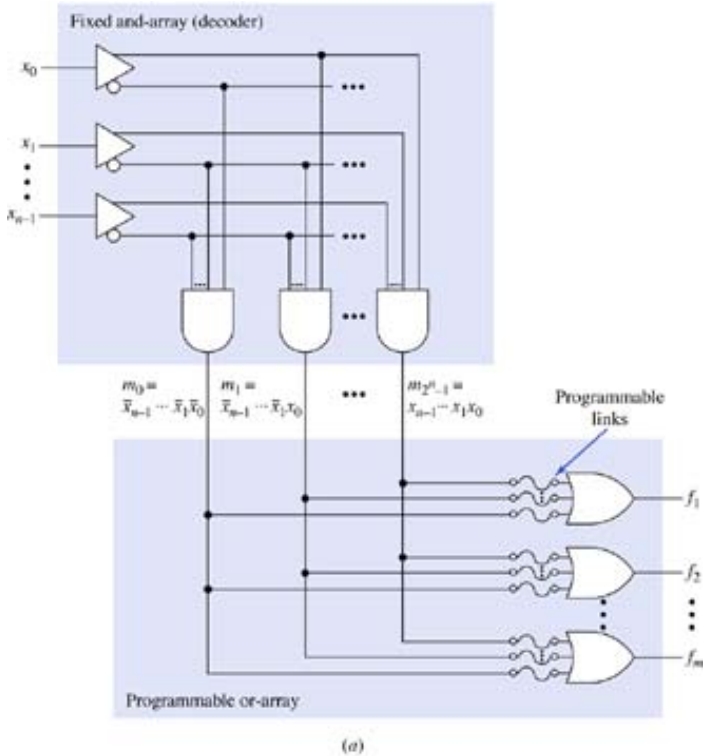
Structure of a PROM

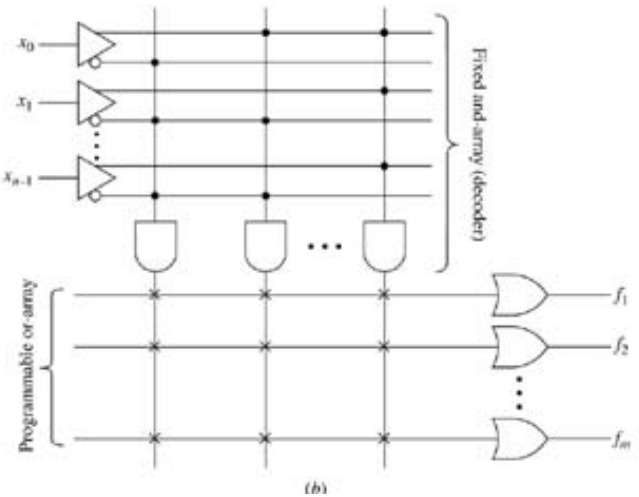


A $2^n \times m$ PROM.

(a) Logic diagram.

(*b*) Representation in PLD notation.

