

# Three final projects on dynamic process simulation

4TTV313U - Introduction to simulation of dynamic process

University of Bordeaux - 2023

Version 1

## 1 Discrete model of a population

The evolution of a population containing hosts and parasites is modeled by :

$$\begin{cases} x_{n+1} = \lambda \cdot x_n \cdot \exp(-ay_n), \\ y_{n+1} = c \cdot x_n \cdot (1 - \exp(-ay_n)), \end{cases}$$

where  $x_n$  is the size of the host population at time  $n$ ,  $y_n$  is the size of the parasite population at time  $n$ ,  $\lambda$  is the growth rate of the host population,  $c$  is the average number of parasites by host, and  $a$  is a parameter where  $\exp(-ay_n)$  represents the host proportion that is not parasited and that contributes to the next generation.

This model is the Nicholson-Bailey model and this is an example coming from the book : *Modélisation mathématique en écologie*, P. Auger, C. Lett and J.C. Poggiale, 2015, DUNOD.

1. For  $a = 0.15$ ,  $\lambda = 2$ ,  $c = 1$ , write a program that displays, in the same graphic, the evolution of different populations, starting with different sizes in  $[0, 200[$  with a simulation step number of  $N = 30$ . Explain the simulation result for  $N = 1000$ . In this graphic, the  $x$ -coordinates of the points represent the time, and the  $y$ -coordinates represent the size of the populations.
2. Display the same evolution in a 2D drawing where each sample  $(x_n, y_n)$  is represented by a point in the 2D drawing.
3. Does the population stabilize towards a limit value?
4. Suppose that populations can stabilize toward a limit value. Calculate, mathematically that value by setting  $y_n = y$  and  $x_n = x$ .
5. try (if possible) to reach these values experimentally by changing initial conditions and model parameters.

Answer to these questions for that model now :

$$\begin{cases} x_{n+1} = \lambda \cdot x_n \cdot \left(1 + \frac{ay_n}{b}\right)^{-b}, \\ y_{n+1} = c \cdot x_n \cdot \left(1 - \left(1 + \frac{ay_n}{b}\right)^{-b}\right). \end{cases}$$

## 2 Continuous model of an adversarial population

Now, We study the growth model of two adversarial populations defined by :

$$\begin{cases} \partial_t x = r_1 \cdot x \cdot \left(1 - \frac{x}{K_1} - \alpha \cdot \frac{y}{K_2}\right), \\ \partial_t y = r_2 \cdot y \cdot \left(1 - \frac{y}{K_2} - \beta \cdot \frac{x}{K_1}\right), \end{cases}$$

where  $K_1$  and  $K_2$  are the maximal capacity of the two populations,  $x$  and  $y$  are the size of the two populations and  $\alpha, \beta$  are the strength of the competition.

This example is from the book : *Modélisation mathématique en écologie*, P. Auger, C. Lett and J.C. Poggiale, 2015, DUNOD.

In this exercise, we suppose that  $K_1 = 200$  and  $K_2 = 300$ .

1. For  $r_1 = r_2 = 1$ ,  $\alpha \cdot \frac{K_2}{K_1} = 0.1$  and  $\beta \cdot \frac{K_1}{K_2} = 0.2$ , write a program that display, in the same graphic, the evolution of different populations, starting with different sizes  $(x, y)$  in  $[0, 200[ \times [0, 200[$  during 100 seconds. You will display the evolution of the populations in 2D, where

- the x-axis represents the size of the population of  $x$ ,
- the y-axis represents the size of the population of  $y$ .

Do the different population sizes stabilize towards some limit values ? There is a coexistence between the populations ?

2. Make the same work with  $r_1 = r_2 = 1$ ,  $\alpha \cdot \frac{K_2}{K_1} = 2.5$  and  $\beta \cdot \frac{K_1}{K_2} = 1.1$ . Comment the different results.
3. Are there parameters where, whatever the initial populations, a single population remains ? If so, give an example of parameters where just  $x$  remains.

### 3 Motor identification

We will use some data to identify an electric linear cylinder. The input of the linear cylinder is an electric voltage. The output is the position of the cylinder.

In the file [https://www.labri.fr/perso/boussica/archives/cours/4TTV313U/linear\\_cylinder\\_2022.csv](https://www.labri.fr/perso/boussica/archives/cours/4TTV313U/linear_cylinder_2022.csv), you will find some data of an unknown electric linear cylinder, containing

- the electric voltage applied to the linear cylinder,
- the resulting linear position of the linear cylinder,

sampled at 200Hz.

A simple model of the motor in movement is given by the following equation:

$$w_{n+1} = \alpha_1 \cdot w_n + \alpha_2 \cdot \epsilon(w_n) + \alpha_3 \cdot u_n$$

where  $w$  is the velocity of the motor,  $u$  is the electric voltage of the motor, and  $\epsilon(w)$  is the sign of  $w$ .

Write a program that computes the parameters that minimise the average quadratic error between the data and the simulation.

Display, on the same graphic, the evolution of  $u$  and  $w$  for the data and for the simulation by using the best parameters you have obtained.