

# A Verified Information-Flow Architecture for SAFE

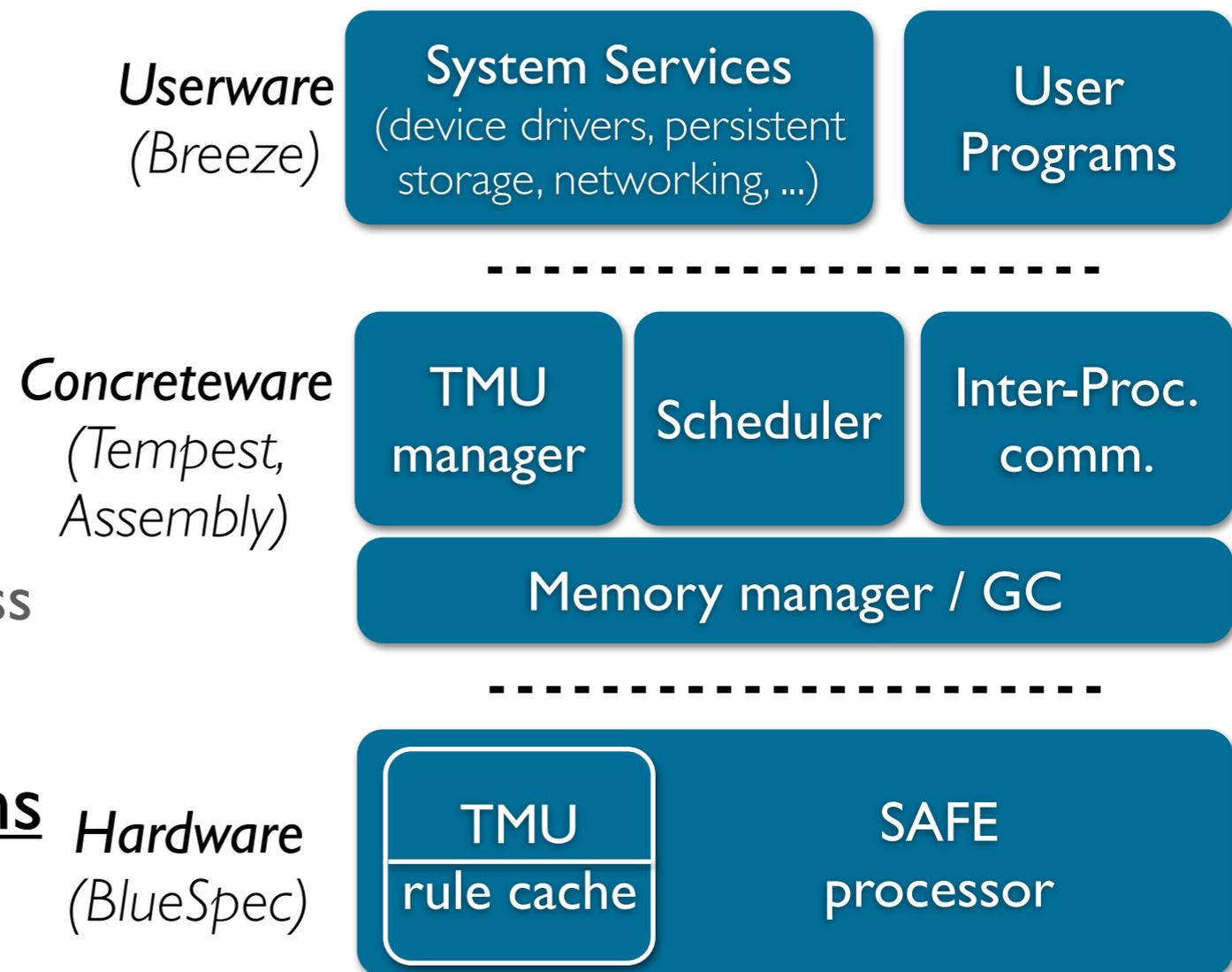
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André DeHon, **Delphine Demange**, Cătălin Hrițcu,  
David Pichardie, Benjamin C. Pierce,  
Randy Pollack, Andrew Tolmach

LTP - Bordeaux - 18 Nov. 2013

# Crash/SAFE



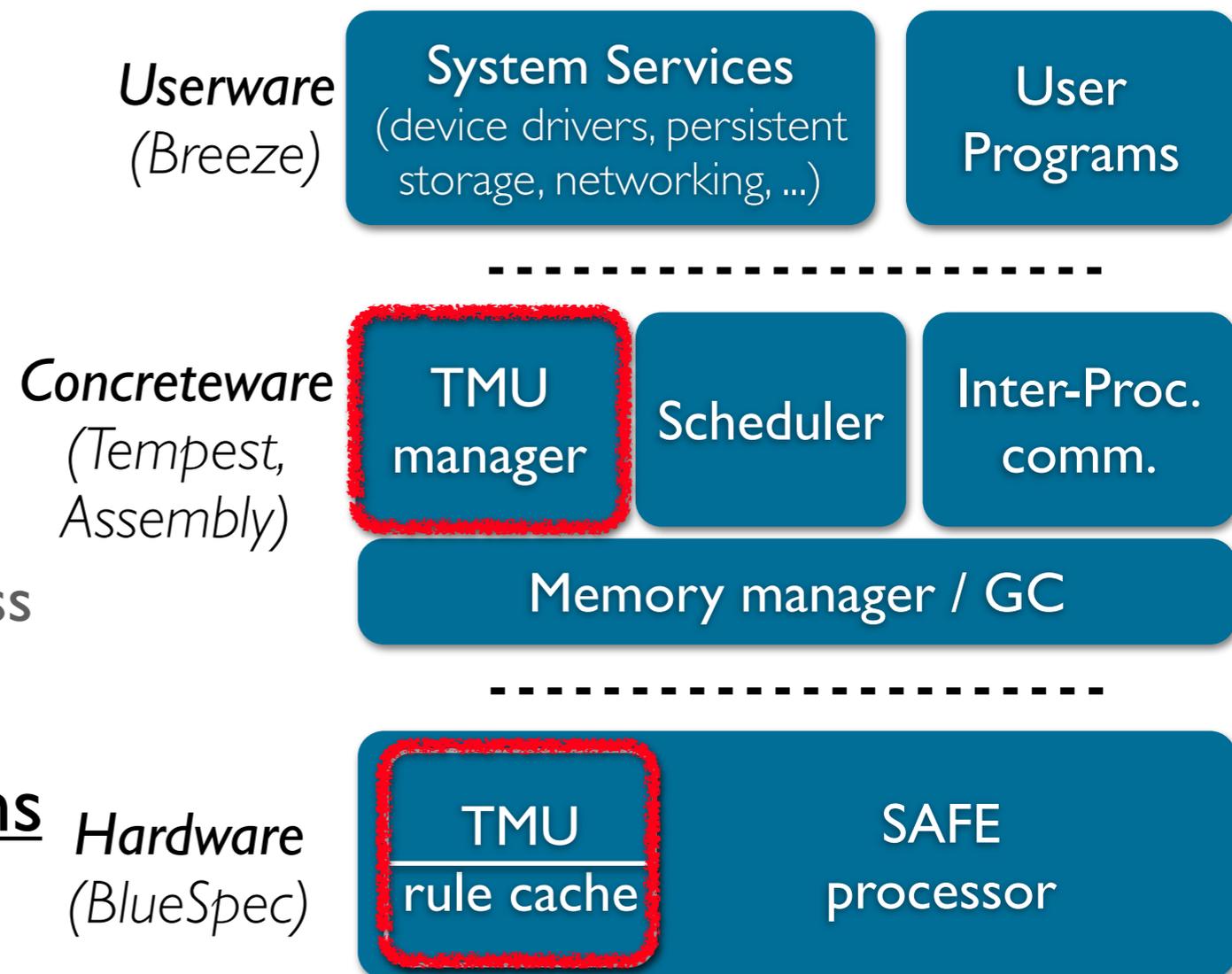
- Clean-slate redesign of entire system stack
  - Hardware, system software, programming languages
- Critical security primitives at all levels
  - Memory safety, strong dynamic typing, information flow and access control
- Verification of key mechanisms
  - deeply integrated into design process



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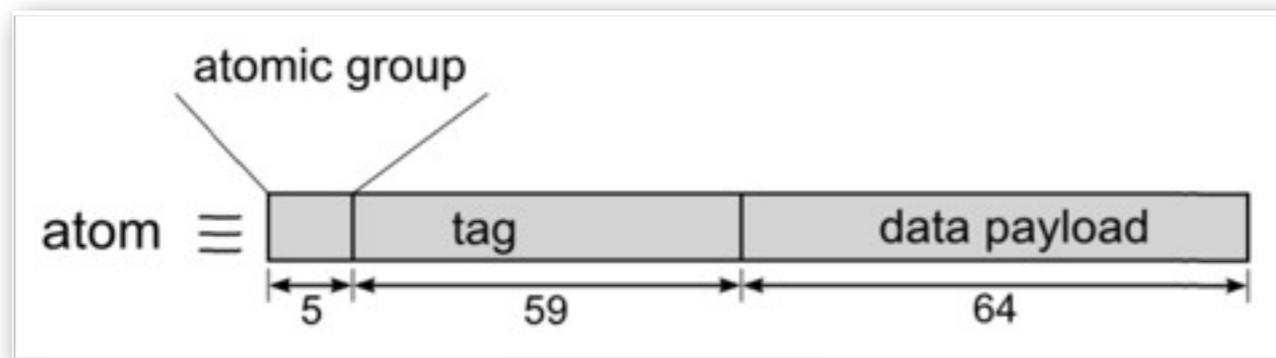
# SAFE hardware (glimpse)

- Hardware types (“atomic groups”)

- instruction  $\neq$  integer  $\neq$  pointer

- Hardware tagging

- *atom* = payload + atomic group + software-defined tag



- Hardware *rule cache*: tag propagation with every machine step

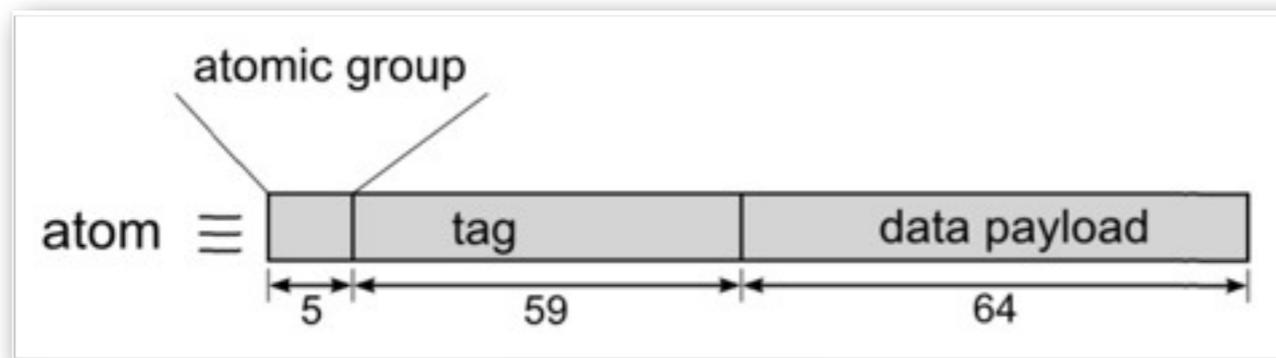
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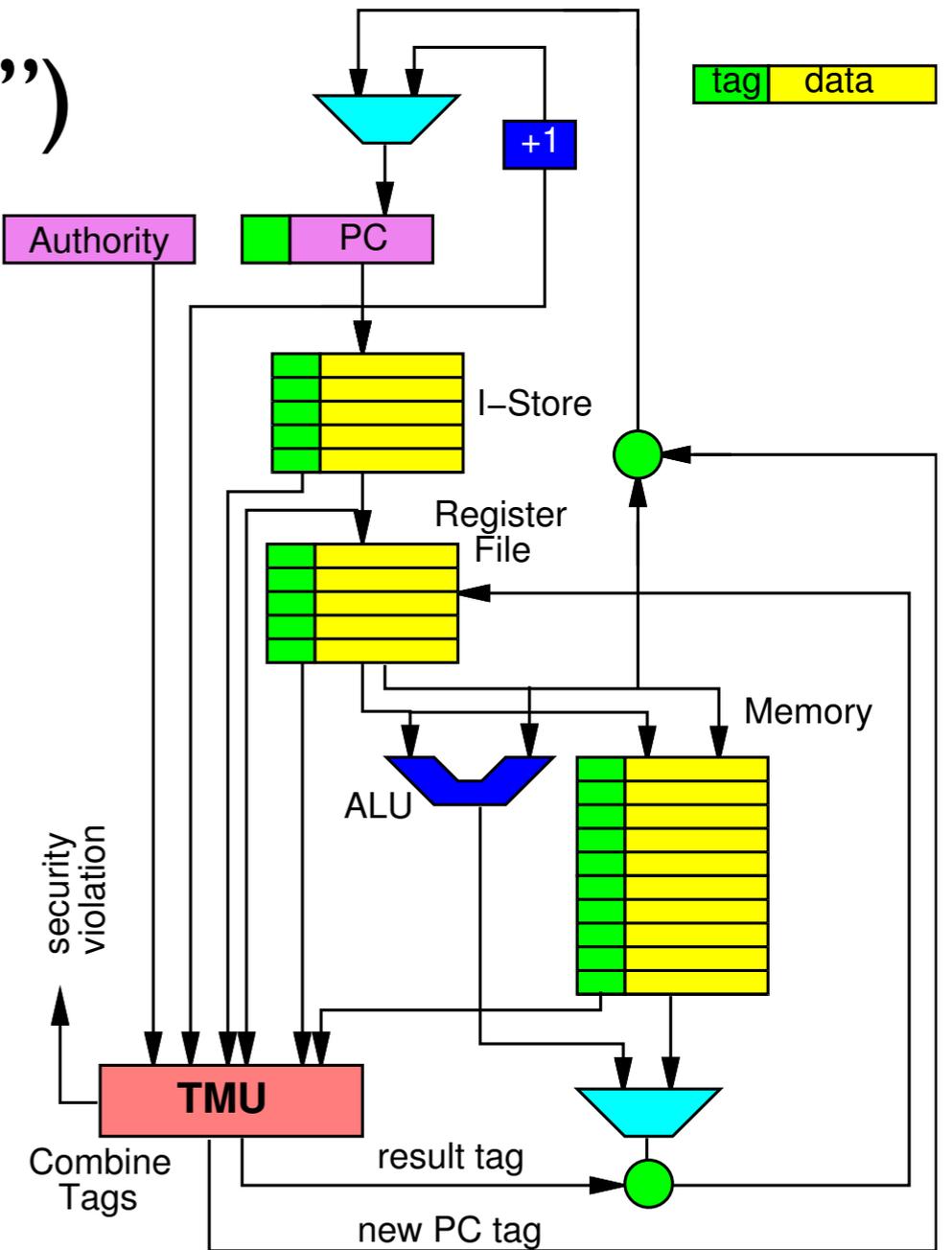
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# This talk: formalizing SAFE's hardware tags for information flow control

- **Abstraction supported by HW tags**
  - labels over a IFC lattice
  - dynamic non-interference
- **We formally state and verify these properties**
  - (hardware + TMU manager)  $\approx$  abstract machine
- **Simplifications**
  - deterministic, single-threaded, stack machine
  - no downgrading, public labels, dynamic principal generation, ...
  - no exception handling (security violation halts the machine)
  - single-line rule cache



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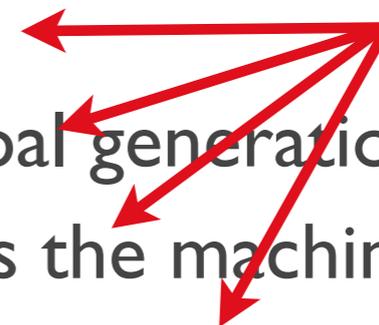
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*Minor*