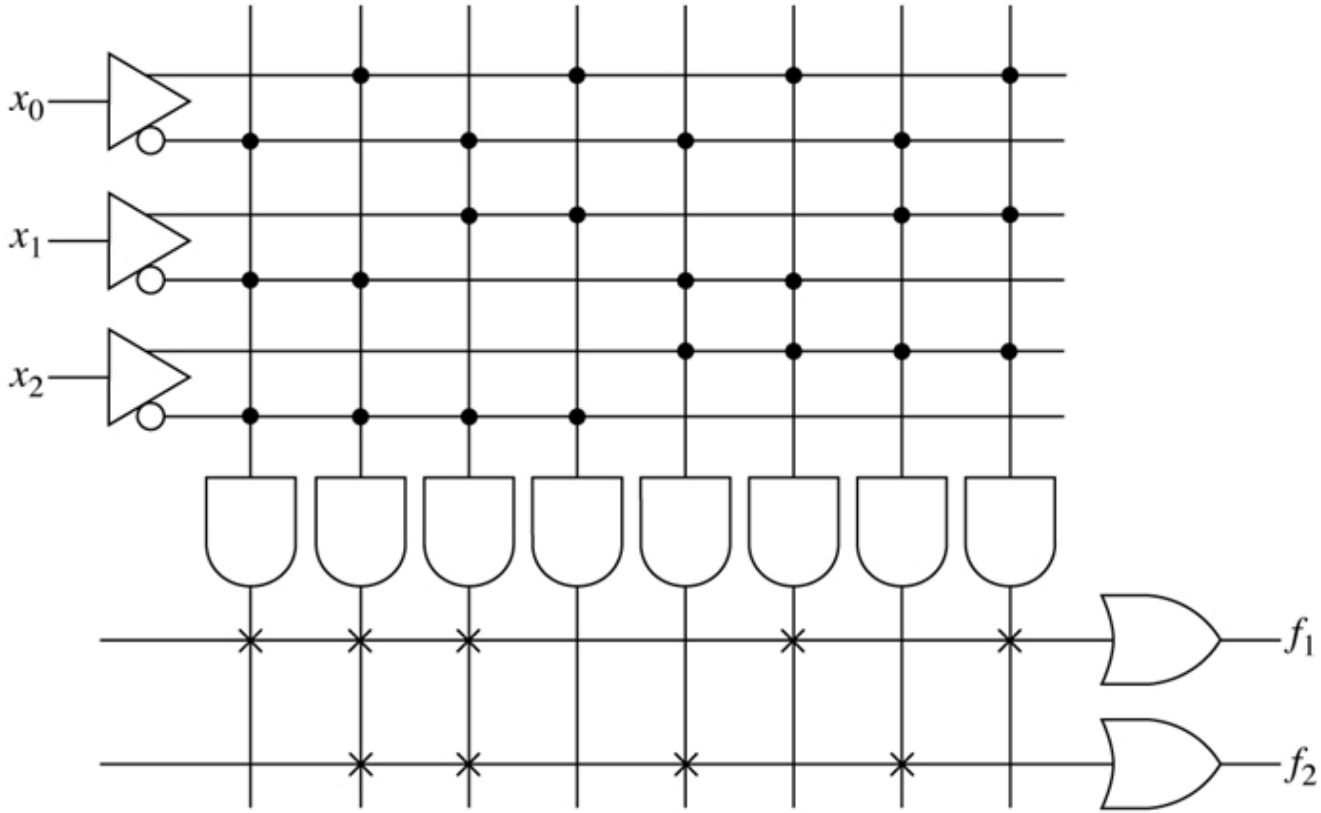




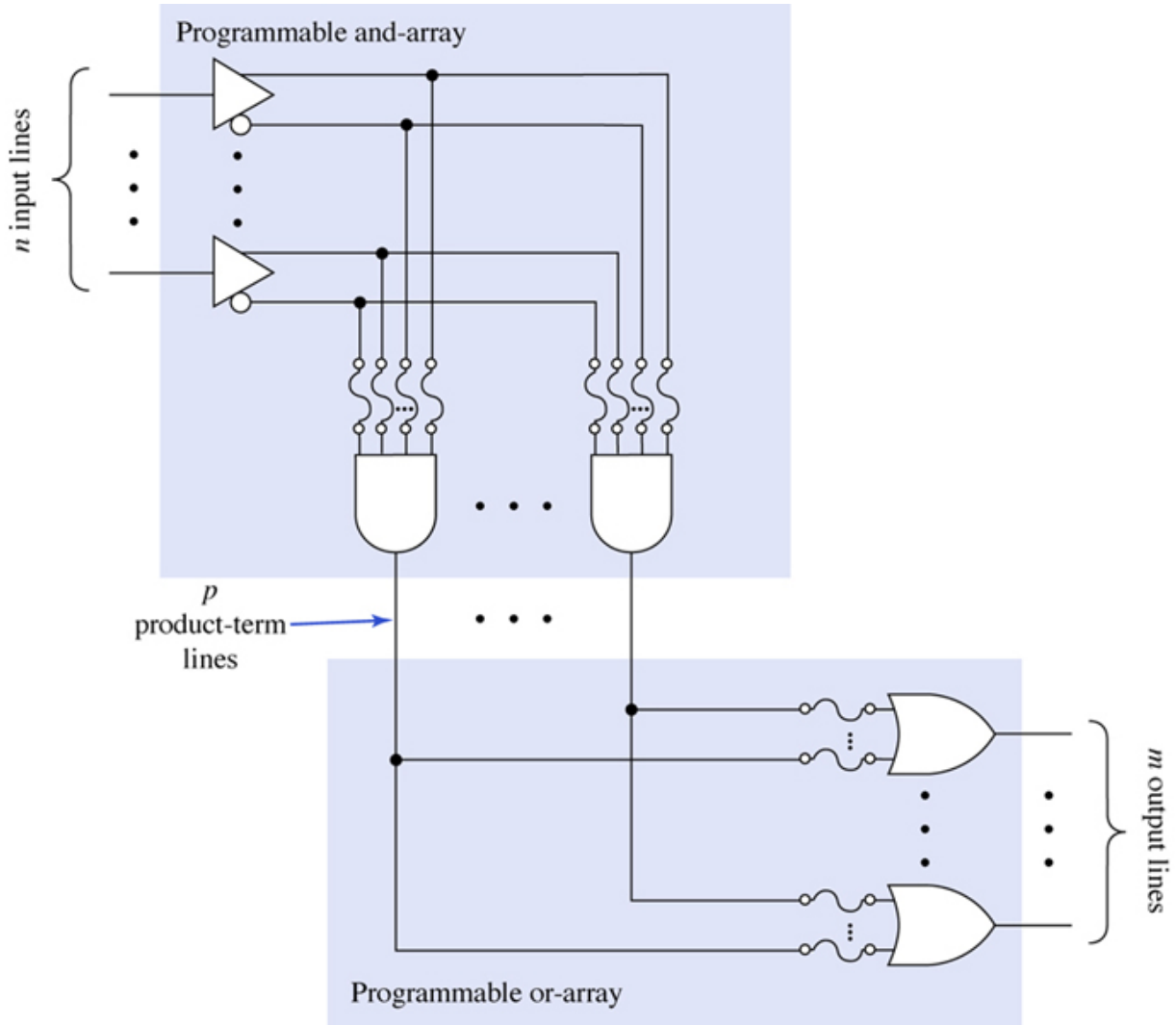
Using a PROM for logic design. (*a*) Truth table. (*b*) PROM realization.

x_2	x_1	x_0	f_1	f_2
0	0	0	1	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

(*a*)

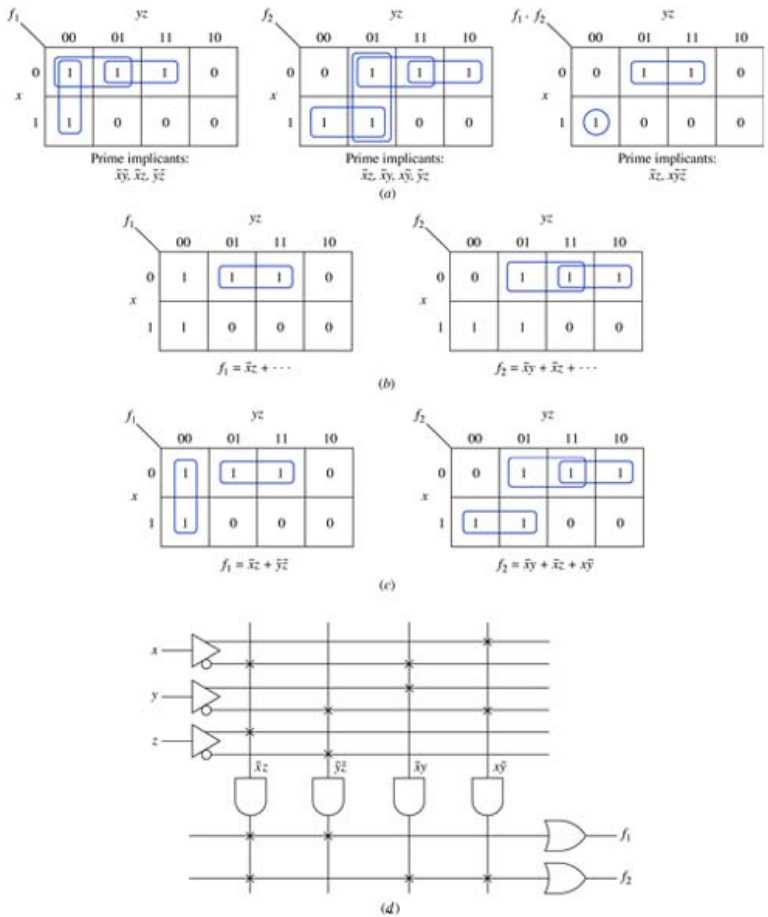


(*b*)

