

Continuous time models: multiple variable

4TTV313U - Introduction to simulation of dynamic process

University of Bordeaux - 2022

Epidemiological model

Until now, we have worked with a single-state variable model. We will now extend to several-compartment models.

For this, we will consider a very simple epidemiological model, the SIR model. The acronym SIR stands for:

- Susceptible, the fraction of people who can contract the disease if they are in contact with it
- Infectious, the fraction of people who have the disease and can spread it
- Recovered, the fraction of people who have had the disease but are now immune to reinfection.

$$\begin{aligned}\frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I\end{aligned}$$

Transitions between one state to another are parameterised through the β and γ parameters.

We know that people with the disease are infectious for 4 days on average: in the absence of individual information, we can expect 1 person out of 4 to recover on any particular day. The recovery rate γ is therefore 0.25. We assume that a susceptible person is in contact with an infected one every 3 days. With the same logic, the infection rate β is 0.33.

The following code can be used to model the system.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy import integrate

def sir_der(X,t,sys):
    s,i,r=X
    beta,gamma=sys['beta'],sys['gamma']
    dsdt=-beta*s*i
    didt=beta*s*i-gamma*i
    drdt=gamma*i
    dXdt=[dsdt,didt,drdt]
    return dXdt

tc=3
tr=4
beta=1.0/tc
gamma=1.0/tr
```

```

syst={"beta":beta,"gamma":gamma}

t=np.linspace(0,int(365/2),int(365/2))

### Initialization
S_0=89
I_0=1
R_0=0
T_0=S_0+I_0+R_0

X_0=[S_0/T_0,I_0/T_0,R_0/T_0]

y=integrate.odeint(sir_der,X_0,t,args=(syst,))

```

Assuming that the population at the beginning is made of 90 persons (the total population, $T_0 = 90$) and that there is only one person infected while all others have never been in contact with the disease, model and plot the spread of the disease for 6 months (i.e., the fraction of infected people during 6 months).

Intervention

Models such as this one can be used to explore different intervention scenarios. The goal is to minimise the maximum number of infected persons during a certain period ($\max(N_{I,t}), t \in [t_0, t_{end}]$, where $N_{I,t} = T_0 * I_t$), and the total number of persons that have been infected during the period, N_{TI} (another way to look at it is the difference between the number of persons susceptible at the beginning and at the end of the period). Do not forget that you cannot have fractions of people. We will explore two of these scenarios: immunisation through vaccination, and a balance between immunisation and campaigning for other preventive measures.

Immunisation We will consider that immunisation removes a fraction of the population from the susceptible group to the resistant group. First, run a simulation with a vaccination rate v_r of 10% at the beginning of the simulation. Then, look at the effect of varying the initial vaccination rate between 0 and 100% on the maximum of infected persons over 6 months (plot $\max(N_{I,t}) = f(v_r)$ and $N_{TI} = f(v_r)$).

Campaigning The campaign mostly impacts the infection rate, that is the β parameter. Observations in other countries show that the effectiveness of the campaign (that is, the reduction in β) does not vary completely linearly with the money spent m_s . It does decrease linearly from 0 to \$500, up to a 20% reduction in β but further investment does not affect the infection rate (see Fig. 1). Define the corresponding function $\beta = f(m_s)$ and compute $\max(N_{I,t})$ and N_{TI} when the campaign cost is \$500.

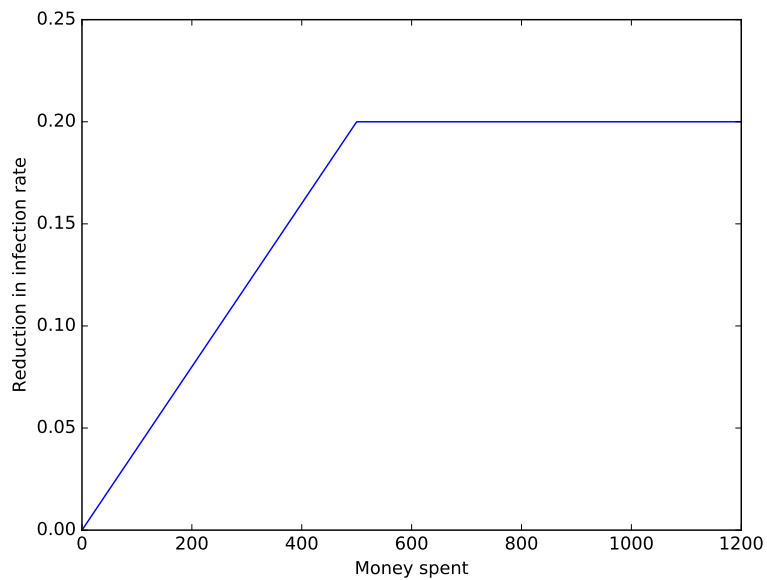


Figure 1: Reduction in the infection parameter β as a function of the money spent on a health campaign

Choosing a strategy To define a public health strategy, you are given \$1200. You have two choices: you can vaccinate as much people as possible (one shot costs \$100) or you can invest in a campaign to remind basic hygiene rules to the students. Of course, you can find a balance between these two investments.

Find the optimal balance between spending money on vaccination or health campaign.

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