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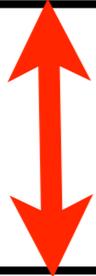
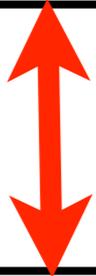
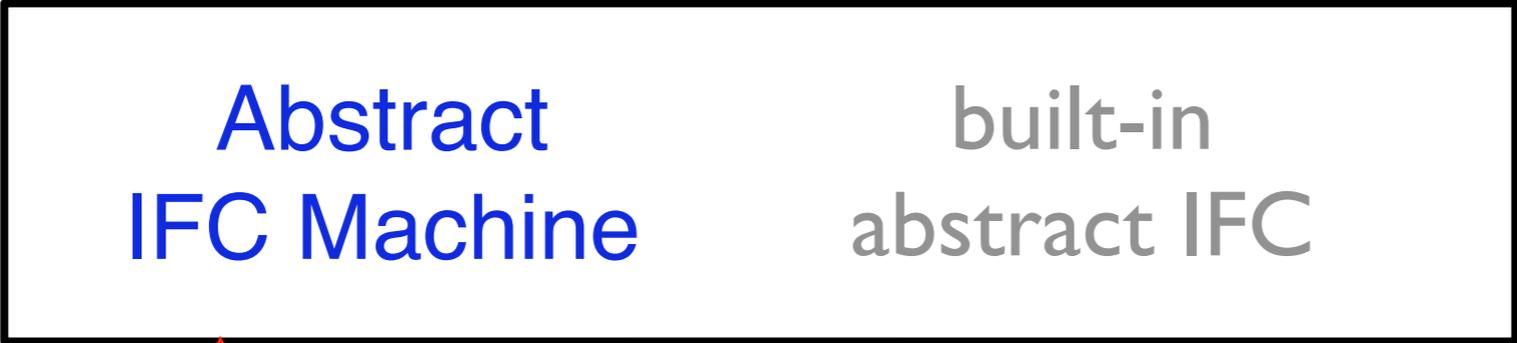
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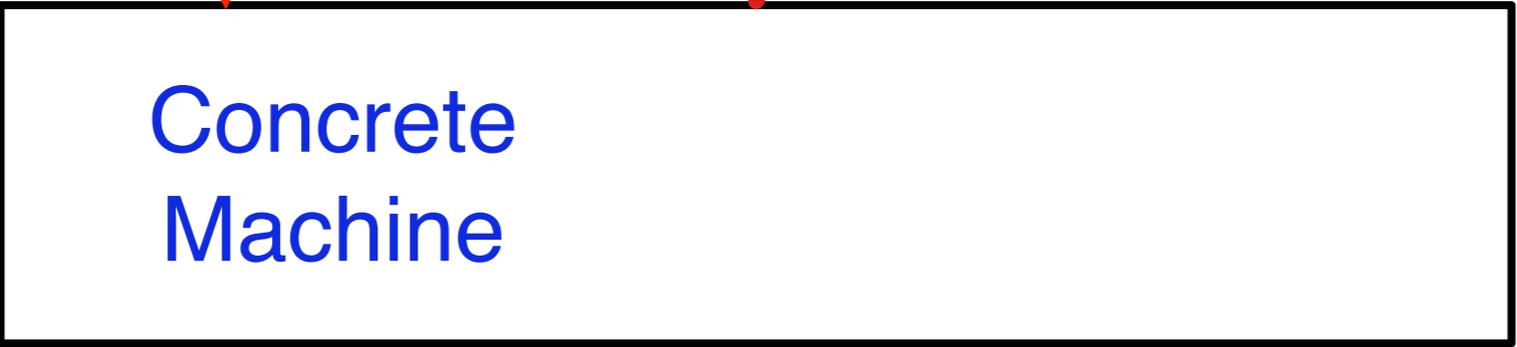
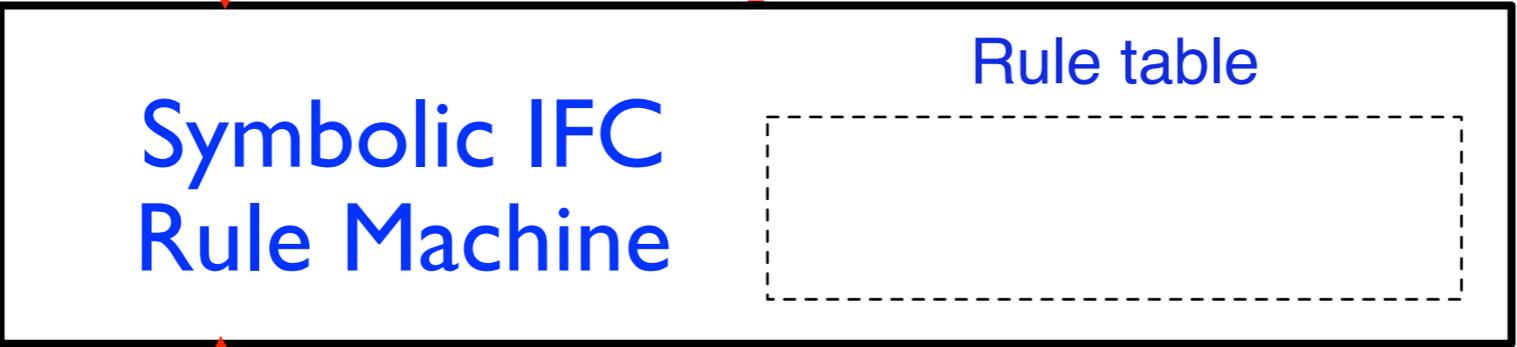
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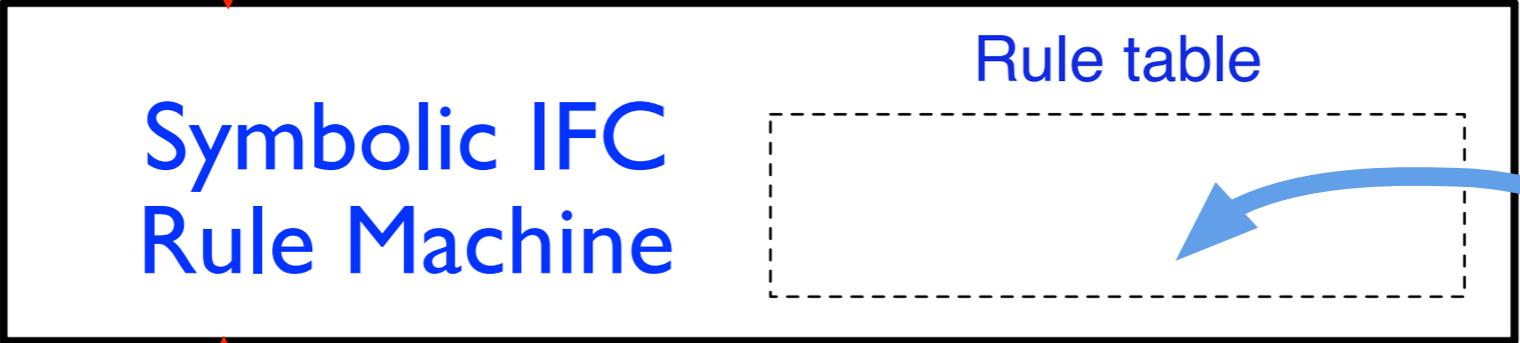
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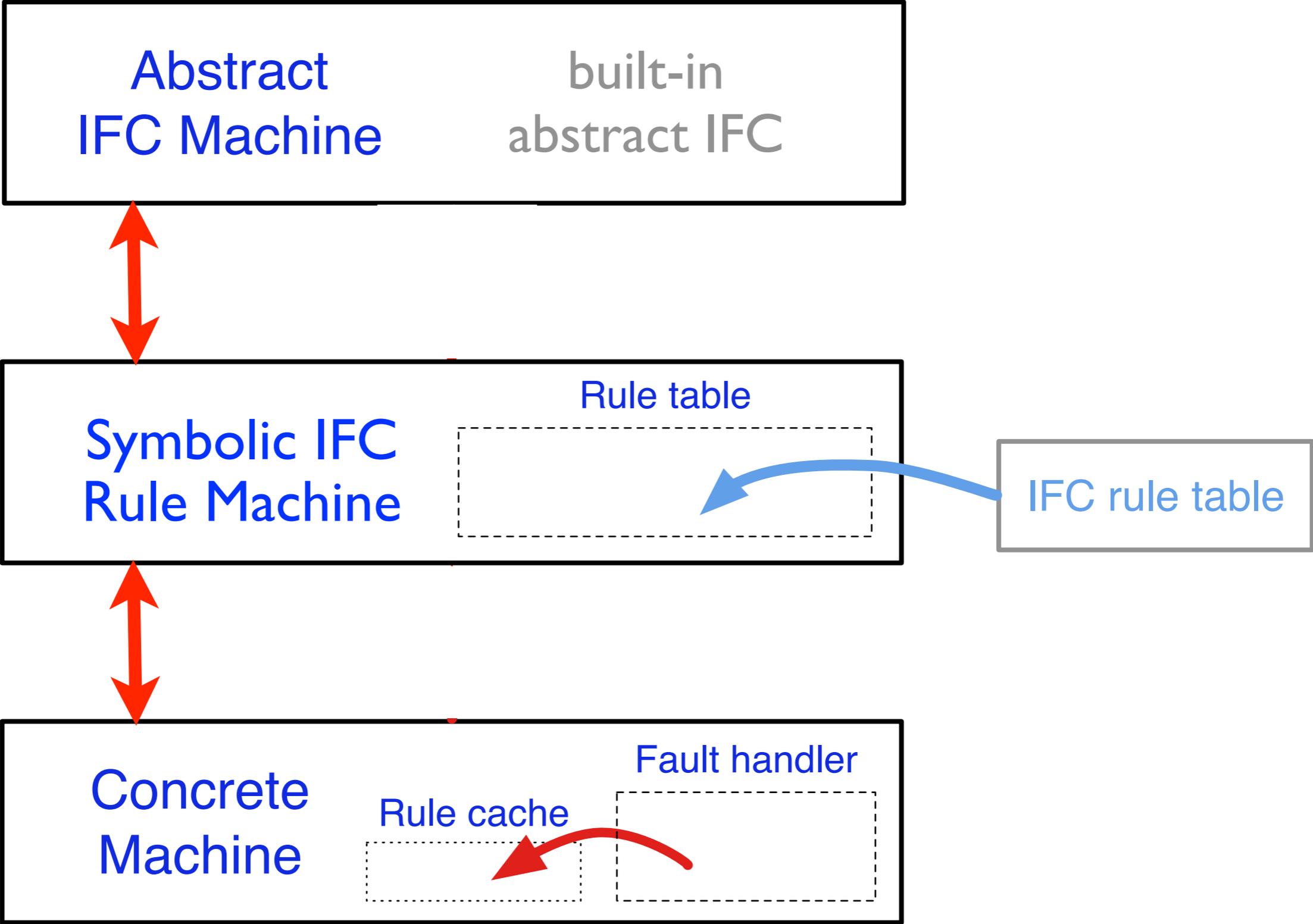
Concrete  
Machine

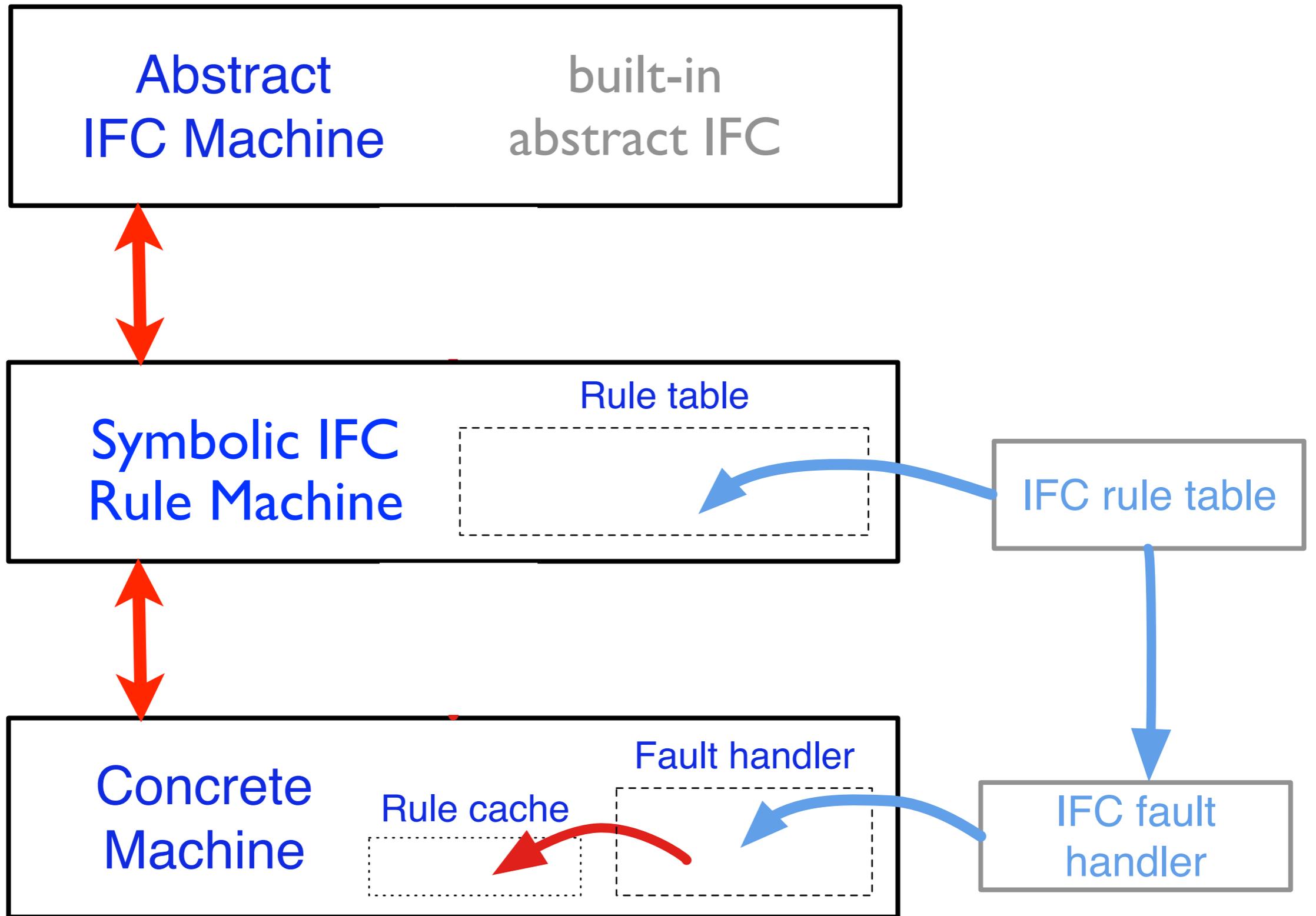












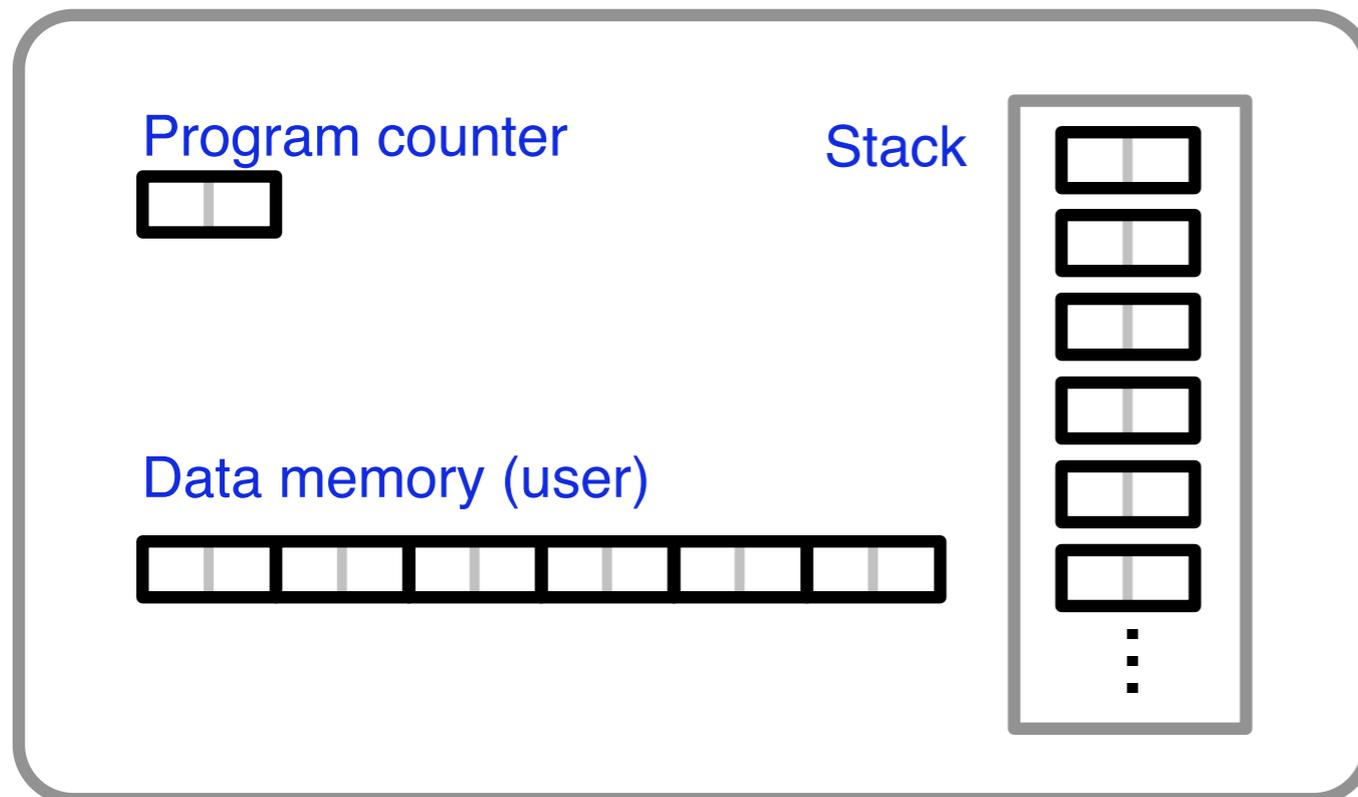
# Abstract Machine

# Abstract Machine

Instruction memory (user)