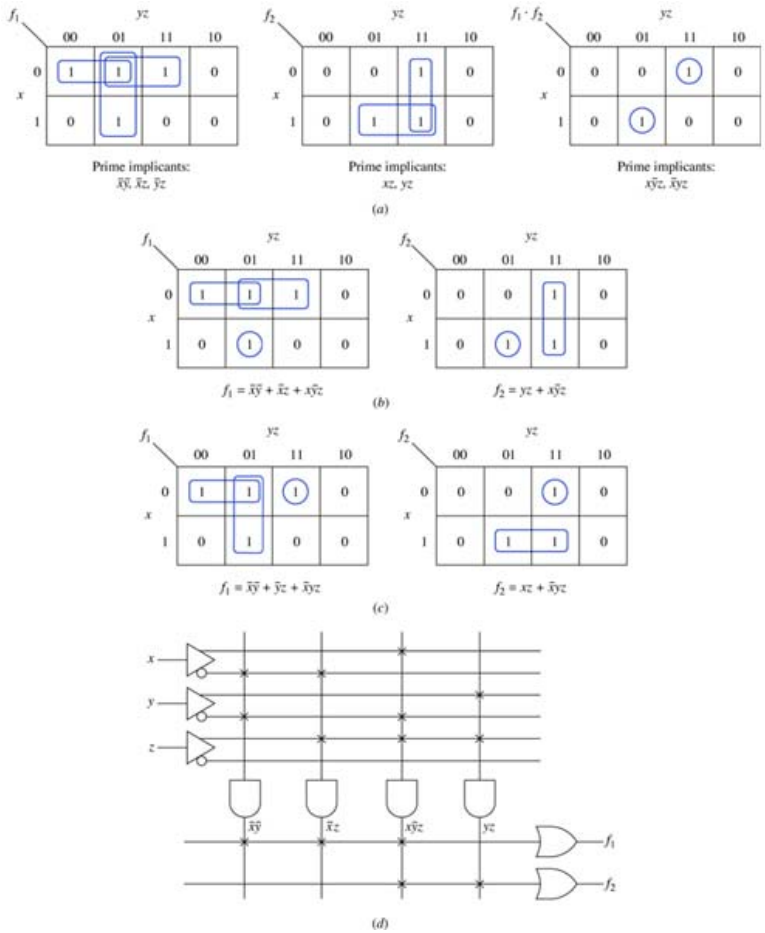


Logic diagram of an $n \times p \times m$ PLA

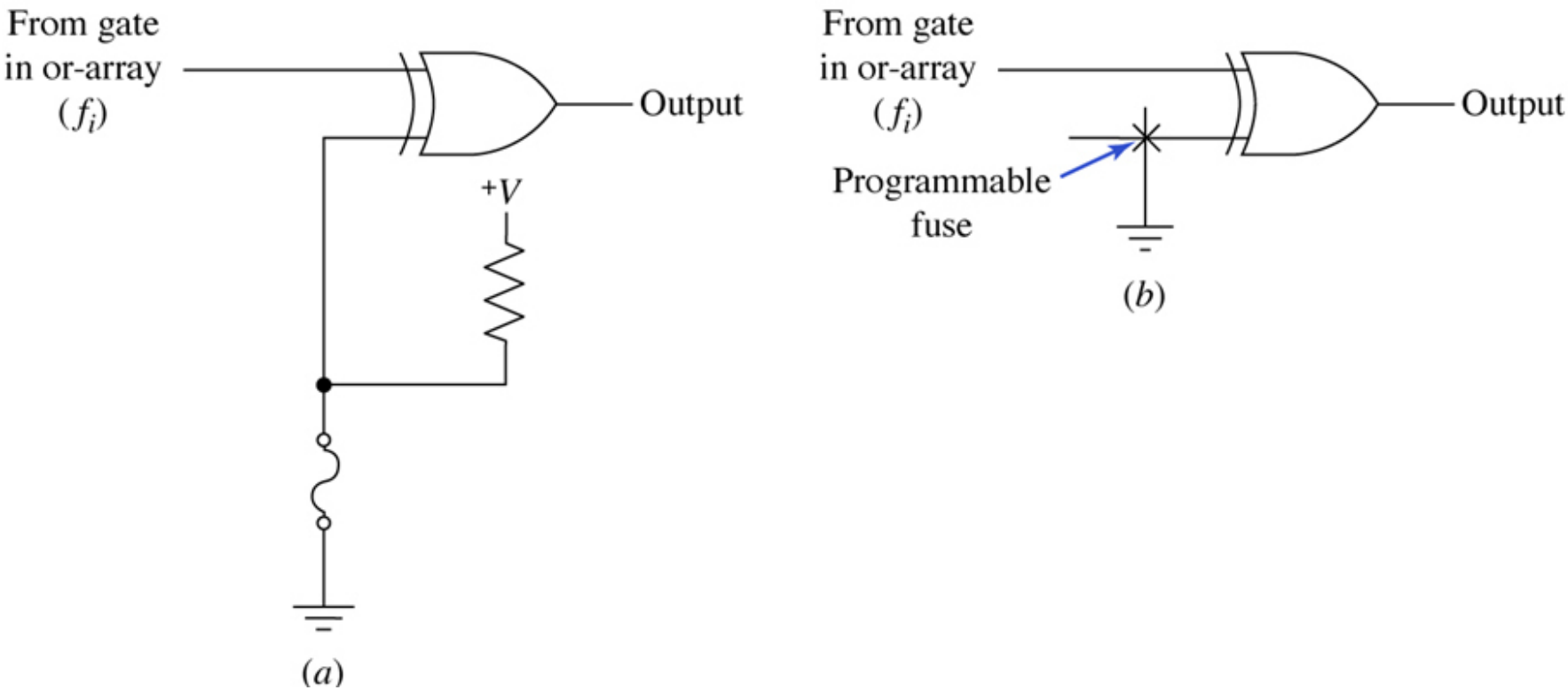
Example of combinational logic design using a PLA. (*a*) Maps showing the multiple-output prime implicants. (*b*) Partial covering of the f_1 and f_2 maps. (*c*) Maps for the multiple-output minimal sum. (*d*) Realization using a $3 \times 4 \times 2$ PLA.



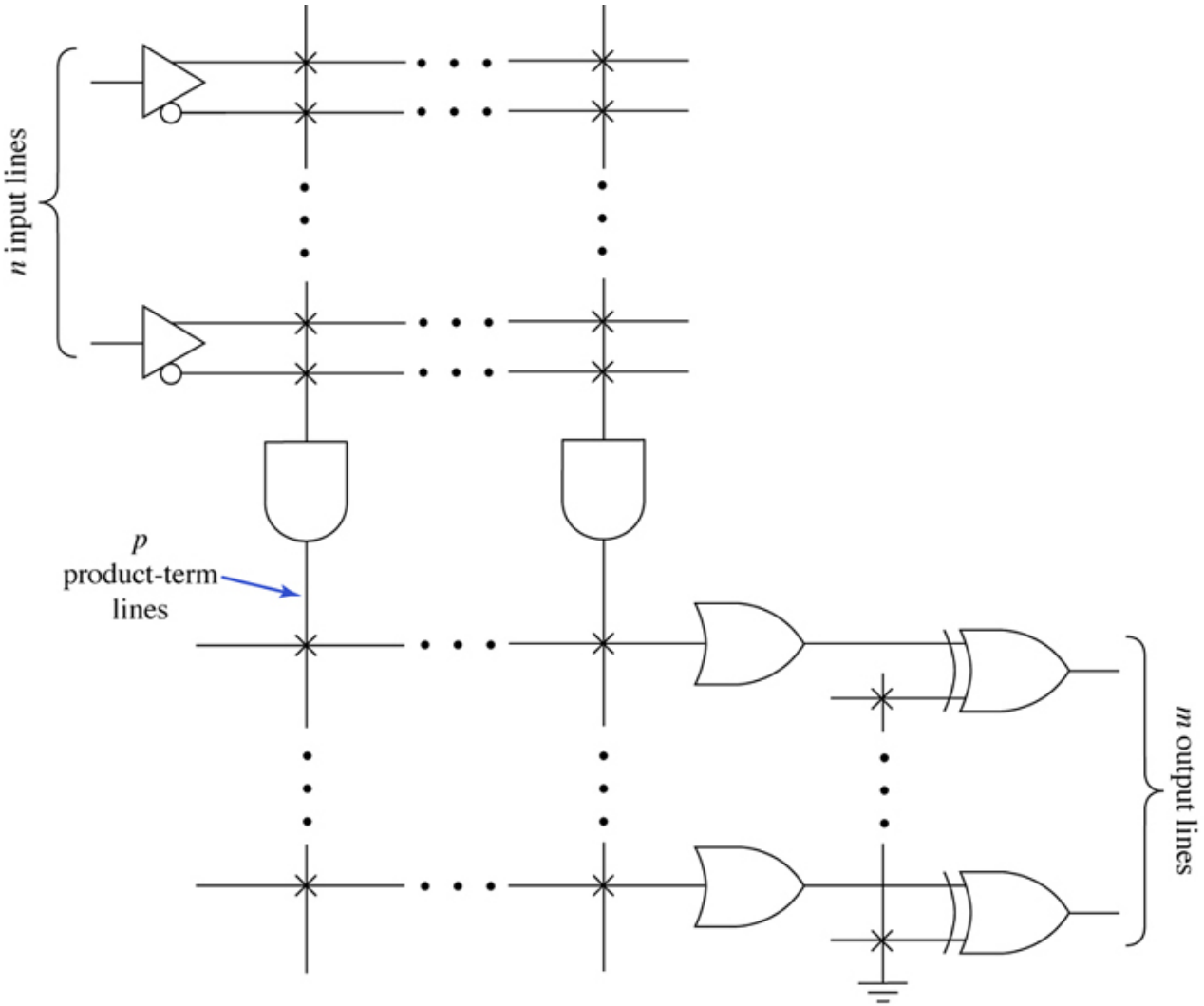
Example of combinational logic design using a PLA. (*a*) Maps showing the multiple-output prime implicants. (*b*) A multiple-output minimal sum covering. (*c*) Alternative multiple-output minimal sum covering. (*d*) Realization using a $3 \times 4 \times 2$ PLA.



