

Three Final projects on dynamic process simulation

4TTV313U - Introduction to simulation of dynamic process

University of Bordeaux - 2021

1 Discrete model of a population

We model the evolution of a population with the logistic equation defined by :

$$x_{n+1} = r \cdot x_n \cdot \left(1 - \frac{x_n}{K}\right),$$

where x_n is the size of the population at time n , r is the growth rate, and K is the maximal capacity of the population.

This example is from the book : *Modélisation mathématique en écologie*, P. Auger, C. Lett and J.C. Poggiale, 2015, DUNOD.

1. For $r = 1.5$, and $K = 200$, write a program that displays, in the same graphic, the evolution of different populations, starting with different sizes in $[0, 200[$ with a simulation step number of $N = 1000$. In this graphic, the x -coordinates of the points represent the time, and the y -coordinates represent the size of the population.

Does the population stabilize towards a limit value?

2. For what values of r does the population die out?
3. Are there values of r for which the population does not stabilize ? If so, does it oscillate between several steady-state values ?
4. Write a program that displays the bifurcation curve. It is a program that produces a 2D picture containing the different steady-states of the system : for each growth rate r , display the steady-state values of the population evolution. The x -axis represents the growth rate and the y -axis represents the size of the population.

2 Continuous model of an adversarial population

Now, We study the growth model of two adversarial populations defined by :

$$\begin{cases} \partial_t x = r_1 \cdot x \cdot \left(1 - \frac{x}{K_1} - \alpha \cdot \frac{y}{K_2}\right), \\ \partial_t y = r_2 \cdot y \cdot \left(1 - \frac{y}{K_2} - \beta \cdot \frac{x}{K_1}\right), \end{cases}$$

where K_1 and K_2 are the maximal capacity of the two populations, x and y are the size of the two populations and α, β are the strength of the competition.

This example is from the book : *Modélisation mathématique en écologie*, P. Auger, C. Lett and J.C. Poggiale, 2015, DUNOD.

In this exercise, we suppose that $K_1 = 200$ and $K_2 = 300$.

1. For $r_1 = r_2 = 1$, $\alpha \cdot \frac{K_2}{K_1} = 0.1$ and $\beta \cdot \frac{K_1}{K_2} = 0.2$, write a program that display, in the same graphic, the evolution of different populations, starting with different sizes (x, y) in $[0, 200[\times [0, 200[$ during 100 seconds. You will display the evolution of the populations in 2D, where

- the x -axis represents the size of the population of x ,
- the y -axis represents the size of the population of y .

Do the different population sizes stabilize towards some limit values ? There is a coexistence between the populations ?

2. Make the same work with $r_1 = r_2 = 1$, $\alpha \cdot \frac{K_2}{K_1} = 2.5$ and $\beta \cdot \frac{K_1}{K_2} = 1.1$. Comment the different results.
3. Are there parameters where, whatever the initial populations, a single population remains ? If so, give an example of parameters where just x remains.

3 Motor identification

We will use some data to identify an electric motor. The input of the motor is an electric voltage. The output is the velocity of the motor.

In the file https://www.labri.fr/perso/boussica/archives/cours/4TTV313U/motor_2021.csv, you will find some data of an unknown electric motor, containing

- the electric voltage applied to the motor,
- the resulting velocity of the motor,

sampled at 5 KHz.

A simple model of the motor in movement is given by the following equation:

$$\partial_t w = -\alpha_1 \cdot w - \alpha_2 \cdot \epsilon(w) + \alpha_3 \cdot u$$

where w is the velocity of the motor, u is the electric voltage of the motor, and $\epsilon(w)$ is the sign of w .

Write a program that computes the parameters that minimise the average quadratic error between the data and the simulation.

Display, on the same graphic, the evolution of u and w for the data and for the simulation by using the best parameters you have obtained.