Machine state



Output

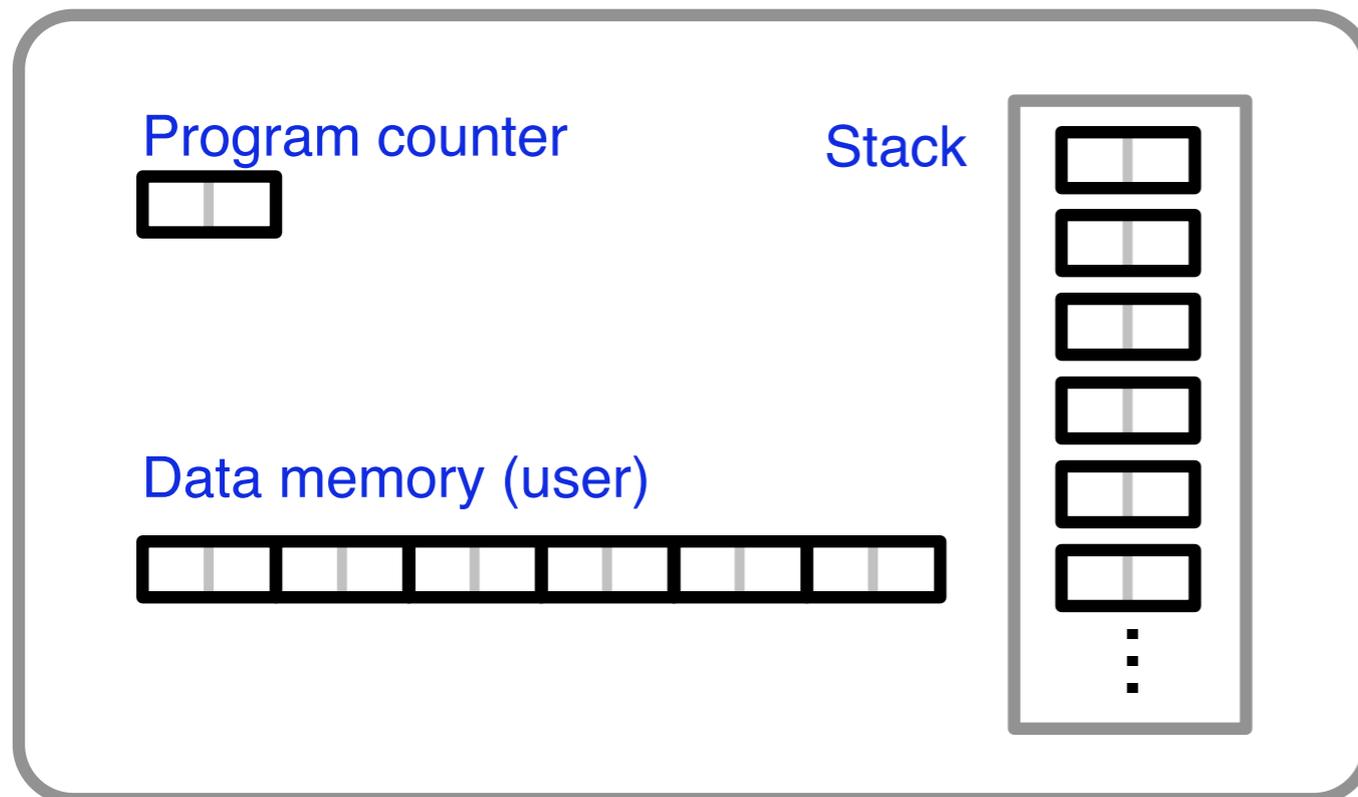


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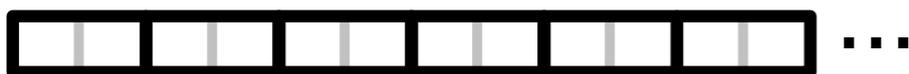
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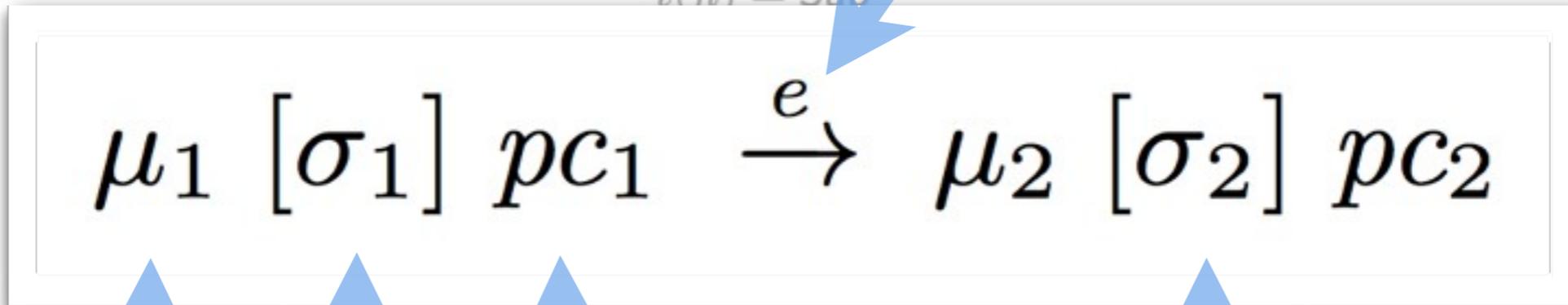
<i>instr</i>	::=	Instructions
		Output
		Sub
		Push <i>n</i>
		Load
		Store
		Jump
		Bnz <i>n</i>
		Call
		Ret

Output



$$\begin{array}{c}
\iota(n) = \text{Sub} \\
\hline
\frac{\mu \quad [n_1 @ L_1, n_2 @ L_2, \sigma] \quad n @ L_{pc} \quad \xrightarrow{\tau}}{\mu \quad [(n_1 - n_2) @ (L_1 \vee L_2), \sigma] \quad n+1 @ L_{pc}} \\
\\
\iota(n) = \text{Output} \\
\hline
\mu \quad [m @ L_1, \sigma] \quad n @ L_{pc} \quad \xrightarrow{m @ L_1 \vee L_{pc}} \mu \quad [\sigma] \quad n+1 @ L_{pc} \\
\\
\iota(n) = \text{Push } m \\
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\mu \quad [\sigma] \quad n @ L_{pc} \quad \xrightarrow{\tau} \mu \quad [m @ \perp, \sigma] \quad n+1 @ L_{pc} \\
\\
\iota(n) = \text{Load} \quad \mu(p) = m @ L_2 \\
\hline
\mu \quad [p @ L_1, \sigma] \quad n @ L_{pc} \quad \xrightarrow{\tau} \mu \quad [m @ L_1 \vee L_2, \sigma] \quad n+1 @ L_{pc} \\
\\
\iota(n) = \text{Store} \quad \mu(p) = k @ L_3 \quad L_1 \vee L_{pc} \leq L_3 \\
\mu(p) \leftarrow (m @ L_1 \vee L_2 \vee L_{pc}) = \mu' \\
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\mu \quad [n' @ L_1, a, \sigma] \quad n @ L_{pc} \quad \xrightarrow{\tau} \mu \quad [a, n+1 @ L_{pc}; \sigma] \quad n' @ (L_1 \vee L_{pc}) \\
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\mu \quad [n' @ L_1; \sigma] \quad n @ L_{pc} \quad \xrightarrow{\tau} \mu \quad [\sigma] \quad n' @ L_1
\end{array}$$

*output*

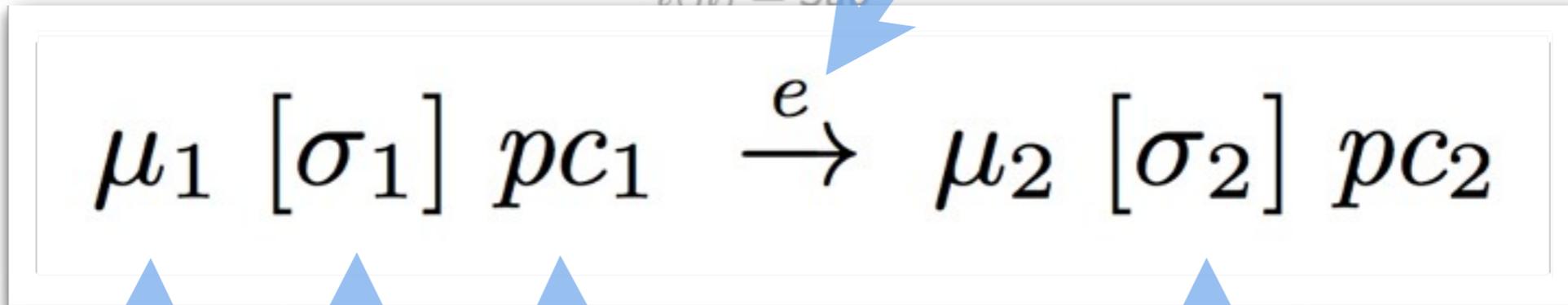


*memory stack pc*

*next state*

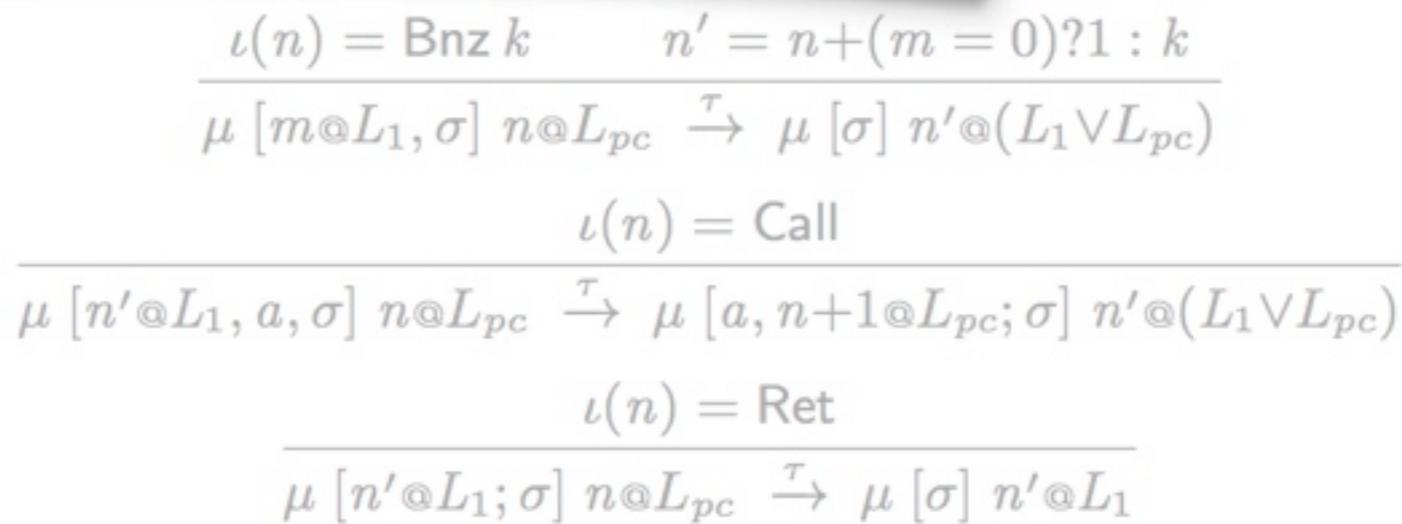
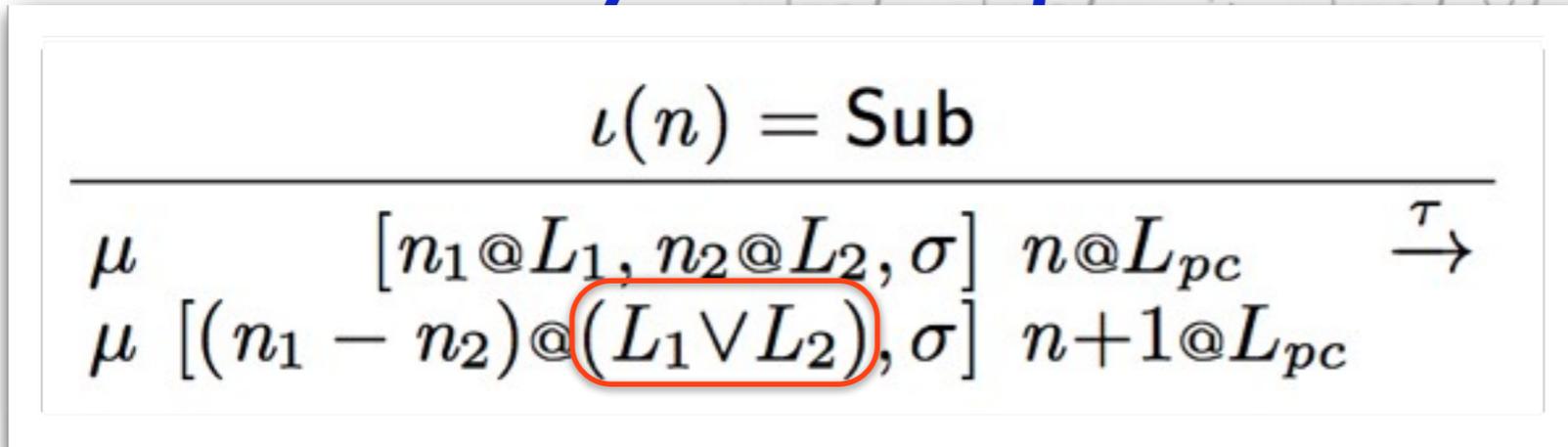
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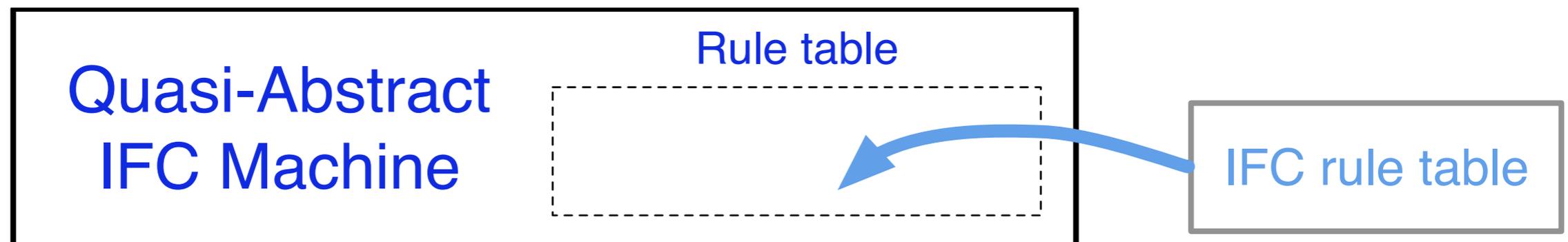
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standard assumptions: attacker cannot distinguish successful termination, failing with an error, and unproductive infinite loop

# Symbolic IFC Rule Machine

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- Alternative presentation of abstract machine
  - Same machine states
  - Same step relation
- IFC side conditions factored out into a separate, explicit *rule table*



$$\begin{array}{c}
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\mu [m @ L_1, \sigma] \ n @ L_{pc} \xrightarrow{m @ L_r} \mu [\sigma] \ (n+1) @ L_{rpc} \\
\iota(n) = \text{Push } m \quad \vdash_{\mathcal{R}} (L_{pc}, -, -, -) \rightsquigarrow_{\text{push}} L_{rpc}, L_r}{\mu [\sigma] \ n @ L_{pc} \xrightarrow{\tau} \mu [m @ L_r, \sigma] \ (n+1) @ L_{rpc}} \\
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\iota(n) = \text{Jump} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{jump}} L_{rpc}, -}{\mu [n' @ L_1, \sigma] \ n @ L_{pc} \xrightarrow{\tau} \mu [\sigma] \ n' @ L_{rpc}} \\
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\frac{\iota(n) = \text{Ret} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{ret}} L_{rpc}, -}{\mu [n' @ L_1; \sigma] \ n @ L_{pc} \xrightarrow{\tau} \mu [\sigma] \ n' @ L_{rpc}}
\end{array}$$

$$\frac{\iota(n) = \text{Sub} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, L_2, -) \rightsquigarrow_{\text{sub}} L_{rpc}, L_r}{\begin{array}{l} \mu [n_1 @ L_1, n_1 @ L_2, \sigma] \quad n @ L_{pc} \\ \mu [(n_1 - n_2) @ L_r, \sigma] \quad (n+1) @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Output} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -) \rightsquigarrow_{\text{push}} L_{rpc}, L_r}{\begin{array}{l} \mu [m @ L_1, \sigma] \quad n @ L_{pc} \\ \mu [m @ L_r, \sigma] \quad (n+1) @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