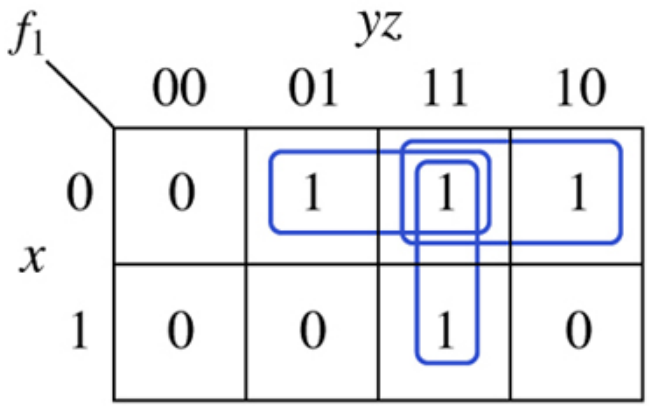
Exclusive-or-gate with a programmable fuse.
(*a*) Circuit diagram. (*b*) Symbolic representation.



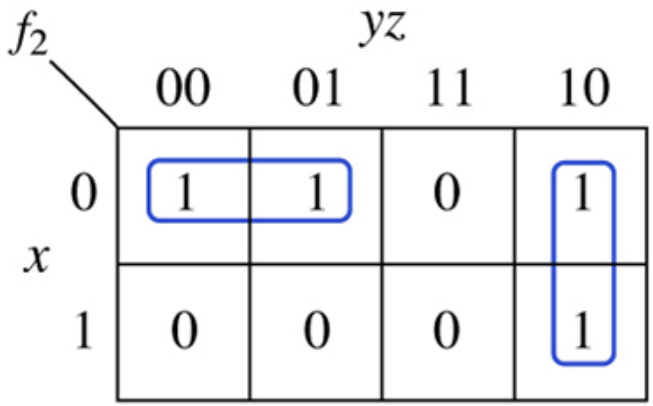
General structure of a PLA having true and complemented output capability



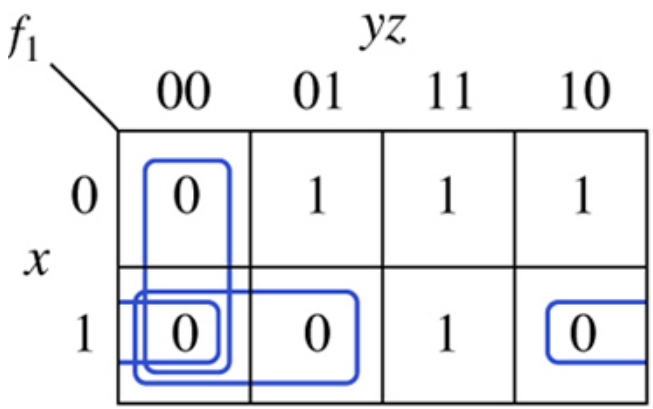
Karnaugh maps for the functions $f_1(x,y,z) = \Sigma m(1,2,3,7)$ and $f_2(x,y,z) = \Sigma m(0,1,2,6)$



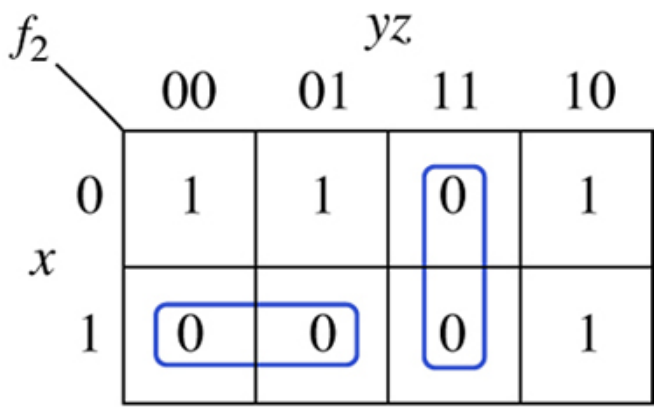
$f_1 = \bar{x}z + \bar{x}y + yz$



$f_2 = \bar{x}\bar{y} + y\bar{z}$

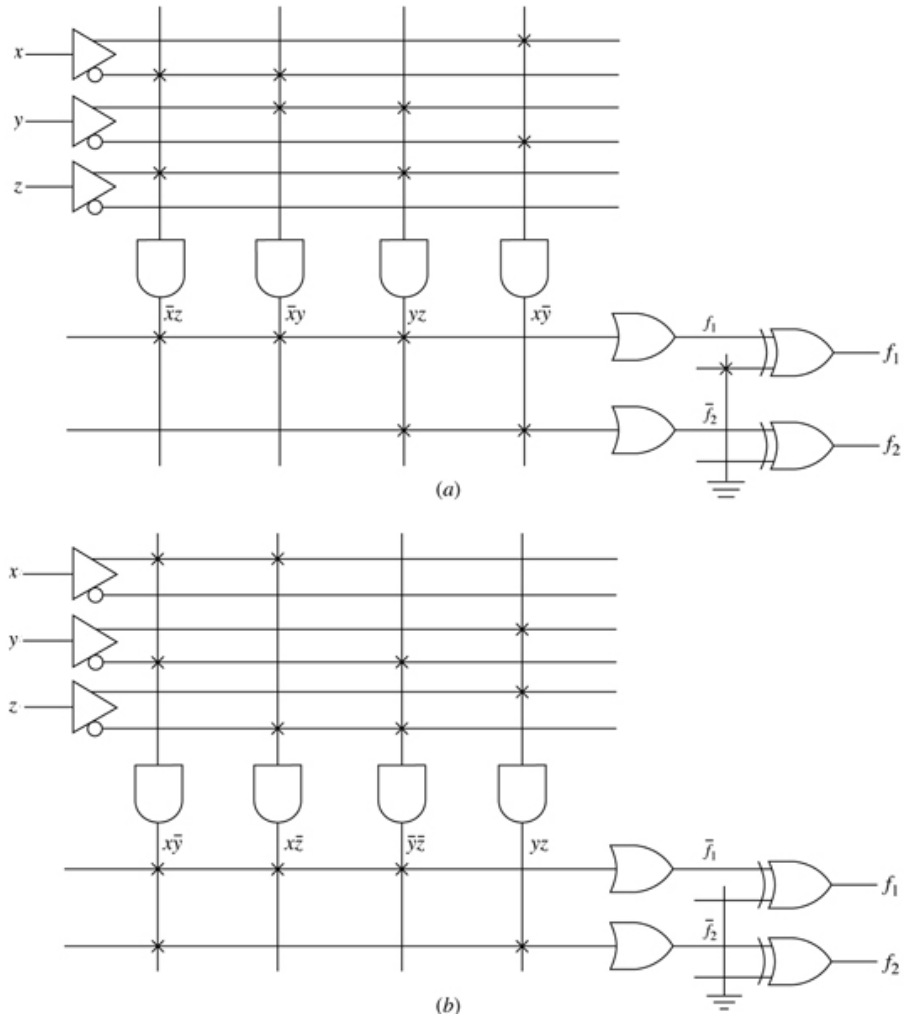


$\bar{f}_1 = x\bar{y} + x\bar{z} + \bar{y}\bar{z}$

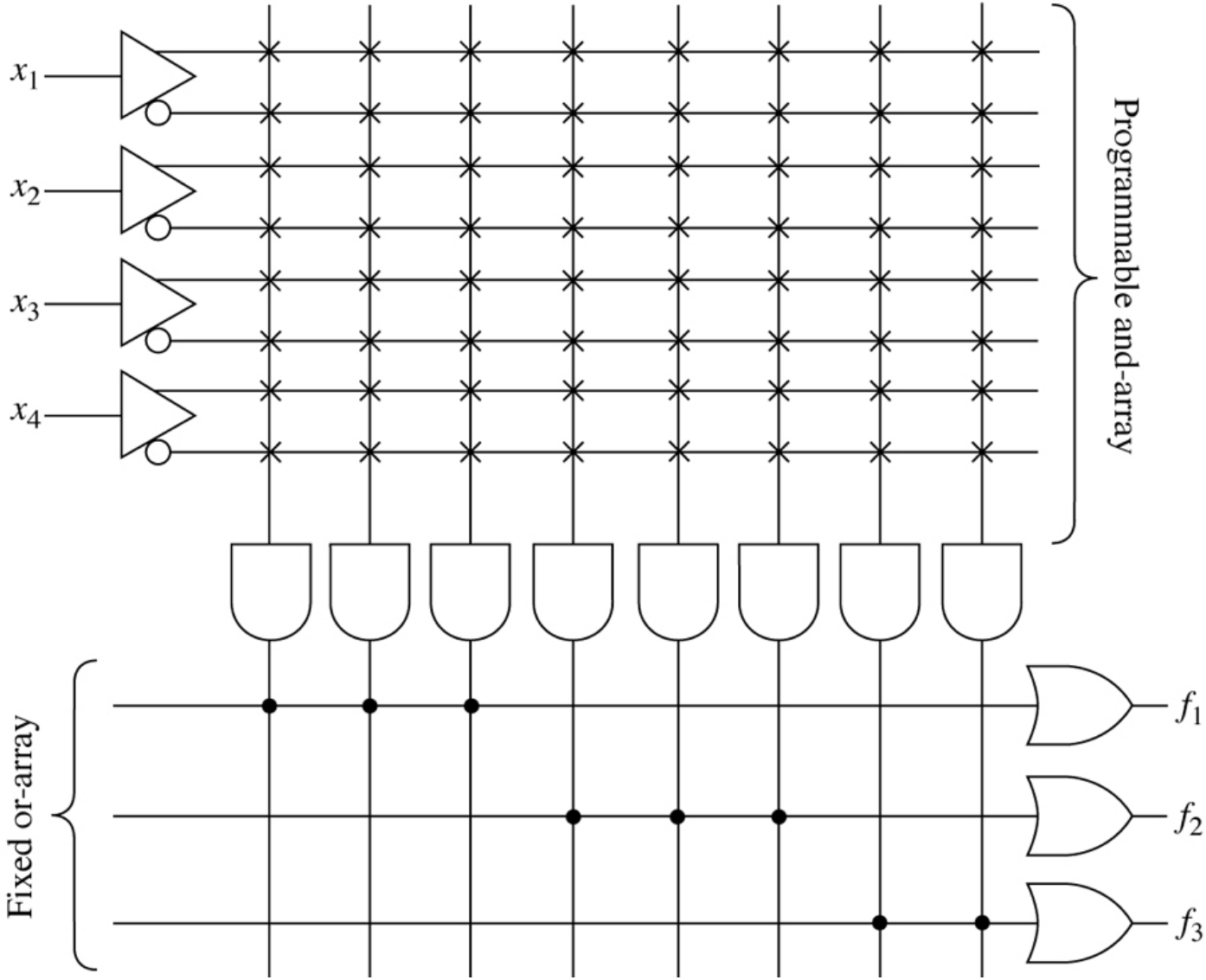


$\bar{f}_2 = x\bar{y} + yz$

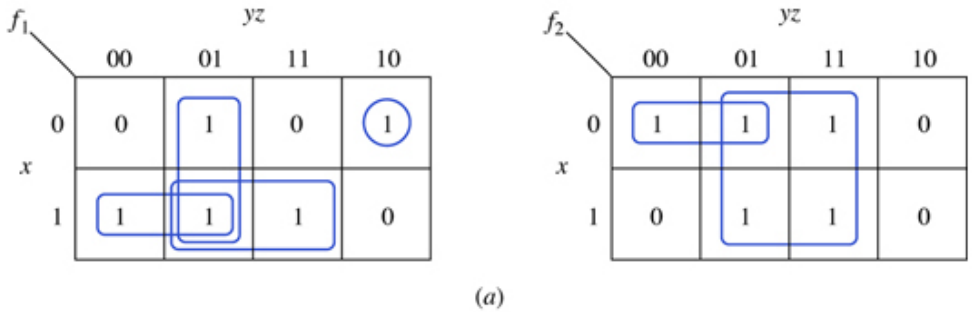
Two realizations of $f_1(x,y,z) = \Sigma m(1,2,3,7)$ and $f_2(x,y,z) = \Sigma m(0,1,2,6)$.



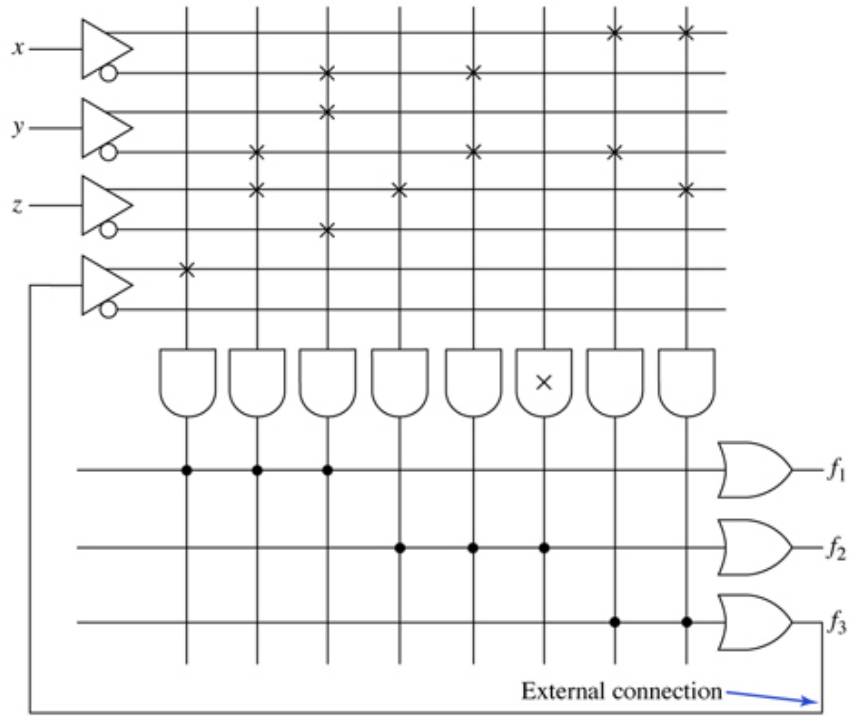
A simple four-input, three-output PAL device.



An example of using a PAL device to realize two Boolean functions. (*a*) Karnaugh maps. (*b*) Realization.