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*consult rule table... and opcode... to obtain result tags... for tags...*

$$\frac{\iota(n) = \text{Sub} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, L_2, -) \rightsquigarrow_{\text{sub}} L_{rpc}, L_r}{\begin{array}{l} \mu [n_1 @ L_1, n_1 @ L_2, \sigma] \quad n @ L_{pc} \\ \mu [(n_1 - n_2) @ L_r, \sigma] \quad (n+1) @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{BNZ} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{bnz}} L_{rpc}, -}{\begin{array}{l} \mu [m @ L_1, \sigma] \quad n @ L_{pc} \\ \mu [\sigma] \quad n' @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Call} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{call}} L_{rpc}, L_r}{\begin{array}{l} \mu [n' @ L_1, a, \sigma] \quad n @ L_{pc} \\ \mu [a, (n+1) @ L_r; \sigma] \quad n' @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Ret} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{ret}} L_{rpc}, -}{\begin{array}{l} \mu [n' @ L_1; \sigma] \quad n @ L_{pc} \\ \mu [\sigma] \quad n' @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Sub} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, L_2, -) \rightsquigarrow_{\text{sub}} L_{rpc}, L_r}{\begin{array}{l} \mu [n_1 @ L_1, n_1 @ L_2, \sigma] \quad n @ L_{pc} \\ \mu [(n_1 - n_2) @ L_r, \sigma] \quad (n+1) @ L_{rpc} \end{array} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Output} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -) \rightsquigarrow_{\text{out}} L_{rpc}, L_r}{\mu [m @ L_1, \sigma] \quad n @ L_{pc} \xrightarrow{\tau} \mu [m @ L_r, \sigma] \quad (n+1) @ L_{rpc}}$$

$$\frac{\iota(n) = \text{Push} \quad \vdash_{\mathcal{R}} (L_{pc}, -, -) \rightsquigarrow_{\text{push}} L_{rpc}, L_r}{\mu [n @ L_r, \sigma] \quad (n+1) @ L_{rpc}}$$

$$\frac{\iota(n) = \text{Load} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -) \rightsquigarrow_{\text{load}} L_{rpc}, L_r}{\mu [n @ L_1, \sigma] \quad (n+1) @ L_{rpc}}$$

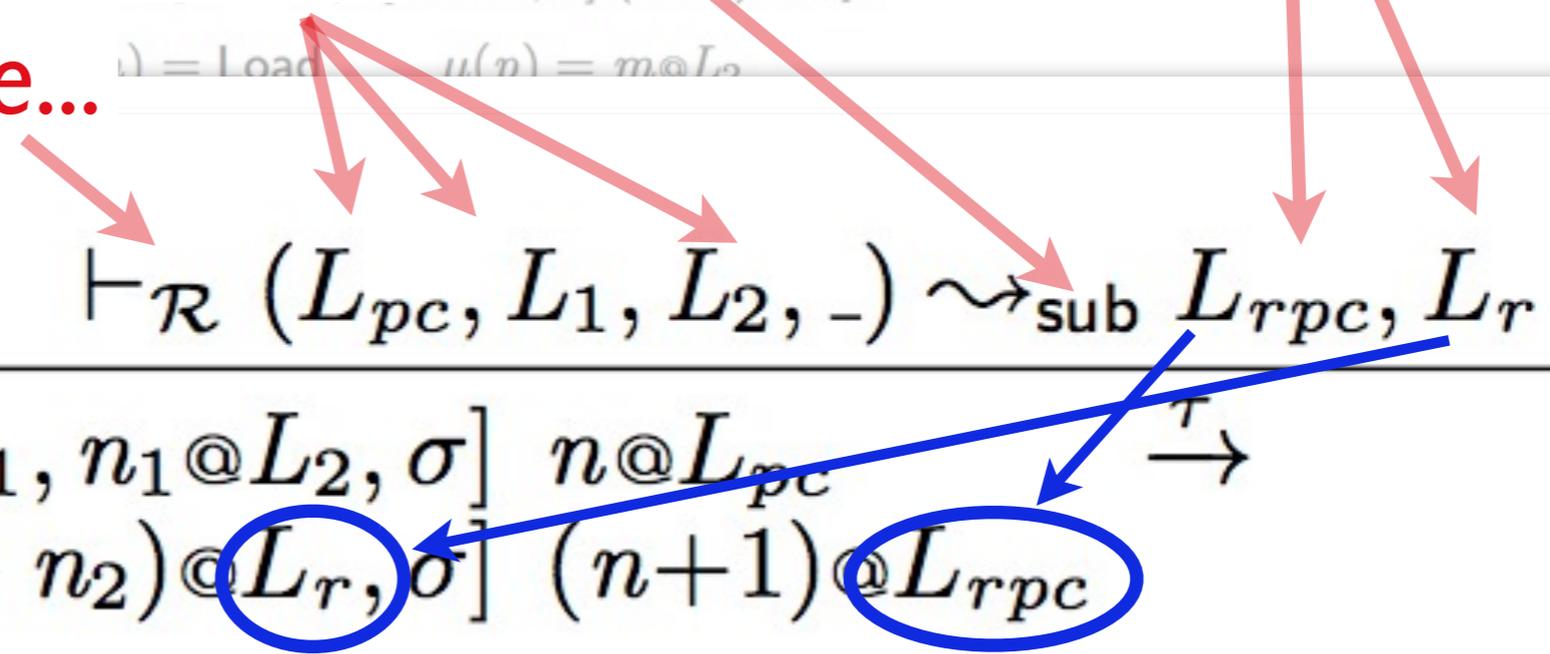
to obtain result tags...

and opcode...

for tags...

consult rule table...

$$\frac{\iota(n) = \text{Sub} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, L_2, -) \rightsquigarrow_{\text{sub}} L_{rpc}, L_r}{\begin{array}{l} \mu [n_1 @ L_1, n_1 @ L_2, \sigma] \quad n @ L_{pc} \\ \mu [(n_1 - n_2) @ L_r, \sigma] \quad (n+1) @ L_{rpc} \end{array} \xrightarrow{\tau}}$$



$$\frac{\iota(n) = \text{Bnz} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{bnz}} L_{rpc}, -}{\mu [m @ L_1, \sigma] \quad n @ L_{pc} \xrightarrow{\tau} \mu [\sigma] \quad n' @ L_{rpc}}$$

$$\frac{\iota(n) = \text{Call} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{call}} L_{rpc}, L_r}{\mu [n' @ L_1, a, \sigma] \quad n @ L_{pc} \xrightarrow{\tau} \mu [a, (n+1) @ L_r; \sigma] \quad n' @ L_{rpc}}$$

$$\frac{\iota(n) = \text{Ret} \quad \vdash_{\mathcal{R}} (L_{pc}, L_1, -, -) \rightsquigarrow_{\text{ret}} L_{rpc}, -}{\mu [n' @ L_1; \sigma] \quad n @ L_{pc} \xrightarrow{\tau} \mu [\sigma] \quad n' @ L_{rpc}}$$

# IFC Rule Table

$\mathcal{R} =$

<i>opcode</i>	<i>allow</i>	<i>erpc</i>	<i>er</i>
sub	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
output	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_{pc}$
push	TRUE	$LAB_{pc}$	BOT
load	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
store	$LAB_1 \sqcup LAB_{pc} \sqsubseteq LAB_3$	$LAB_{pc}$	$LAB_1 \sqcup LAB_2 \sqcup LAB_{pc}$
jump	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
bnz	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
call	TRUE	$LAB_1 \sqcup LAB_{pc}$	$LAB_{pc}$
ret	TRUE	$LAB_1$	--

# IFC Rule Table

*is this operation allowed?*

$\mathcal{R} =$

<i>opcode</i>	<i>allow</i>	<i>erpc</i>	<i>er</i>
sub	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
output	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_{pc}$
push	TRUE	$LAB_{pc}$	BOT
load	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
store	$LAB_1 \sqcup LAB_{pc} \sqsubseteq LAB_3$	$LAB_{pc}$	$LAB_1 \sqcup LAB_2 \sqcup LAB_{pc}$
jump	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
bnz	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
call	TRUE	$LAB_1 \sqcup LAB_{pc}$	$LAB_{pc}$
ret	TRUE	$LAB_1$	--

*new pc label*

*label for result*

# IFC Rule Table

*is this operation allowed?*

$\mathcal{R} =$

<i>opcode</i>	<i>allow</i>	<i>e<sub>rpc</sub></i>	<i>e<sub>r</sub></i>
sub	TRUE	LAB <sub>pc</sub>	LAB <sub>1</sub> $\sqcup$ LAB <sub>2</sub>
output	TRUE	LAB <sub>pc</sub>	LAB <sub>1</sub> $\sqcup$ LAB <sub>pc</sub>
push	TRUE	LAB <sub>pc</sub>	BOT
load	TRUE	LAB <sub>pc</sub>	
store	LAB <sub>1</sub> $\sqcup$ LAB <sub>pc</sub> $\sqsubseteq$ LAB		
jump	TRUE		

*new pc label*

*label for result*

*subtraction is always allowed*

*pc label is unchanged*

*result label is join of arg labels*

# Concrete Machine

# Concrete machine

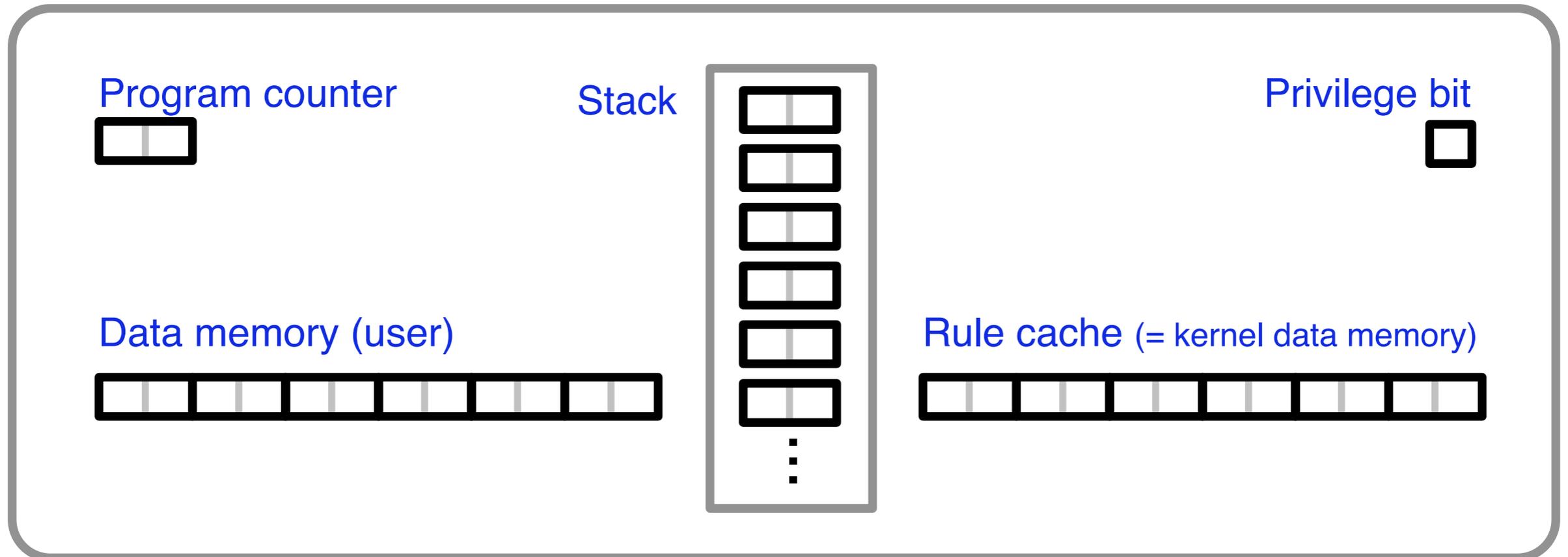
Instruction memory (user)



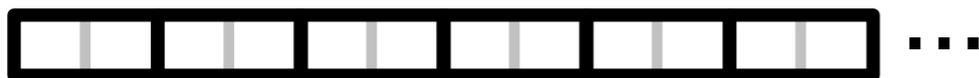
Instruction memory (kernel)



Machine state



Output



# Concrete machine

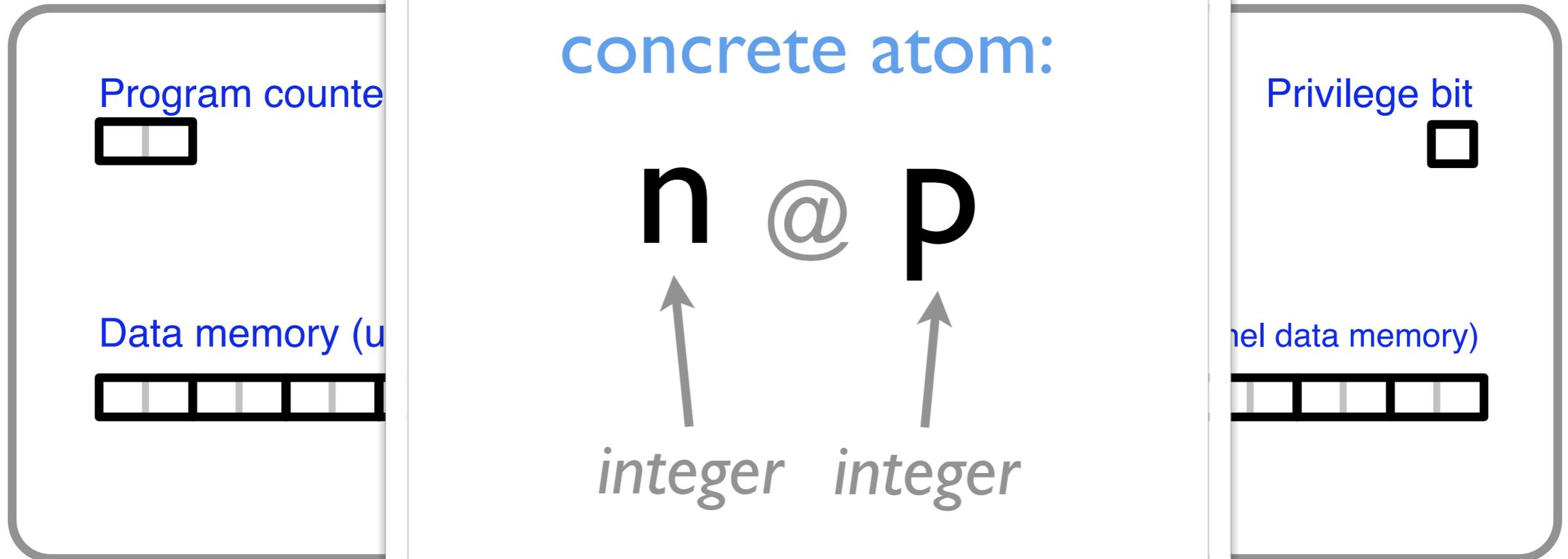
Instruction memory (user)



Instruction memory (kernel)



Machine state



Output



# Concrete machine

Instruction memory (user)

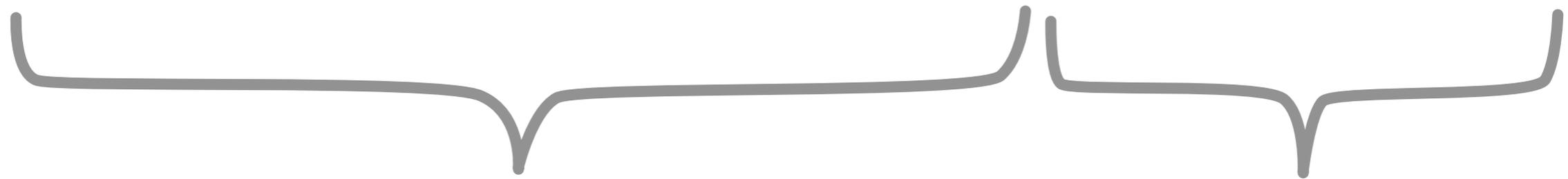
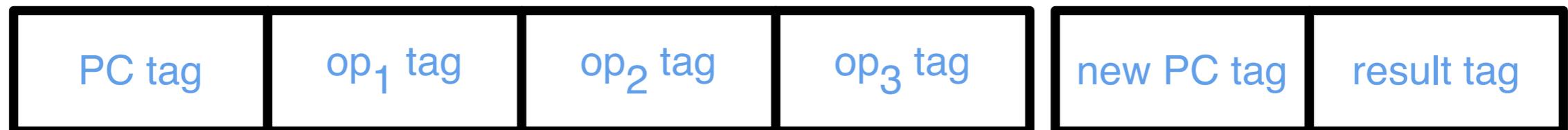


Instruction memory (kernel)



*(single-line)*

**Rule cache:**



**Inputs**

**Outputs**

Output



# User mode (cache hit)

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa = \boxed{\text{sub } T_{pc} \ T_1 \ T_2 \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Output} \\
 \kappa = \boxed{\text{output } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\
 u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Push } m \\
 \kappa = \boxed{\text{push } T_{pc} \ - \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 u \ \kappa \ \mu \ [\sigma] \ n @ T_{pc} \xrightarrow{\tau} u \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Load} \\
 \mu(p) = m @ T_2 \\
 \kappa = \boxed{\text{load } T_{pc} \ T_1 \ T_2 \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 u \ \kappa \ \mu \ [p @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Store} \quad \mu(p) = k @ T_3 \\
 \kappa = \boxed{\text{store } T_{pc} \ T_1 \ T_2 \ T_3 \ | \ T_{rpc} \ T_r} \\
 \mu(p) \leftarrow (m @ T_r) = \mu' \\
 \hline
 u \ \kappa \ \mu \ [p @ T_1, m @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu' \ [\sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Jump} \\
 \kappa = \boxed{\text{jump } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 \hline
 u \ \kappa \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Bnz } k \\
 \kappa = \boxed{\text{bnz } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 n' = n + (m = 0) ? 1 : k \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Call} \\
 \kappa = \boxed{\text{call } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Ret} \\
 \kappa = \boxed{\text{ret } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 \hline
 u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