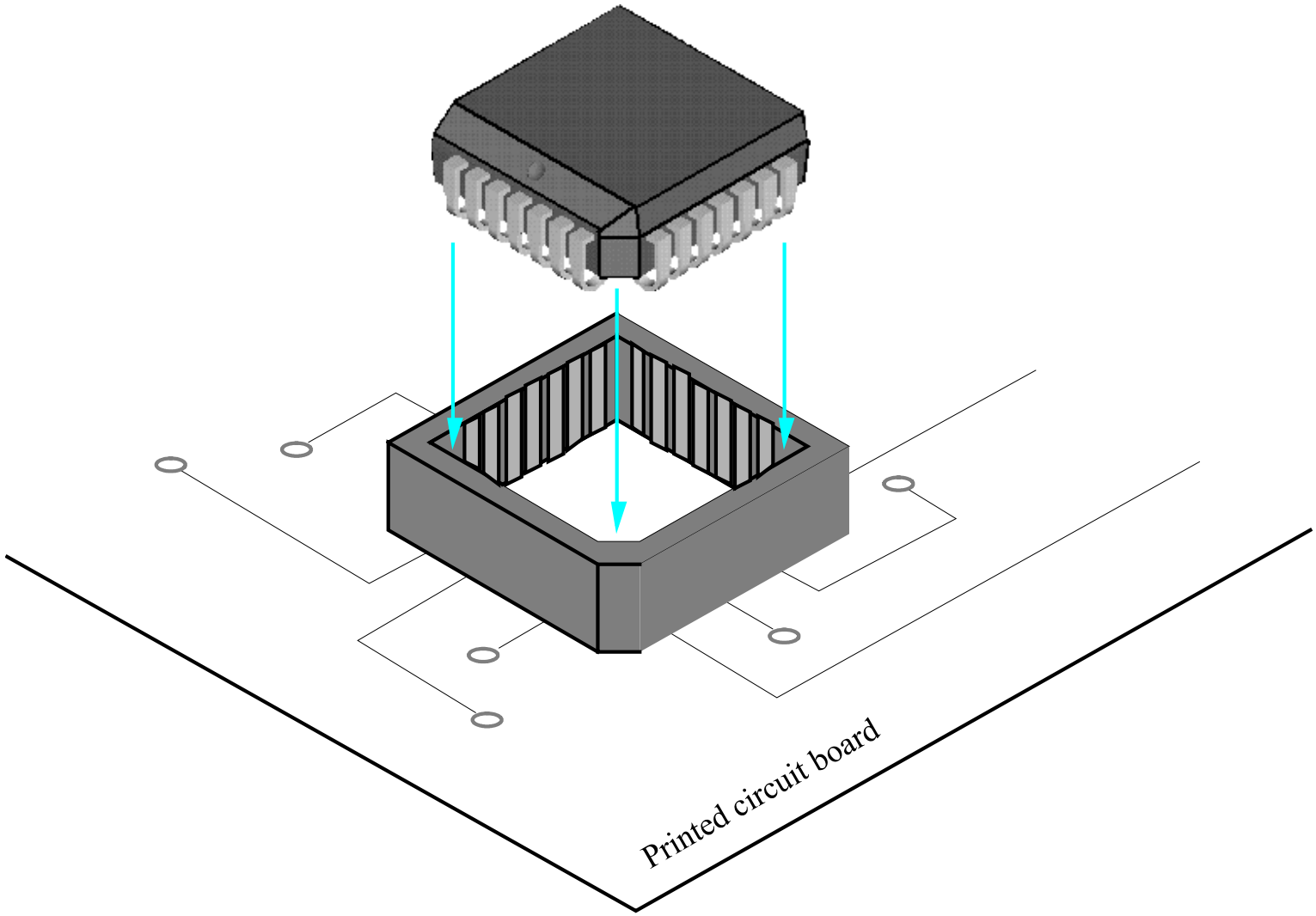


(*a*)



(*b*)

A PLD programming unit



A PLCC package with socket

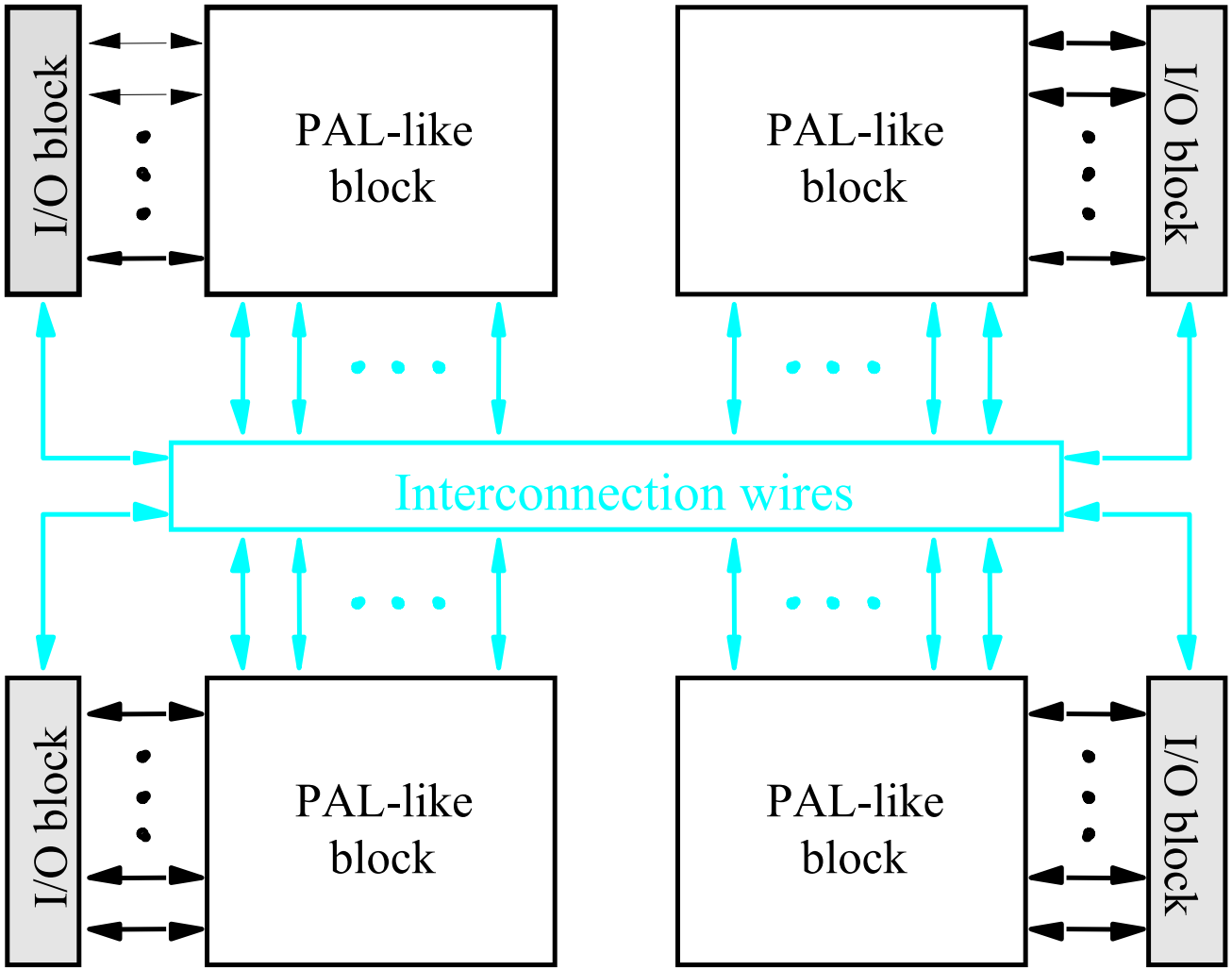
Limitations of PLAs and PALs

These chips are limited to fairly modest size, typically supporting a combined number of inputs plus outputs of not more than 32.

Complex Programmable Logic Devices (CPLDs)

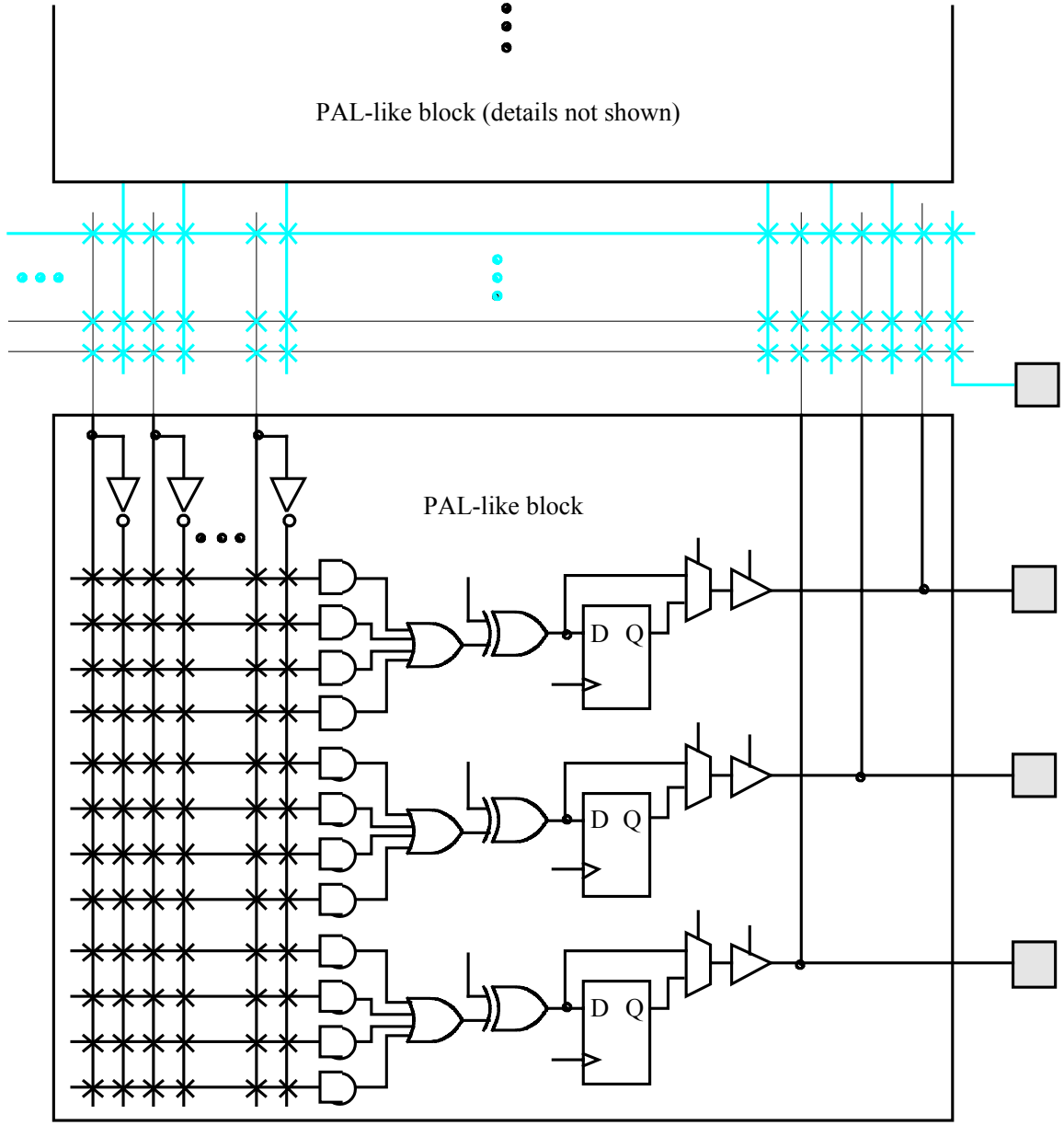
A CPLD comprises multiple PAL-like blocks on a single chip with internal wiring resources to connect the circuit blocks.

It is made to implement complex circuits that cannot be done on a PAL or PLA.

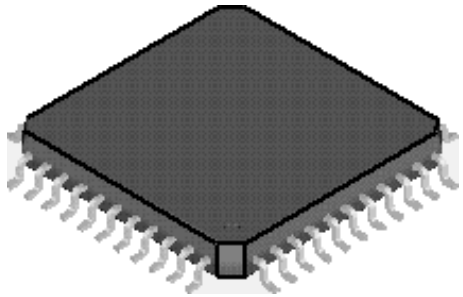


Structure of a CPLD

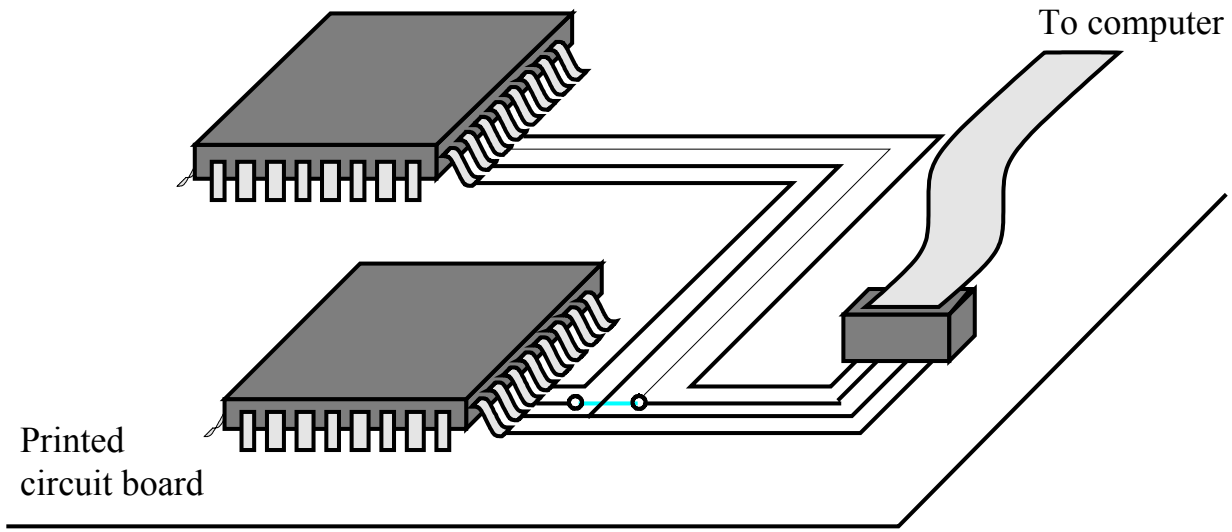
A section of a CPLD



CPLD packaging and programming



(a) CPLD in a Quad Flat Pack (QFP) package



(b) JTAG programming

A Measure of Circuit Size

A commonly used measure is the total number of two-input NAND gates that would be needed to build the circuit.

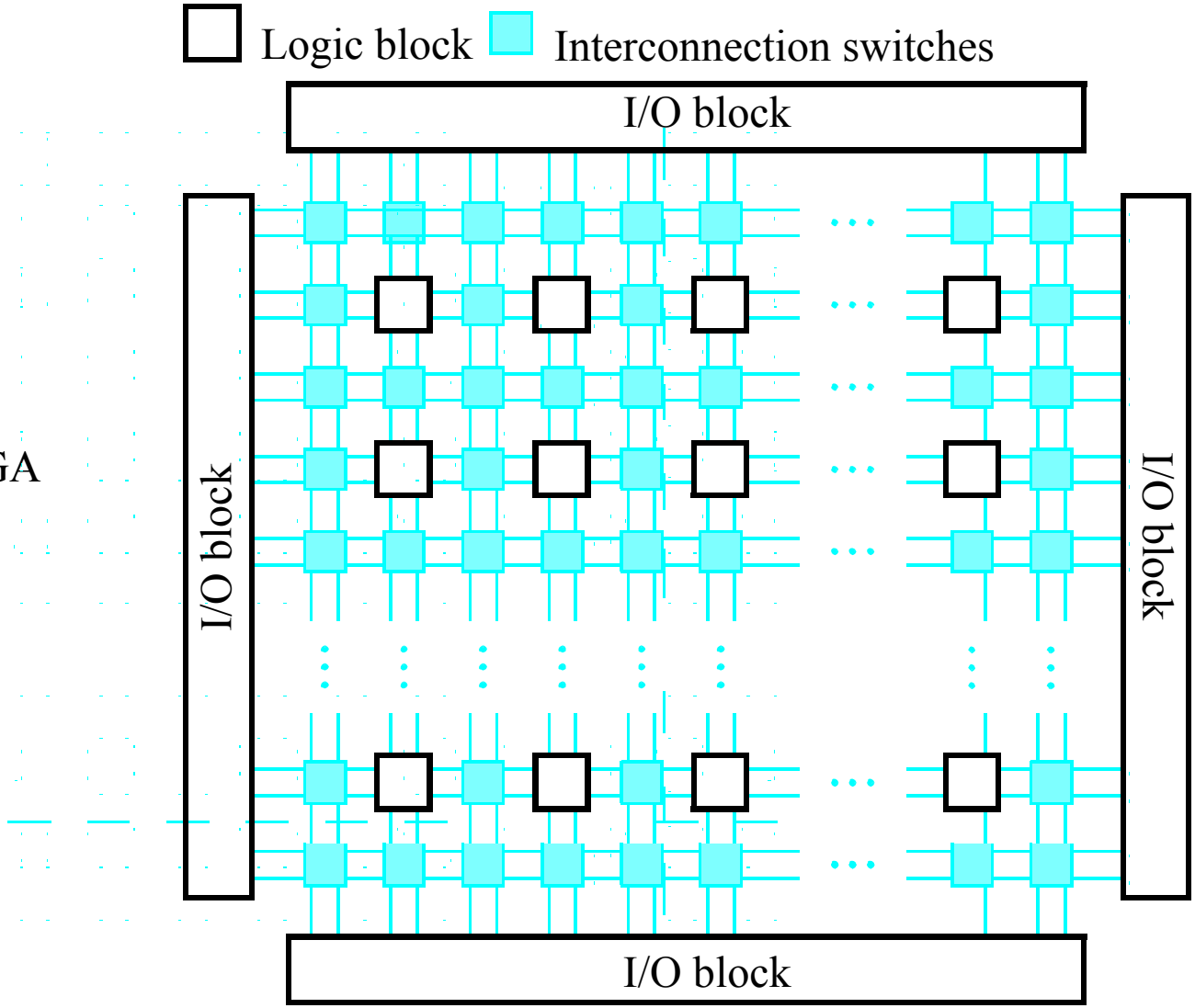
It is called the *number of equivalent gates*.