# User-to-kernel mode (cache miss)

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa_i \neq \boxed{\text{sub } T_{pc} \ T_1 \ T_2 \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Output} \\
 \kappa_i \neq \boxed{\text{output } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Push } m \\
 \kappa_i \neq \boxed{\text{push } T_{pc} \ - \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [\sigma] \ n @ T_{pc} \xrightarrow{\tau} k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Load} \\
 \mu(p) = m @ T_2 \\
 \kappa_i \neq \boxed{\text{load } T_{pc} \ T_1 \ T_2 \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [p @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); p @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Store} \quad \mu(p) = k @ T_3 \\
 \kappa_i \neq \boxed{\text{store } T_{pc} \ T_1 \ T_2 \ T_3} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [p @ T_1, m @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); p @ T_1, m @ T_2, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Jump} \\
 \kappa_i \neq \boxed{\text{jump } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Bnz } k \\
 \kappa_i \neq \boxed{\text{bnz } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Call} \\
 \kappa_i \neq \boxed{\text{call } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Ret} \\
 \kappa_i \neq \boxed{\text{ret } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T.
 \end{array}$$

# Kernel mode

$$\begin{array}{c}
 \phi(n) = \text{Sub} \\
 \hline
 k \ \kappa \ \mu \ [n_1 @ -, n_1 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Push } m \\
 \hline
 k \ \kappa \ \mu \ [\sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\
 \hline
 k \ \kappa \ \mu \ [p @ -, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_1, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Store} \quad \text{store } \kappa \ p \ (m @ T_1) = \kappa' \\
 \hline
 k \ \kappa \ \mu \ [p @ -, m @ T_1, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa' \ \mu \ [\sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Jump} \\
 \hline
 k \ \kappa \ \mu \ [n' @ -, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [\sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Bnz } k \quad n' = n + (m = 0) ? 1 : k \\
 \hline
 k \ \kappa \ \mu \ [m @ -, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [\sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Call} \\
 \hline
 k \ \kappa \ \mu \ [n' @ -, a, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [a, (n + 1 @ T_r, k); \sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Ret} \\
 \hline
 k \ \kappa \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ - \xrightarrow{\tau} \pi \ \kappa \ \mu \ [\sigma] \ n' @ T_1
 \end{array}$$

# User mode (cache hit)

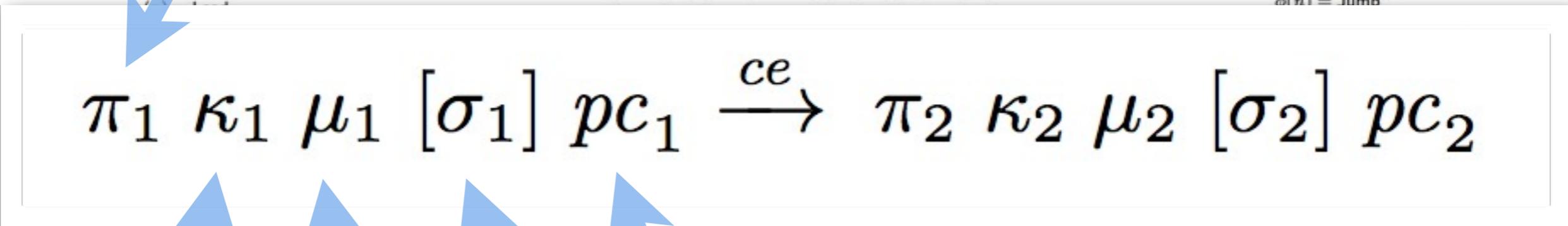
# User-to-kernel mode (cache miss)

# Kernel mode

*privilege bit*

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa = \text{sub } T_{pc} \ T_1 \ T_2 \ - \ - \ T_{rpc} \ T_r \\ \hline u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \\ \hline \iota(n) = \text{Output} \\ \kappa = \text{output } T_{pc} \ T_1 \ - \ - \ T_{rpc} \ T_r \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{m @ T_r} \\ u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc} \\ \hline \iota(n) = \text{Push } m \\ \kappa = \text{push } T_{pc} \ - \ - \ - \ T_{rpc} \ T_r \\ \hline u \ \kappa \ \mu \ [\sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \ u \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa_i \neq \text{sub } T_{pc} \ T_1 \ T_2 \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T. \\ \hline \iota(n) = \text{Output} \\ \kappa_i \neq \text{output } T_{pc} \ T_1 \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\ \hline \iota(n) = \text{Push } m \\ \kappa_i \neq \text{push } T_{pc} \ - \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [\sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \phi(n) = \text{Sub} \\ \hline k \ \kappa \ \mu \ [n_1 @ -, n_1 @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \\ k \ \kappa \ \mu \ [(n_1 - n_2) @ T., \sigma] \ n + 1 @ T. \\ \hline \phi(n) = \text{Push } m \\ \hline k \ \kappa \ \mu \ [\sigma] \ n @ - \ \xrightarrow{\tau} \ k \ \kappa \ \mu \ [m @ T., \sigma] \ n + 1 @ T. \\ \hline \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\ \hline k \ \kappa \ \mu \ [p @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \ k \ \kappa \ \mu \ [m @ T_1, \sigma] \ n + 1 @ T. \\ \hline \phi(n) = \text{Store} \quad \text{store } \kappa \ p \ (m @ T_1) = \kappa' \\ \hline k \ \kappa \ \mu \ [p @ -, m @ T_1, \sigma] \ n @ - \ \xrightarrow{\tau} \ k \ \kappa' \ \mu \ [\sigma] \ n + 1 @ T. \\ \hline \phi(n) = \text{Jump} \end{array}$$


*kernel memory*

*user memory*

*stack*

*pc*

$$\begin{array}{l} \iota(n) = \text{Jump} \\ \kappa = \text{jump } T_{pc} \ T_1 \ - \ - \ T_{rpc} \ - \\ \hline u \ \kappa \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\ \hline \iota(n) = \text{Bnz } k \\ \kappa = \text{bnz } T_{pc} \ T_1 \ - \ - \ T_{rpc} \ - \\ n' = n + (m = 0) ? 1 : k \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n @ T_{rpc} \\ \hline \iota(n) = \text{Call} \\ \kappa = \text{call } T_{pc} \ T_1 \ - \ - \ T_{rpc} \ T_r \\ \hline u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \\ \hline \iota(n) = \text{Ret} \\ \kappa = \text{ret } T_{pc} \ T_1 \ - \ - \ T_{rpc} \ - \\ \hline u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Jump} \\ \kappa_i \neq \text{jump } T_{pc} \ T_1 \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, \sigma] \ 0 @ T. \\ \hline \iota(n) = \text{Bnz } k \\ \kappa_i \neq \text{bnz } T_{pc} \ T_1 \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\ \hline \iota(n) = \text{Call} \\ \kappa_i \neq \text{call } T_{pc} \ T_1 \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \\ \hline \iota(n) = \text{Ret} \\ \kappa_i \neq \text{ret } T_{pc} \ T_1 \ - \ - \ = \ \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T. \end{array}$$

# User mode (cache hit)

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa = \boxed{\text{sub } T_{pc} \ T_1 \ T_2 \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 \text{u } \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Output} \\
 \kappa = \boxed{\text{output } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 \text{u } \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{m @ T_r} \\
 \text{u } \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Push } m \\
 \kappa = \boxed{\text{push } T_{pc} \ - \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 \text{u } \kappa \ \mu \ [\sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \ \text{u } \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Load} \\
 \mu(p) = m @ T_2 \\
 \kappa = \boxed{\text{load } T_{pc} \ T_1 \ T_2 \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 \text{u } \kappa \ \mu \ [p @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Store} \quad \mu(p) = k @ T_3 \\
 \kappa = \boxed{\text{store } T_{pc} \ T_1 \ T_2 \ T_3 \ | \ T_{rpc} \ T_r} \\
 \mu(p) \leftarrow (m @ T_r) = \mu' \\
 \hline
 \text{u } \kappa \ \mu \ [p @ T_1, m @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu' \ [\sigma] \ n + 1 @ T_{rpc} \\
 \\
 \iota(n) = \text{Jump} \\
 \kappa = \boxed{\text{jump } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 \hline
 \text{u } \kappa \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Bnz } k \\
 \kappa = \boxed{\text{bnz } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 n' = n + (m = 0) ? 1 : k \\
 \hline
 \text{u } \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Call} \\
 \kappa = \boxed{\text{call } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ T_r} \\
 \hline
 \text{u } \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \\
 \\
 \iota(n) = \text{Ret} \\
 \kappa = \boxed{\text{ret } T_{pc} \ T_1 \ - \ - \ | \ T_{rpc} \ -} \\
 \hline
 \text{u } \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{u } \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

# User-to-kernel mode (cache miss)

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa_i \neq \boxed{\text{sub } T_{pc} \ T_1 \ T_2 \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Output} \\
 \kappa_i \neq \boxed{\text{output } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Push } m \\
 \kappa_i \neq \boxed{\text{push } T_{pc} \ - \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [\sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \ \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Load} \\
 \mu(p) = m @ T_2 \\
 \kappa_i \neq \boxed{\text{load } T_{pc} \ T_1 \ T_2 \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [p @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); p @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Store} \quad \mu(p) = k @ T_3 \\
 \kappa_i \neq \boxed{\text{store } T_{pc} \ T_1 \ T_2 \ T_3} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [p @ T_1, m @ T_2, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); p @ T_1, m @ T_2, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Jump} \\
 \kappa_i \neq \boxed{\text{jump } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Bnz } k \\
 \kappa_i \neq \boxed{\text{bnz } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Call} \\
 \kappa_i \neq \boxed{\text{call } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \\
 \\
 \iota(n) = \text{Ret} \\
 \kappa_i \neq \boxed{\text{ret } T_{pc} \ T_1 \ - \ -} = \kappa_j \\
 \hline
 \text{u } [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \ \xrightarrow{\tau} \\
 \text{k } [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T.
 \end{array}$$

# Kernel mode

$$\begin{array}{c}
 \phi(n) = \text{Sub} \\
 \hline
 \text{k } \kappa \ \mu \ [n_1 @ -, n_1 @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \\
 \text{k } \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Push } m \\
 \hline
 \text{k } \kappa \ \mu \ [\sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\
 \hline
 \text{k } \kappa \ \mu \ [p @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa \ \mu \ [m @ T_1, \sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Store} \quad \text{store } \kappa \ p \ (m @ T_1) = \kappa' \\
 \hline
 \text{k } \kappa \ \mu \ [p @ -, m @ T_1, \sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa' \ \mu \ [\sigma] \ n + 1 @ T. \\
 \\
 \phi(n) = \text{Jump} \\
 \hline
 \text{k } \kappa \ \mu \ [n' @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa \ \mu \ [\sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Bnz } k \quad n' = n + (m = 0) ? 1 : k \\
 \hline
 \text{k } \kappa \ \mu \ [m @ -, \sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa \ \mu \ [\sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Call} \\
 \hline
 \text{k } \kappa \ \mu \ [n' @ -, a, \sigma] \ n @ - \ \xrightarrow{\tau} \ \text{k } \kappa \ \mu \ [a, (n + 1 @ T_r, k); \sigma] \ n' @ T. \\
 \\
 \phi(n) = \text{Ret} \\
 \hline
 \text{k } \kappa \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ - \ \xrightarrow{\tau} \ \pi \ \kappa \ \mu \ [\sigma] \ n' @ T_1
 \end{array}$$

# User mode (cache hit)

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \end{array}$$
  

$$\begin{array}{l} \iota(n) = \text{Output} \\ \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\ u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc} \end{array}$$

# User-to-kernel mode (cache miss)

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_1] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T. \end{array}$$
  

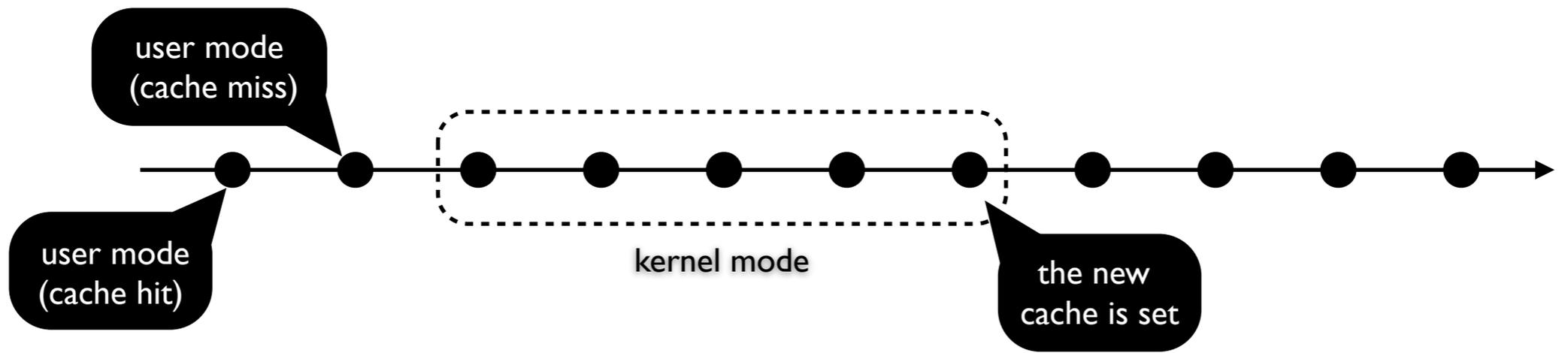
$$\begin{array}{l} \iota(n) = \text{Output} \\ \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_1] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \end{array}$$

# Kernel mode

$$\begin{array}{l} \phi(n) = \text{Sub} \\ \hline k \ \kappa \ \mu \ [n_1 @ -, n_1 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\ k \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T. \end{array}$$
  

$$\begin{array}{l} \phi(n) = \text{Push } m \\ \hline k \ \kappa \ \mu \ [\sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T. \end{array}$$
  

$$\begin{array}{l} \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\ \hline k \ \kappa \ \mu \ [p @ T_1, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T. \end{array}$$



$$\begin{array}{l} u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$
  

$$\begin{array}{l} \iota(n) = \text{Call} \\ \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \end{array}$$
  

$$\begin{array}{l} \iota(n) = \text{Ret} \\ \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\ \hline u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_1] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \end{array}$$
  

$$\begin{array}{l} \iota(n) = \text{Call} \\ \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_1] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \end{array}$$
  

$$\begin{array}{l} \iota(n) = \text{Ret} \\ \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_1] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T. \end{array}$$

User mode  
(cache hit)

User-to-kernel mode  
(cache miss)

Kernel mode

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Output} \\ \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\ u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_2 @ T_2, \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Output} \\ \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \phi(n) = \text{Sub} \\ \hline k \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T. \end{array}$$

$$\begin{array}{l} \phi(n) = \text{Push } m \\ \hline k \ \kappa \ \mu \ [\sigma] \ n @ T_{pc} \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_1, \sigma] \ n + 1 @ T. \end{array}$$

$$\begin{array}{l} \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\ \hline \end{array}$$

$\iota(n) = \text{Sub}$

$\kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r}$

$u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau}$   
 $u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc}$

$$\begin{array}{l} n' = n + (m = 0) ? 1 : k \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Call} \\ \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Ret} \\ \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid -} \\ \hline u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Bnz } k \\ \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Call} \\ \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Ret} \\ \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T. \end{array}$$

User mode  
(cache hit)

User-to-kernel mode  
(cache miss)

Kernel mode

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Sub} \\ \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T. \end{array}$$

$$\begin{array}{l} \phi(n) = \text{Sub} \\ \hline k \ \kappa \ \mu \ [n_1 @ -, n_1 @ -, \sigma] \ n @ -. \xrightarrow{\tau} \\ k \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T. \\ \phi(n) = \text{Push } m \\ \hline k \ \kappa \ \mu \ [\sigma] \ n @ -. \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T. \\ \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \end{array}$$

$\iota(n) = \text{Sub}$

$\kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r}$

$$\begin{array}{l} u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc} \end{array}$$

$$\begin{array}{l} n' = n + (m = 0) ? 1 : k \\ \hline u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \\ \iota(n) = \text{Call} \\ \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc} \\ \iota(n) = \text{Ret} \\ \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid -} \\ \hline u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc} \end{array}$$

$$\begin{array}{l} \iota(n) = \text{Bnz } k \\ \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T. \\ \iota(n) = \text{Call} \\ \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T. \\ \iota(n) = \text{Ret} \\ \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T. \end{array}$$

# User mode (cache hit)

# User-to-kernel mode (cache miss)

# Kernel mode

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc}
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Output} \\
 \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r}
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Sub} \\
 \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_2 @ T_2, \sigma] \ 0 @ T_-
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Output} \\
 \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau}
 \end{array}$$

$$\begin{array}{c}
 \phi(n) = \text{Sub} \\
 \hline
 k \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_r
 \end{array}$$

$$\begin{array}{c}
 \phi(n) = \text{Push } m \\
 \hline
 k \ \kappa \ \mu \ [\sigma] \ n @ T_{pc} \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_r, \sigma] \ n + 1 @ T_r
 \end{array}$$

$$\begin{array}{c}
 \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1
 \end{array}$$

$\iota(n) = \text{Sub}$

$\kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j$

---

$u \ [\kappa_i, \kappa_o] \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau}$   
 $k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_2 @ T_2, \sigma] \ 0 @ T_-$

$$\begin{array}{c}
 u \ \kappa \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Bnz } k \\
 \kappa = \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\
 n' = n + (m = 0) ? 1 : k \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Call} \\
 \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Ret} \\
 \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\
 \hline
 u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{c}
 \kappa_i \neq \boxed{\text{jump} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n' @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, \sigma] \ 0 @ T_-
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Bnz } k \\
 \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T_-
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Call} \\
 \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T_-
 \end{array}$$

$$\begin{array}{c}
 \iota(n) = \text{Ret} \\
 \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa_-] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T_-
 \end{array}$$