Field-Programmable Gate Arrays (FPGAs)

An FPGA is a PLD that supports implementation of large logic circuits.

It is different from others in that it does not contain AND or OR planes. Instead, it contains logic blocks as depicted in the next slide.

Structure of an FPGA



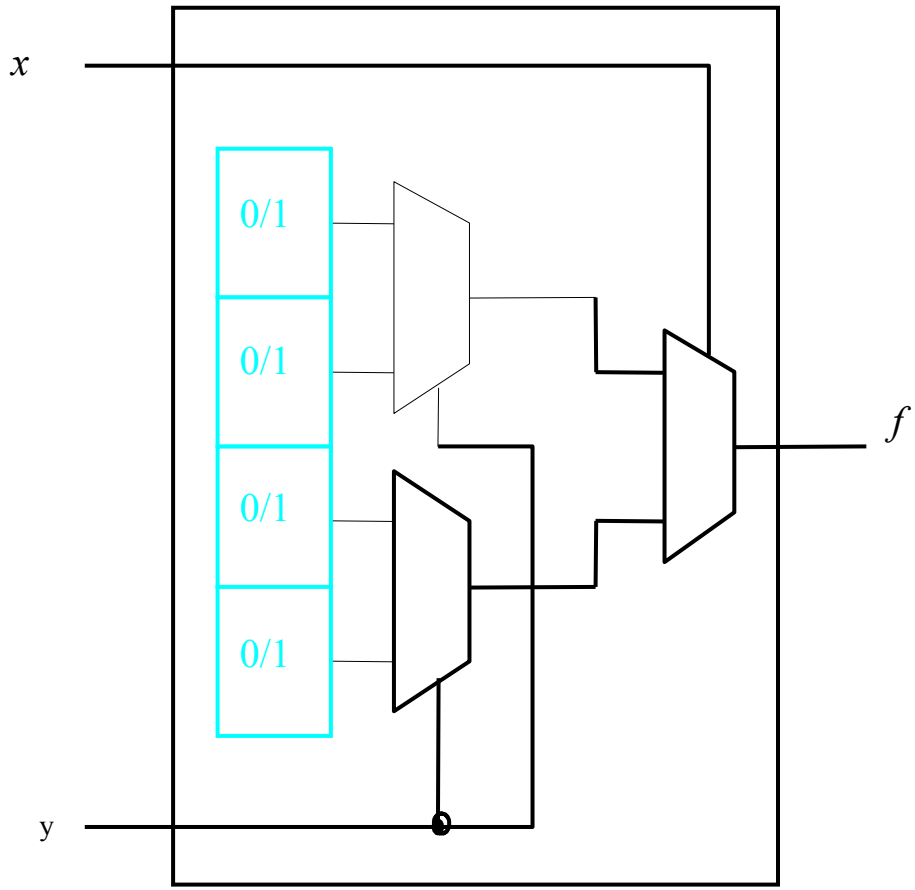
Typical FPGAs

FPGAs can be used to implement logic circuits of more than a few hundred thousand equivalent gates in size.

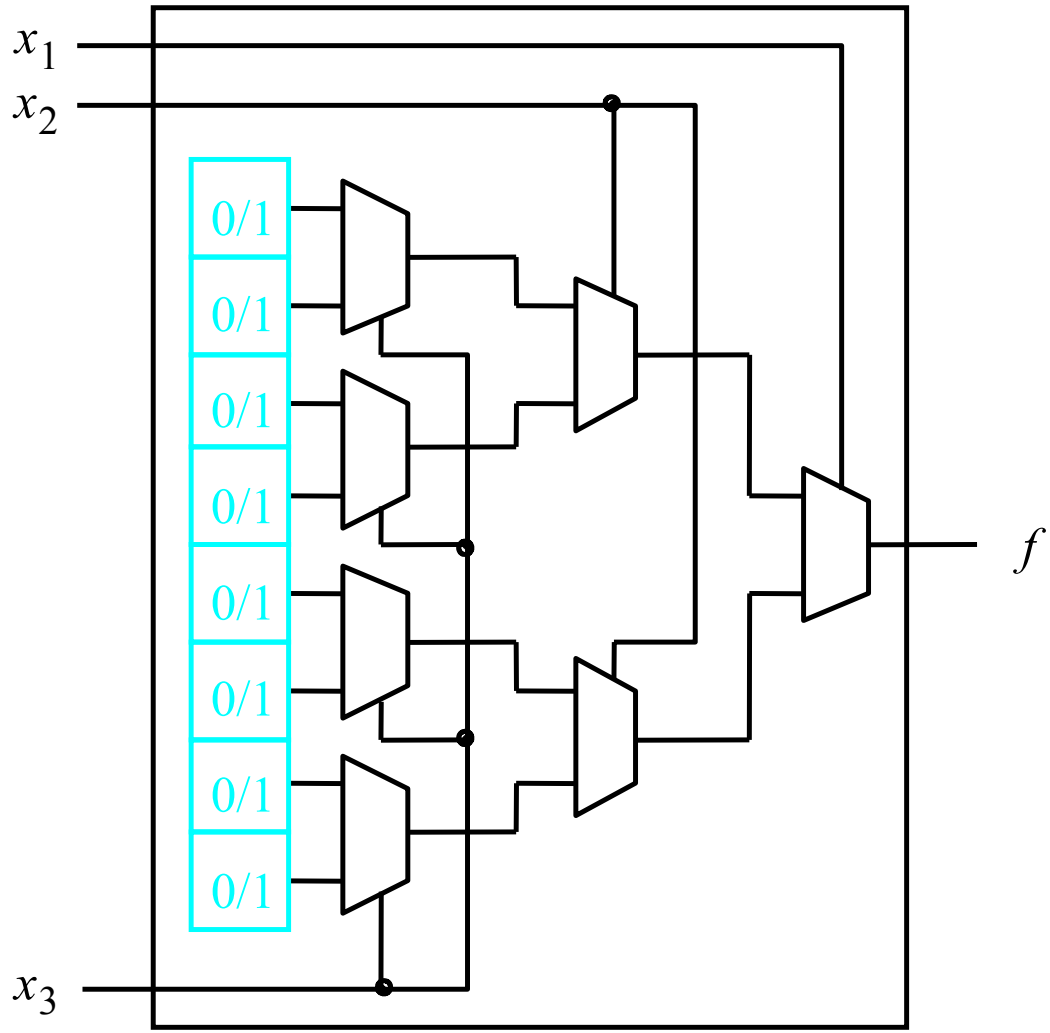
The most commonly used logic block is a *lookup table* (*LUT*) as depicted in Fig. 3.36.

Slide 3.35.1

A two-input lookup table



A three-input LUT



A section of a programmed FPGA