User mode  
(cache hit)

User-to-kernel mode  
(cache miss)

Kernel mode

$$\begin{array}{c} \iota(n) = \text{Sub} \\ \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n_1@T_1, n_2@T_2, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [(n_1 - n_2)@T_r, \sigma] \ n+1@T_{rpc} \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Output} \\ \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [m@T_1, \sigma] \ n@T_{pc} \xrightarrow{m@T_r} \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Sub} \\ \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n_1@T_1, n_2@T_2, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); n_1@T_1, n_2@T_2, \sigma] \ 0@T_- \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Output} \\ \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n@T_1, \sigma] \ n@T_{pc} \xrightarrow{\tau} \end{array}$$

$$\begin{array}{c} \phi(n) = \text{Sub} \\ \hline k \ \kappa \ \mu \ [n_1@T_-, n_2@T_-, \sigma] \ n@T_- \xrightarrow{\tau} \\ k \ \kappa \ \mu \ [(n_1 - n_2)@T_-, \sigma] \ n+1@T_- \end{array}$$

$$\begin{array}{c} \phi(n) = \text{Push } m \\ \hline k \ \kappa \ \mu \ [\sigma] \ n@T_- \xrightarrow{\tau} k \ \kappa \ \mu \ [m@T_-, \sigma] \ n+1@T_- \end{array}$$

$$\begin{array}{c} \phi(n) = \text{Load} \quad \kappa(p) = m@T_1 \end{array}$$

$\iota(n) = \text{Sub}$

$\kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j$

$$\begin{array}{c} u \ [\kappa_i, \kappa_o] \ \mu \ [n_1@T_1, n_2@T_2, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); n_1@T_1, n_2@T_2, \sigma] \ 0@T_- \end{array}$$

$$\begin{array}{c} u \ \kappa \ \mu \ [n'@T_1, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n'@T_{rpc} \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Bnz } k \\ \kappa = \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\ n' = n + (m = 0) ? 1 : k \\ \hline u \ \kappa \ \mu \ [m@T_1, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n'@T_{rpc} \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Call} \\ \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\ \hline u \ \kappa \ \mu \ [n'@T_1, a, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [a, (n+1)@T_r, u]; \sigma] \ n'@T_{rpc} \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Ret} \\ \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\ \hline u \ \kappa \ \mu \ [(n'@T_1, u); \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ u \ \kappa \ \mu \ [\sigma] \ n'@T_{rpc} \end{array}$$

$$\begin{array}{c} \kappa_i \neq \boxed{\text{jump} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n'@T_1, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); n'@T_1, \sigma] \ 0@T_- \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Bnz } k \\ \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [m@T_1, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); m@T_1, \sigma] \ 0@T_- \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Call} \\ \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [n'@T_1, a, \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); n'@T_1, a, \sigma] \ 0@T_- \end{array}$$

$$\begin{array}{c} \iota(n) = \text{Ret} \\ \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\ \hline u \ [\kappa_i, \kappa_o] \ \mu \ [(n'@T_1, \pi); \sigma] \ n@T_{pc} \xrightarrow{\tau} \\ k \ [\kappa_j, \kappa_-] \ \mu \ [(n@T_{pc}, u); (n'@T_1, \pi); \sigma] \ 0@T_- \end{array}$$

User mode  
(cache hit)

User-to-kernel mode  
(cache miss)

Kernel mode

$$\begin{array}{l}
 \iota(n) = \text{Sub} \\
 \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc}
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Output} \\
 \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\
 u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Sub} \\
 \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_2 @ T_2, \sigma] \ 0 @ T_1
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Output} \\
 \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T_1
 \end{array}$$

$$\begin{array}{l}
 \phi(n) = \text{Sub} \\
 \hline
 k \ \kappa \ \mu \ [n_1 @ -, n_2 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$
  

$$\begin{array}{l}
 \phi(n) = \text{Push } m \\
 \hline
 k \ \kappa \ \mu \ [\sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$
  

$$\begin{array}{l}
 \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\
 \hline
 k \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_-
 \end{array}$$

$\phi(n) = \text{Sub}$

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$$\begin{array}{l}
 k \ \kappa \ \mu \ [n_1 @ -, n_2 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Bnz } k \\
 \kappa = \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid -} \\
 n' = n + (m = 0) ? 1 : k \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Call} \\
 \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc}
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Ret} \\
 \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid T_{rpc} \mid -} \\
 \hline
 u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Bnz } k \\
 \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T_1
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Call} \\
 \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T_1
 \end{array}$$
  

$$\begin{array}{l}
 \iota(n) = \text{Ret} \\
 \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_o] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T_1
 \end{array}$$

User mode  
(cache hit)

User-to-kernel mode  
(cache miss)

Kernel mode

$$\begin{array}{l}
 \iota(n) = \text{Sub} \\
 \kappa = \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n_1 @ T_1, n_2 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [(n_1 - n_2) @ T_r, \sigma] \ n + 1 @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Output} \\
 \kappa = \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\
 u \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Sub} \\
 \kappa_i \neq \boxed{\text{sub} \mid T_{pc} \mid T_1 \mid T_2 \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [n_1 @ T_1, n_1 @ T_2, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n_1 @ T_1, n_1 @ T_2, \sigma] \ 0 @ T.
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Output} \\
 \kappa_i \neq \boxed{\text{output} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{m @ T_r} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T.
 \end{array}$$

$$\begin{array}{l}
 \phi(n) = \text{Sub} \\
 \hline
 k \ \kappa \ \mu \ [n_1 @ -, n_2 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$

$$\begin{array}{l}
 \phi(n) = \text{Push } m \\
 \hline
 k \ \kappa \ \mu \ [\sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [m @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$

$$\begin{array}{l}
 \phi(n) = \text{Load} \quad \kappa(p) = m @ T_1 \\
 \hline
 k \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ - \xrightarrow{\tau} k \ \kappa \ \mu \ [\sigma] \ n + 1 @ T_-
 \end{array}$$

$\phi(n) = \text{Sub}$

$$\begin{array}{l}
 k \ \kappa \ \mu \ [n_1 @ -, n_2 @ -, \sigma] \ n @ - \xrightarrow{\tau} \\
 k \ \kappa \ \mu \ [(n_1 - n_2) @ T_-, \sigma] \ n + 1 @ T_-
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Bnz } k \\
 \kappa = \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\
 n' = n + (m = 0) ? 1 : k \\
 \hline
 u \ \kappa \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

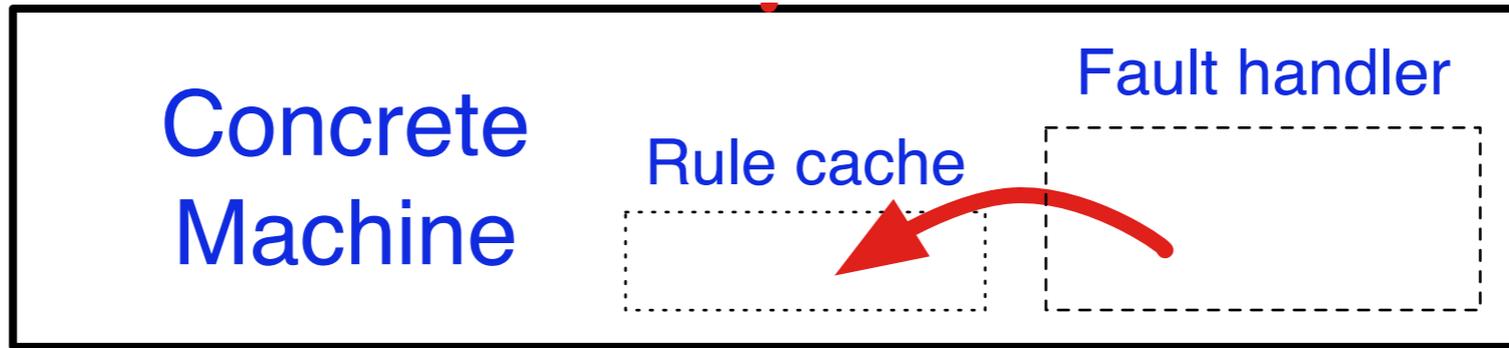
$$\begin{array}{l}
 \iota(n) = \text{Call} \\
 \kappa = \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid T_r} \\
 \hline
 u \ \kappa \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [a, (n + 1 @ T_r, u); \sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Ret} \\
 \kappa = \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid - \mid T_{rpc} \mid -} \\
 \hline
 u \ \kappa \ \mu \ [(n' @ T_1, u); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 u \ \kappa \ \mu \ [\sigma] \ n' @ T_{rpc}
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Bnz } k \\
 \kappa_i \neq \boxed{\text{bnz} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [m @ T_1, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); m @ T_1, \sigma] \ 0 @ T.
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Call} \\
 \kappa_i \neq \boxed{\text{call} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [n' @ T_1, a, \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); n' @ T_1, a, \sigma] \ 0 @ T.
 \end{array}$$

$$\begin{array}{l}
 \iota(n) = \text{Ret} \\
 \kappa_i \neq \boxed{\text{ret} \mid T_{pc} \mid T_1 \mid - \mid -} = \kappa_j \\
 \hline
 u \ [\kappa_i, \kappa_0] \ \mu \ [(n' @ T_1, \pi); \sigma] \ n @ T_{pc} \xrightarrow{\tau} \\
 k \ [\kappa_j, \kappa] \ \mu \ [(n @ T_{pc}, u); (n' @ T_1, \pi); \sigma] \ 0 @ T.
 \end{array}$$



# Fault Handler

<i>opcode</i>	<i>allow</i>	<i>e<sub>rpc</sub></i>	<i>e<sub>r</sub></i>
sub	TRUE	LAB <sub>pc</sub>	LAB <sub>1</sub> ⊔ LAB <sub>2</sub>
output	TRUE	LAB <sub>pc</sub>	LAB <sub>1</sub> ⊔ LAB <sub>pc</sub>
push	TRUE	LAB <sub>pc</sub>	BOT
load	TRUE	LAB <sub>pc</sub>	LAB <sub>1</sub> ⊔ LAB <sub>2</sub>
store	LAB <sub>1</sub> ⊔ LAB <sub>pc</sub> ⊆ LAB <sub>3</sub>	LAB <sub>pc</sub>	LAB <sub>1</sub> ⊔ LAB <sub>2</sub> ⊔ LAB <sub>pc</sub>
jump	TRUE	LAB <sub>1</sub> ⊔ LAB <sub>pc</sub>	--
bnz	TRUE	LAB <sub>1</sub> ⊔ LAB <sub>pc</sub>	--
call	TRUE	LAB <sub>1</sub> ⊔ LAB <sub>pc</sub>	LAB <sub>pc</sub>
ret	TRUE	LAB <sub>1</sub>	--

IFC RULE TABLE



RULE COMPILER



FAULT HANDLER

# Handler Generation

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- IFC rule table entries form a small DSL for computing labels and booleans
  - parameterized over lattice  $\perp$ ,  $\sqcup$  and  $\sqsubseteq$

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Tag:  $\text{Label} \rightarrow \text{Int}$    Lab:  $\text{Int} \rightarrow \text{Label}$

genBot, genJoin, genFlows : code sequence computing on integers

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Tag:  $\text{Label} \rightarrow \text{Int}$    Lab:  $\text{Int} \rightarrow \text{Label}$   
genBot, genJoin, genFlows : code sequence computing on integers
- We use structured code generators to simplify verification

```
genFaultHandler  $\mathcal{R}$  = genComputeResults  $\mathcal{R}$  ++
  genStoreResults ++
  genIf [Ret] [Push (-1); Jump]
```

```
genComputeResults  $\mathcal{R}$  =
  genIndexedCases genMatchOp (genApplyRule  $\circ$   $Rule_{\mathcal{R}}$ ) opcodes
```

```
genMatchOp  $op$  =
  [Push  $op$ ] ++ genLoadFrom addrOpLabel ++ genEqual
genEqual = [Sub] ++ genNot
```



Match opcode

```
genApplyRule  $\langle allow, e_{rpc}, e_r \rangle$  = genBool  $allow$  ++
  genIf (genSome (genELab  $e_{rpc}$  ++ genELab  $e_r$ )) genNone
```



Compute results of rule

```
genELab BOT = genBot
  LAB $_i$  = genLoadFrom addrTag $_i$ 
  LE $_1$   $\sqcup$  LE $_2$  = genELab LE $_2$  ++ genELab LE $_1$  ++ genJoin
```

```
genBool TRUE = genTrue
  LE $_1$   $\sqsubseteq$  LE $_2$  = genELab LE $_2$  ++ genELab LE $_1$  ++ genFlows
```

```
genStoreResults =
  genIf (genStoreAt addrTag $_r$  ++ genStoreAt addrTag $_{rpc}$  ++ genTrue)
  genFalse
```



Update the cache

```
genFalse = [Push 0]
genTrue = [Push 1]
genAnd = genIf [] (genPop ++ genFalse)
genOr = genIf (genPop ++ genTrue) []
genNot = genIf genFalse genTrue
genImpl = genNot ++ genOr
genSome  $c$  =  $c$  ++ genTrue
genNone = genFalse
```

```
genIndexedCases  $genDefault$   $genGuard$   $genBody$  =  $g$ 
  where  $g$  nil =  $genDefault$ 
         $g$  ( $n :: ns$ ) =  $genGuard$   $n$  ++ genIf ( $genBody$   $n$ ) ( $g$   $ns$ )
```

```
genIf  $t$   $f$  = genSkipIf (length  $f'$ ) ++  $f'$  ++  $t$ 
  where  $f' = f$  ++ genSkip(length  $t$ )
```

```
genSkip  $n$  = genTrue ++ genSkipIf  $n$ 
genSkipIf  $n$  = [Bnz ( $n+1$ )]
genStoreAt  $p$  = [Push  $p$ ; Store]
genLoadFrom  $p$  = [Push  $p$ ; Load]
genPop = [Bnz 1]
```

```
opcodes = add :: output :: ... :: ret :: nil
```

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genFaultHandler  $\mathcal{R}$  = genComputeResults  $\mathcal{R}$  ++
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```
genELab BOT          = genBot
  LABi             = genLoadFrom addrTagi
  LE1  $\sqcup$  LE2    = genELab LE2 ++ genELab LE1 ++ genJoin

genBool TRUE        = genTrue
  LE1  $\sqsubseteq$  LE2 = genELab LE2 ++ genELab LE1 ++ genFlows
```

```
genIf (genStoreAt addr Tagr ++ genStoreAt addr Tagrpc ++ genTrue)
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genNot   = genIf genFalse genTrue
genImpl  = genNot ++ genOr
genSome c = c ++ genTrue
genNone  = genFalse
```

```
genIndexedCases genDefault genGuard genBody = g
  where g nil      = genDefault
        g (n :: ns) = genGuard n ++ genIf (genBody n) (g ns)
```

```
genIf t f      = genSkipIf (length f') ++ f' ++ t
  where f' = f ++ genSkip(length t)
genSkip n     = genTrue ++ genSkipIf n
genSkipIf n  = [Bnz (n+1)]
genStoreAt p = [Push p; Store]
genLoadFrom p = [Push p; Load]
genPop        = [Bnz 1]
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Update the cache

```
genFalse = [Push 0]
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```

```
genIf  $t$   $f$  = genSkiplf (length  $f'$ ) ++  $f'$  ++  $t$ 
  where  $f' = f$  ++ genSkip(length  $t$ )
  genSkip  $n$  = genTrue ++ genSkiplf  $n$ 
  genSkiplf  $n$  = [Bnz ( $n+1$ )]
```

```
genStoreAt  $p$  = [Push  $p$ ; Store]
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genPop = [Bnz 1]
```

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genPop = [Bnz 1]
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```
opcodes = add :: output :: ... :: ret :: nil
```

# Handler Correctness

**Lemma 7.1** (Fault handler correctness, allowed case). Suppose that  $\vdash_{\mathcal{R}} (L_{pc}, \ell_1, \ell_2, \ell_3) \rightsquigarrow_{opcode} L_{rpc}, L_r$  and

$$\kappa_i = \boxed{opcode \mid \text{Tag}(L_{pc}) \mid \text{Tag}(\ell_1) \mid \text{Tag}(\ell_2) \mid \text{Tag}(\ell_3)} .$$

Then  $\phi_{\mathcal{R}} \vdash \langle k \ [ \kappa_i, \kappa_o ] \ \mu \ [ (pc, u); \sigma ] \ 0@T\_ \rangle \rightarrow_k^*$   
 $\langle u \ [ \kappa_i, \kappa'_o ] \ \mu \ [ \sigma ] \ pc \rangle$

with output cache  $\kappa'_o = \boxed{\text{Tag}(L_{rpc}) \mid \text{Tag}(L_r)} .$

and similarly for disallowed case

# Correctness proof

- Based on custom Hoare logic

$$\begin{aligned} \{P\} c \{Q\} &\triangleq \\ c = \phi(n), \dots, \phi(n' - 1) \wedge P(\kappa, \sigma) &\implies \\ \exists \kappa' \sigma'. \quad Q(\kappa', \sigma') & \\ \wedge \phi \vdash \langle k \ \kappa \ \mu \ [\sigma] \ n@T\_ \rangle &\rightarrow_k^* \langle k \ \kappa' \ \mu \ [\sigma'] \ n'@T\_ \rangle \end{aligned}$$

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- Only apply to self-contained code that “falls off end”
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- Only apply to self-contained code that “falls off end”
  - Usual composition law holds
  - We use different form of triple to describe “escaping” code
- Imply total correctness, because deterministic

# Proof style

- Triples for each instruction, each generator, e.g.

$$\frac{P(\kappa, \sigma) := \exists n_1 \mathbf{T}_1 n_2 \mathbf{T}_2 \sigma'. \quad \sigma = n_1 @ \mathbf{T}_1, n_2 @ \mathbf{T}_2, \sigma' \wedge Q(\kappa, ((n_1 + n_2) @ \mathbf{T}_-, \sigma'))}{\{P\} [\text{Add}] \{Q\}}$$

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$$\{P\} [\text{Add}] \{Q\}$$

$$P(\kappa, \sigma) := \exists n \mathbf{T} \sigma'. \sigma = n @ \mathbf{T}, \sigma' \wedge (n \neq 0 \implies P_1(\kappa, \sigma')) \\ \wedge (n = 0 \implies P_2(\kappa, \sigma'))$$

$$\{P_1\} c_1 \{Q\} \quad \{P_2\} c_2 \{Q\}$$

---

$$\{P\} \text{genIf } c_1 \ c_2 \{Q\}$$

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$$\{P_1\} c_1 \{Q\} \quad \{P_2\} c_2 \{Q\}$$

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- Triples for concrete lattice operations (genBot, genJoin, genFlows)

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$$\{P_1\} c_1 \{Q\} \quad \{P_2\} c_2 \{Q\}$$

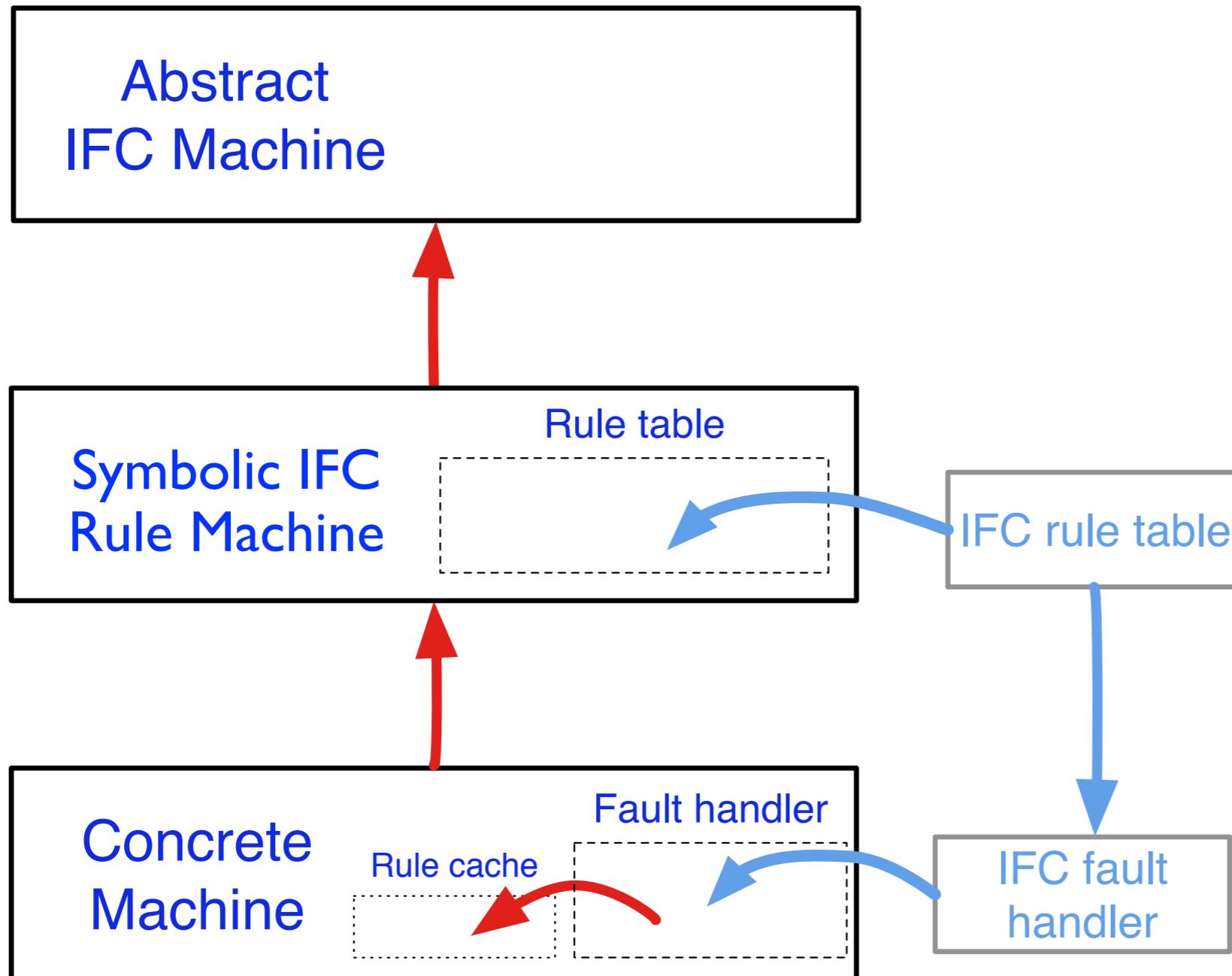
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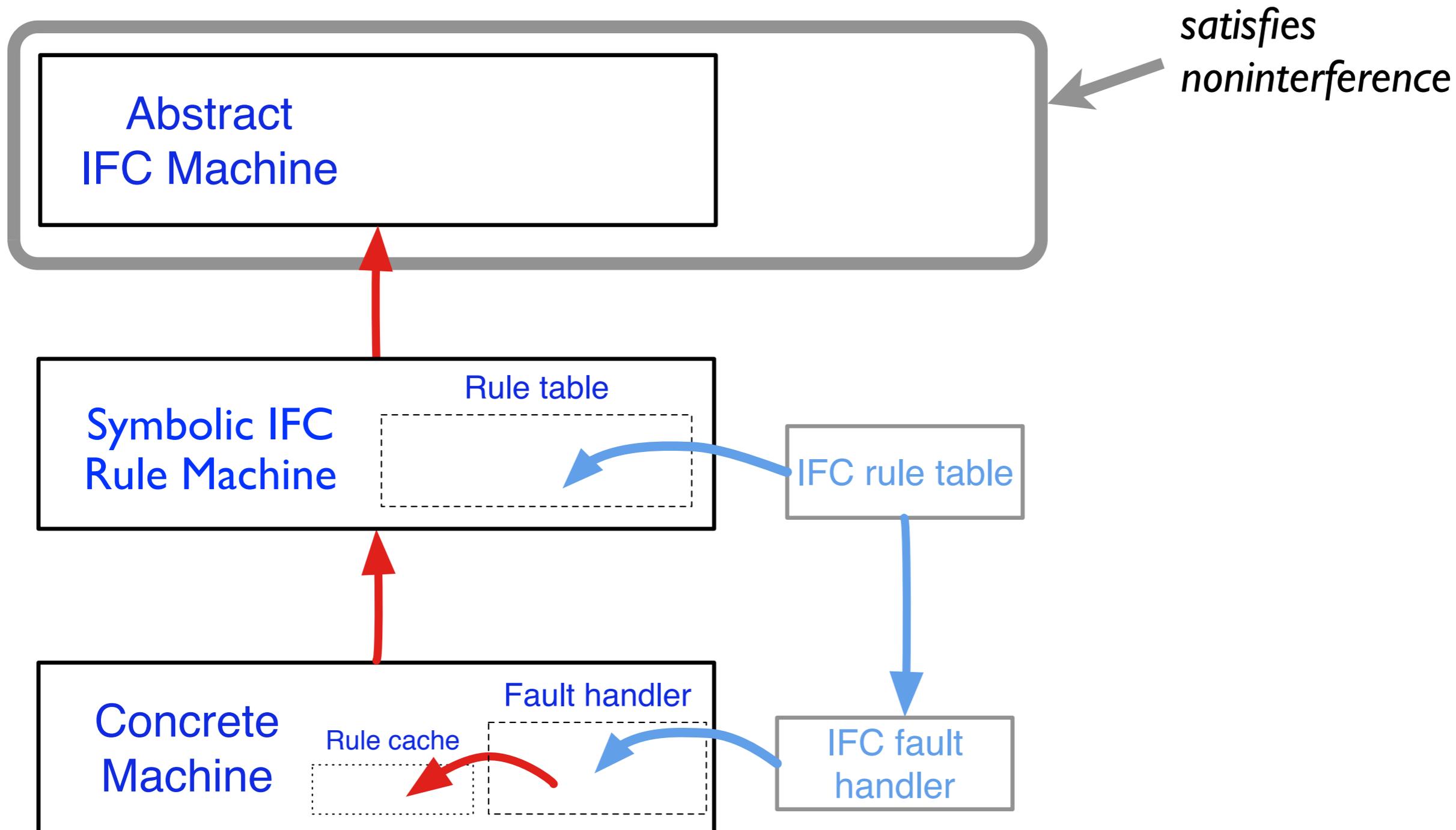
$$\{P\} \text{genIf } c_1 \ c_2 \ \{Q\}$$

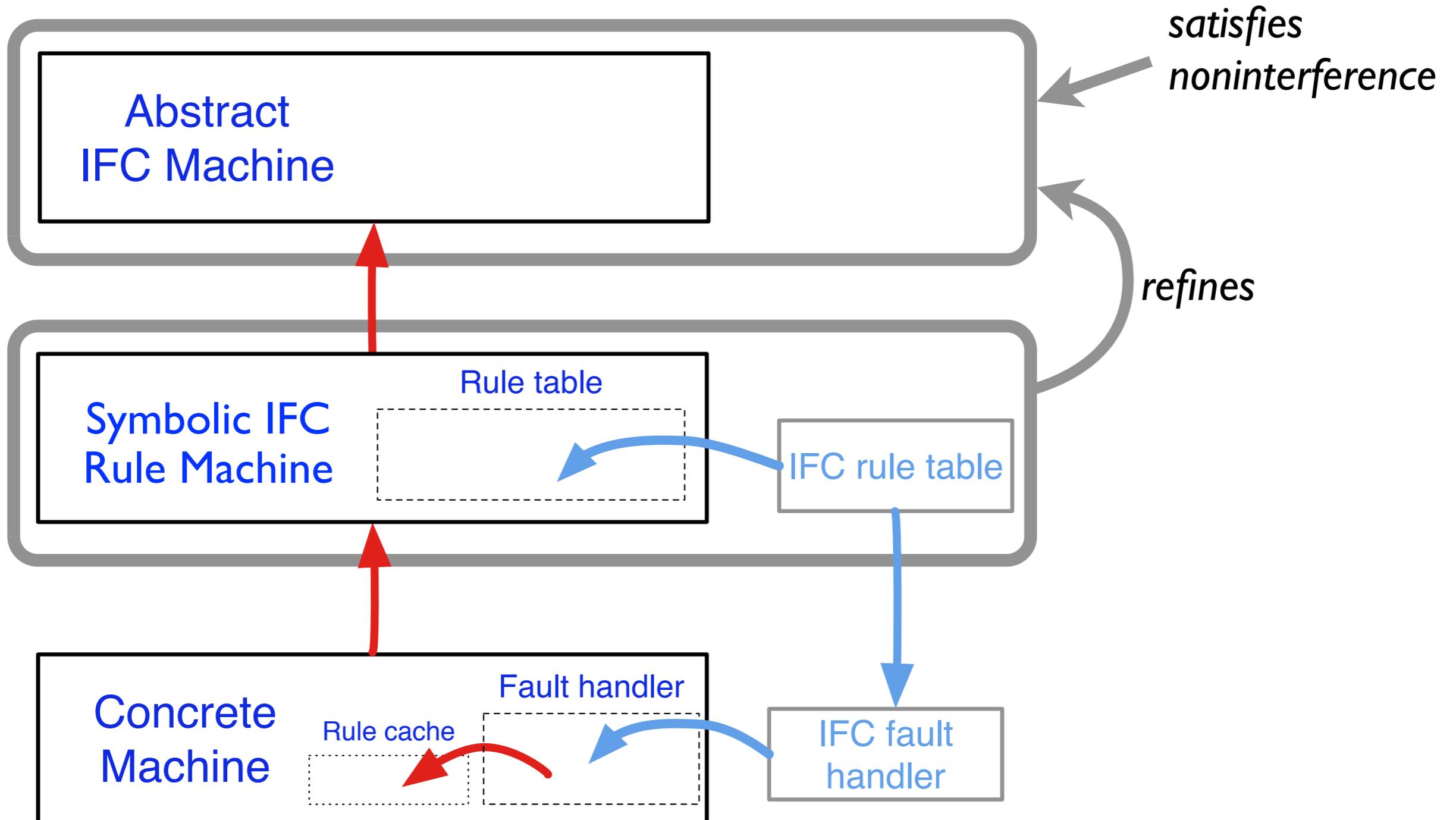
- Triples for concrete lattice operations (genBot, genJoin, genFlows)
- WP style helps automate proofs (straight-line code)

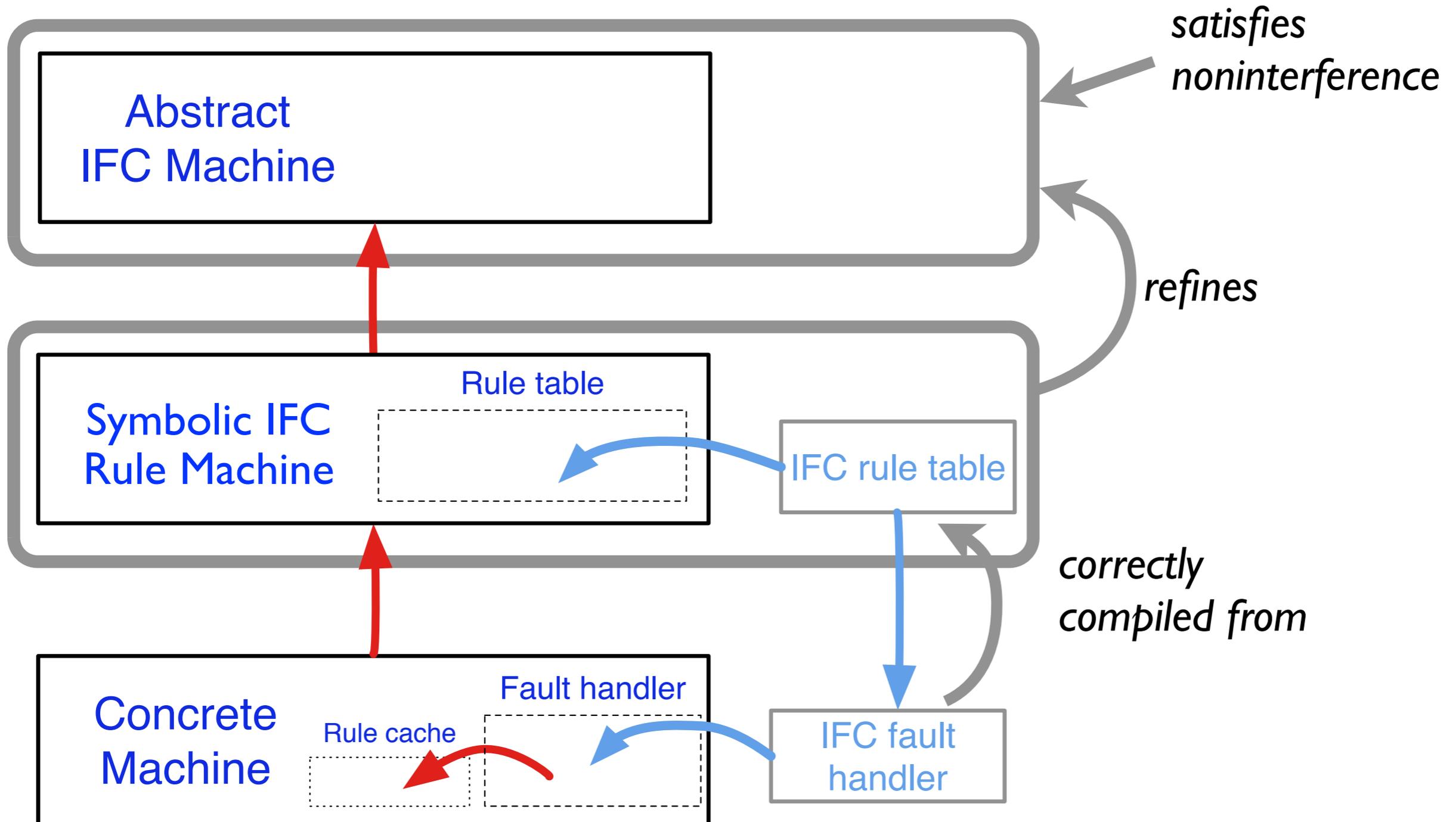
# Non-interference for concrete machine

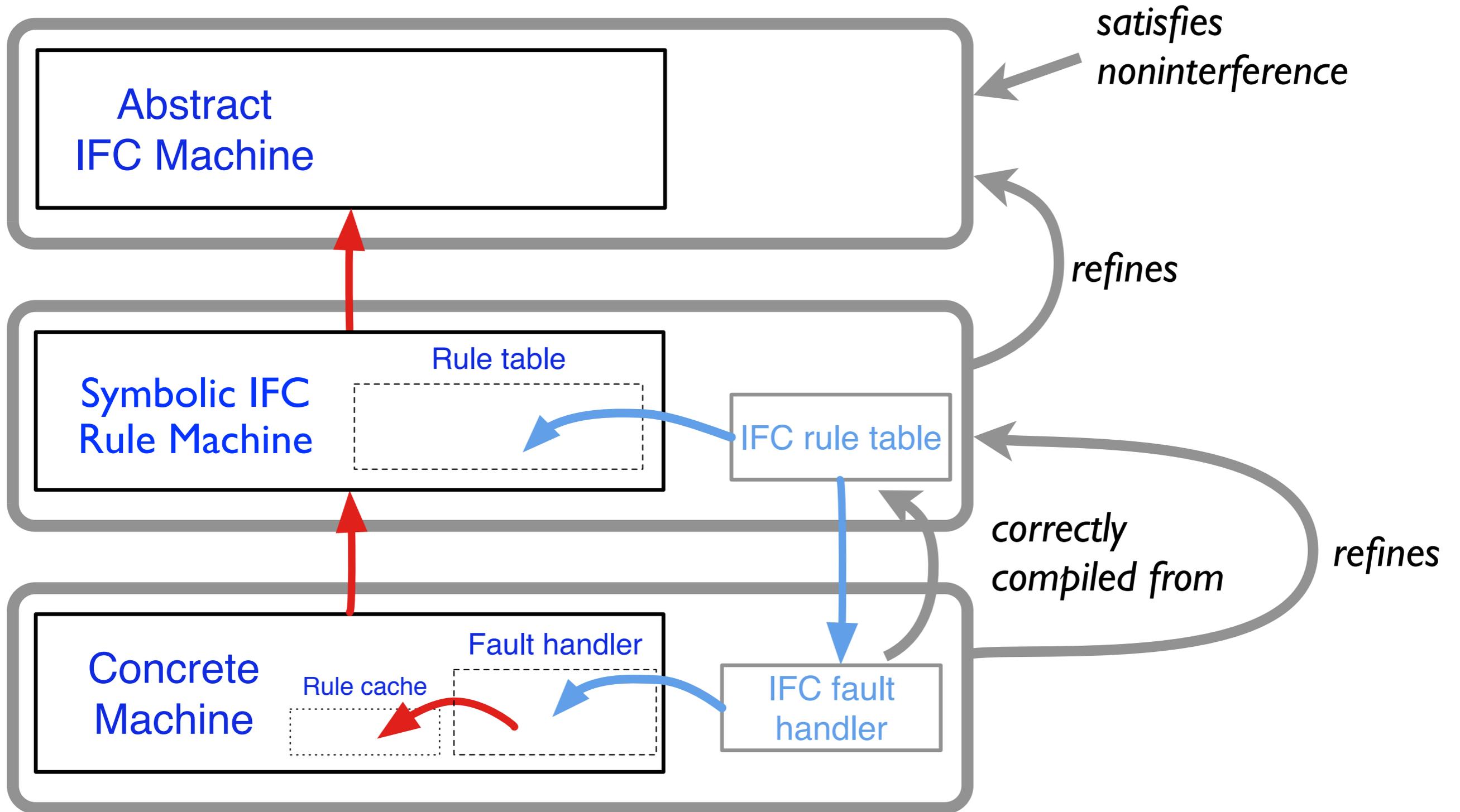
- Running this particular fault handler
- Together with arbitrary user code

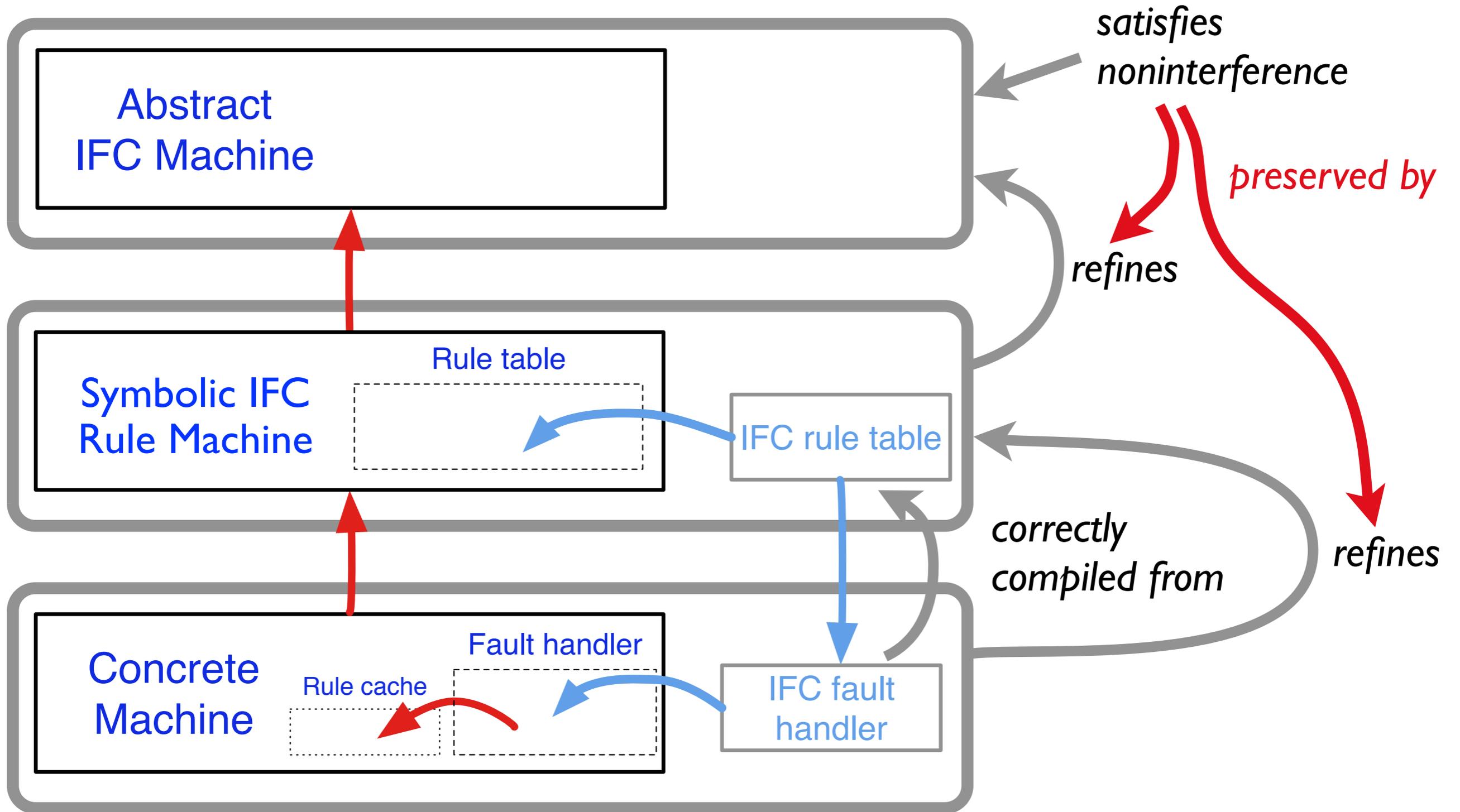


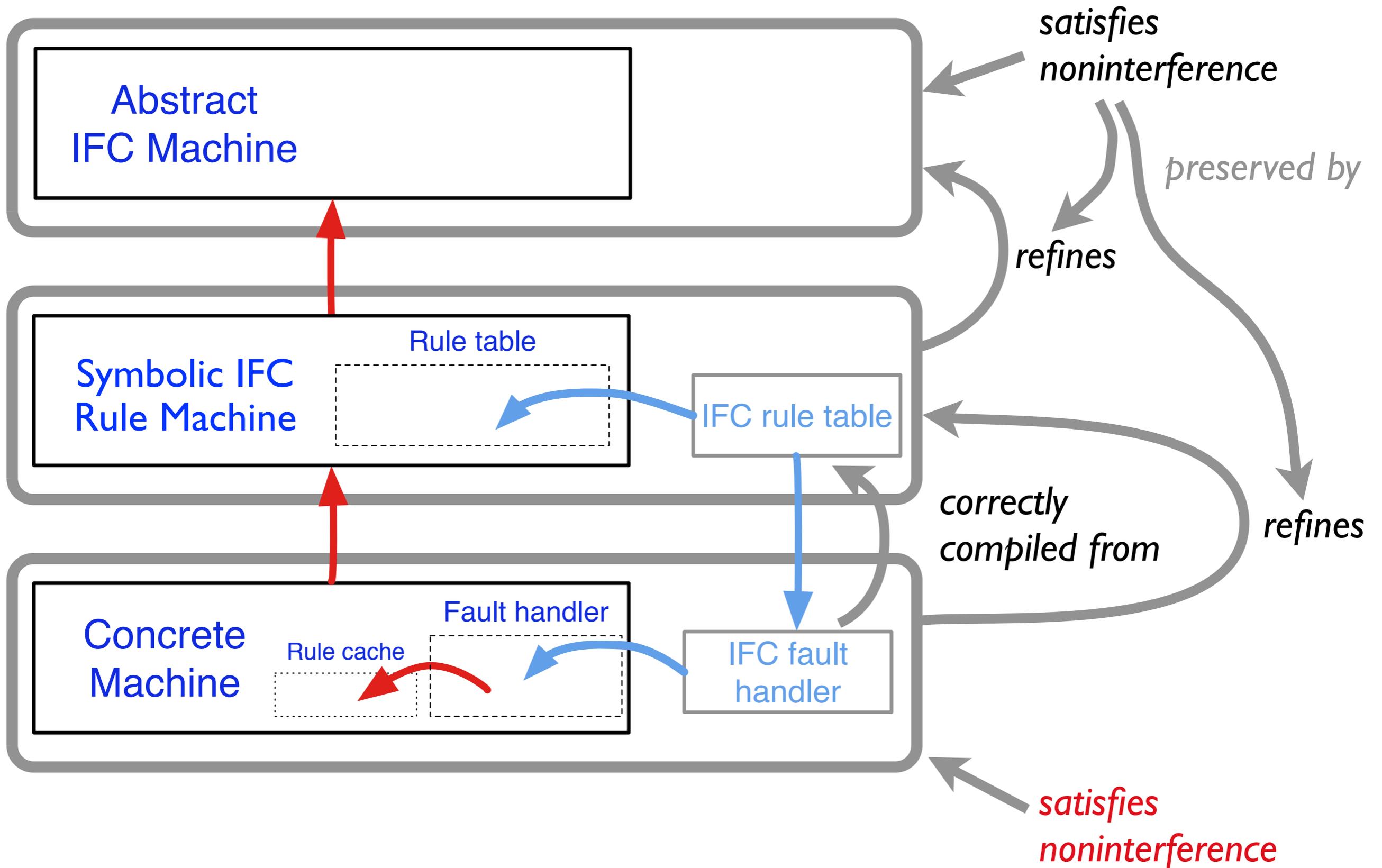












# Points to note

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- Refinement framework *very* useful for reasoning
  - start with concrete object
  - propose abstracted version
    - incorporate convenient structure and annotations
  - prove refinement
  - prove interesting property of abstract object
  - automatically follows for concrete object

# Points to note

- Refinement framework *very* useful for reasoning
  - start with concrete object
  - propose abstracted version
    - incorporate convenient structure and annotations
  - prove refinement
  - prove interesting property of abstract object
  - automatically follows for concrete object
- Need a *generic* notion of noninterference that makes sense for all machines
  - Includes a notion of abstracting concrete tags (and associated memory states) into labels

# Conclusion

Not presented here : richer IFC tag model

More uses for tags

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## More uses for tags

- SAFE architecture is quite generic
  - Can be used to implement a range of IFC label models just by varying the rule table [Montagu CSF '13]

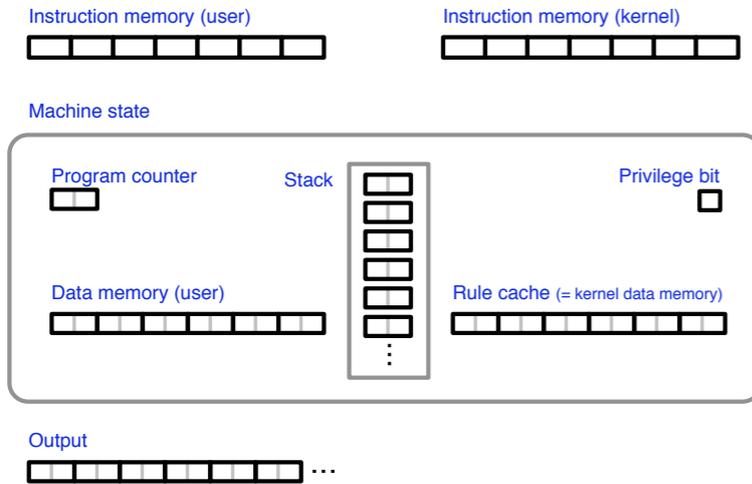
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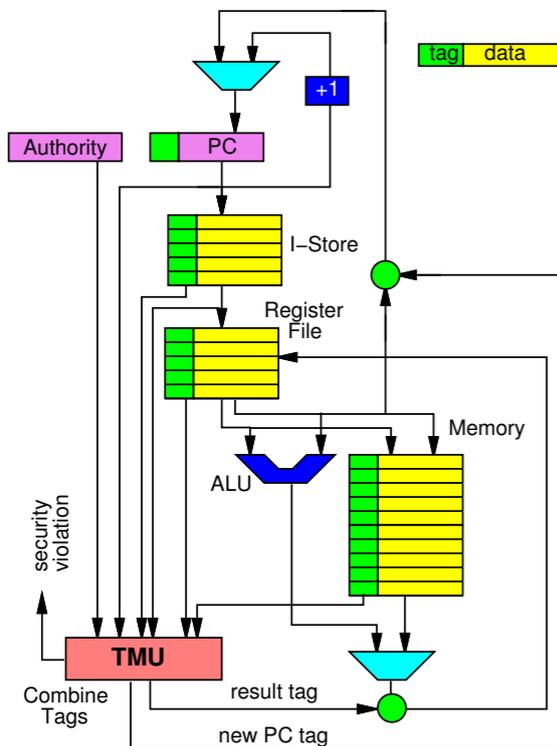
- SAFE architecture is quite generic
  - Can be used to implement a range of IFC label models just by varying the rule table [Montagu CSF '13]
- Other potential uses
  - access control (clearance), memory protection, linearity, dynamic typing



<i>opcode</i>	<i>allow</i>	$e_{r_{pc}}$	$e_r$
sub	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
output	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_{pc}$
push	TRUE	$LAB_{pc}$	BOT
load	TRUE	$LAB_{pc}$	$LAB_1 \sqcup LAB_2$
store	$LAB_1 \sqcup LAB_{pc} \sqsubseteq LAB_3$	$LAB_{pc}$	$LAB_1 \sqcup LAB_2 \sqcup LAB_{pc}$
jump	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
bnz	TRUE	$LAB_1 \sqcup LAB_{pc}$	--
call	TRUE	$LAB_1 \sqcup LAB_{pc}$	$LAB_{pc}$
ret	TRUE	$LAB_1$	--

# Thank you!

Questions??



$$\frac{\iota(n) = \text{Sub}}{\mu \frac{[n_1 \circ L_1, n_2 \circ L_2, \sigma] \ n \circ L_{pc}}{[(n_1 - n_2) \circ (L_1 \vee L_2), \sigma] \ n + 1 \circ L_{pc}} \xrightarrow{\tau}}$$

$$\frac{\iota(n) = \text{Output}}{\mu [m \circ L_1, \sigma] \ n \circ L_{pc} \xrightarrow{m \circ L_1 \vee L_{pc}} \mu [\sigma] \ n + 1 \circ L_{pc}}$$

$$\frac{\iota(n) = \text{Push } m}{\mu [\sigma] \ n \circ L_{pc} \xrightarrow{\tau} \mu [m \circ \perp, \sigma] \ n + 1 \circ L_{pc}}$$

$$\frac{\iota(n) = \text{Load} \quad \mu(p) = m \circ L_2}{\mu [p \circ L_1, \sigma] \ n \circ L_{pc} \xrightarrow{\tau} \mu [m \circ L_1 \vee L_2, \sigma] \ n + 1 \circ L_{pc}}$$

$$\frac{\iota(n) = \text{Store} \quad \mu(p) = k \circ L_3 \quad L_1 \vee L_{pc} \leq L_3}{\mu(p) \leftarrow (m \circ L_1 \vee L_2 \vee L_{pc}) = \mu'}{\mu [p \circ L_1, m \circ L_2, \sigma] \ n \circ L_{pc} \xrightarrow{\tau} \mu' [\sigma] \ n + 1 \circ L_{pc}}$$

$$\frac{\iota(n) = \text{Jump}}{\mu [n' \circ L_1, \sigma] \ n \circ L_{pc} \xrightarrow{\tau} \mu [\sigma] \ n' \circ (L_1 \vee L_{pc})}$$