

Dessins de graphes planaires par intersection ou contact de L

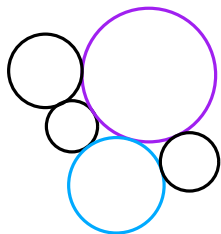
Claire Pennarun
Daniel Gonçalves Lucas Isenmann

JGA - Bordeaux - 17 novembre 2017

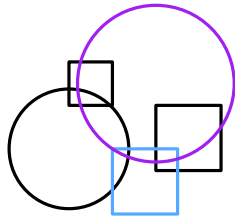
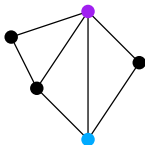
CONTACT / INTERSECTION GRAPHS

Given a family of shapes X , a graph has a **X -intersection** (resp. **X -contact**) representation if:

- each vertex has a shape $x \in X$
- two vertices are adjacent iff their assigned shapes intersect (resp. touch).



$\{\circ\}$ -contact representation



$\{\circ, \square\}$ -intersection representation

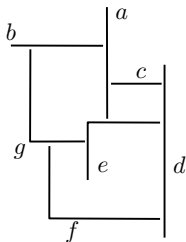
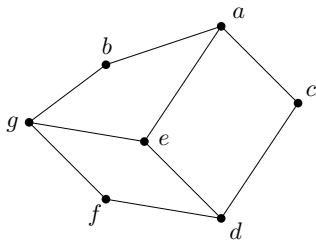
CONTACT / INTERSECTION GRAPHS

Representation by segments

- \triangle -free planar graphs: contacts of segments
- planar graphs: intersections of segments

Representation by paths of horizontal/vertical segments (VPG)

- A graph is B_k -VPG if the paths have at most k bends
- contact-segment $\equiv B_1$ -VPG \rightarrow \triangle -free planar graphs are B_1 -VPG, have a $\{\perp, \lrcorner, |, -\}$ -contact representation
- planar graphs: (intersection) B_2 -VPG



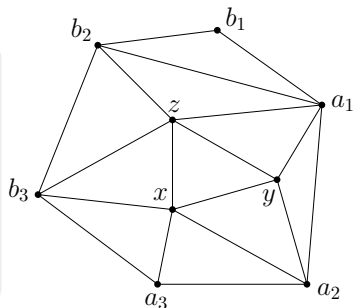
2-SIDED NEAR-TRIANGULATIONS

Near-triangulation: every inner face is a triangle

Definition

A near-triangulation is **2-sided** if it has:

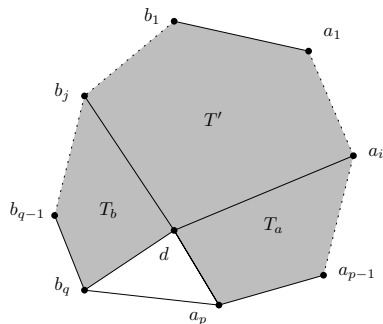
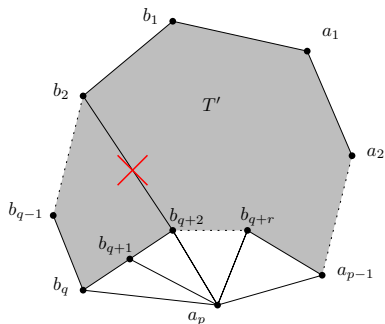
- no separating triangle
- around the outerface: vertices $a_1, a_2, \dots, a_p, b_q, \dots, b_1$
- no chord $a_i a_j$ or $b_i b_j$



2-SIDED NEAR-TRIANGULATIONS

Three decomposition operations:

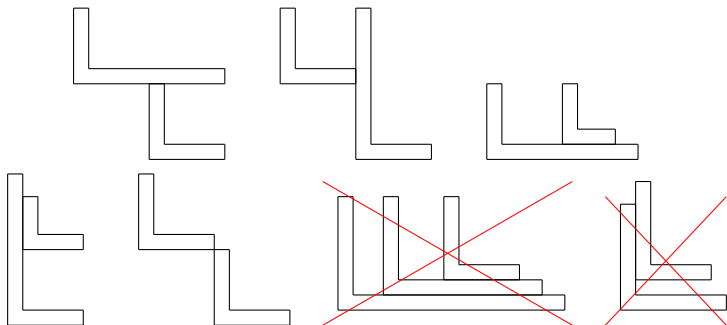
- **a_p -removal** ($p > 1$, a fan around a_p not adjacent to b_j) \rightarrow rename the neighbors of a_p into b_j vertices $\rightarrow T'$
- **b_q -removal** (symmetric)
- **cutting** ($p, q > 1$, d common neighbor of a_p and b_q) $\rightarrow T' + T_a + T_b$



THICK L-CONTACT REPRESENTATION

Contact representation (CR) with thick \perp shapes

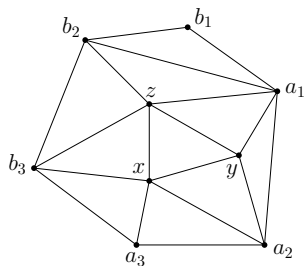
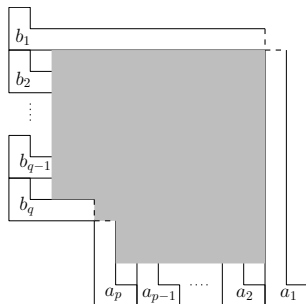
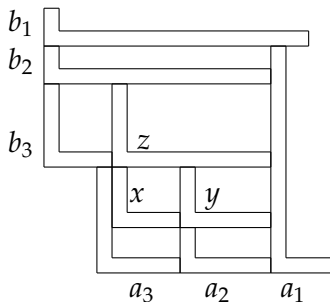
Contacts: exactly one side of a thick \perp , or bottom right + top left corners, + 1 forbidden configuration



THICK \perp -CONTACT REPRESENTATION

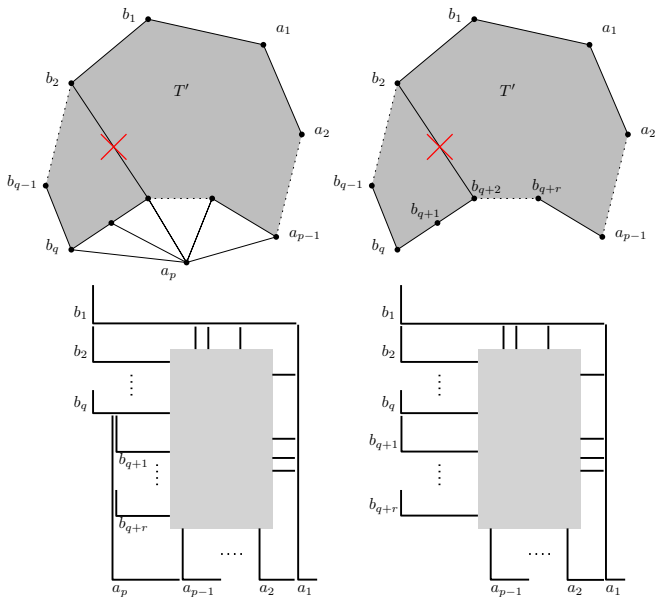
Theorem

Every 2-sided near-triangulation has a thick \perp CR with this general configuration.

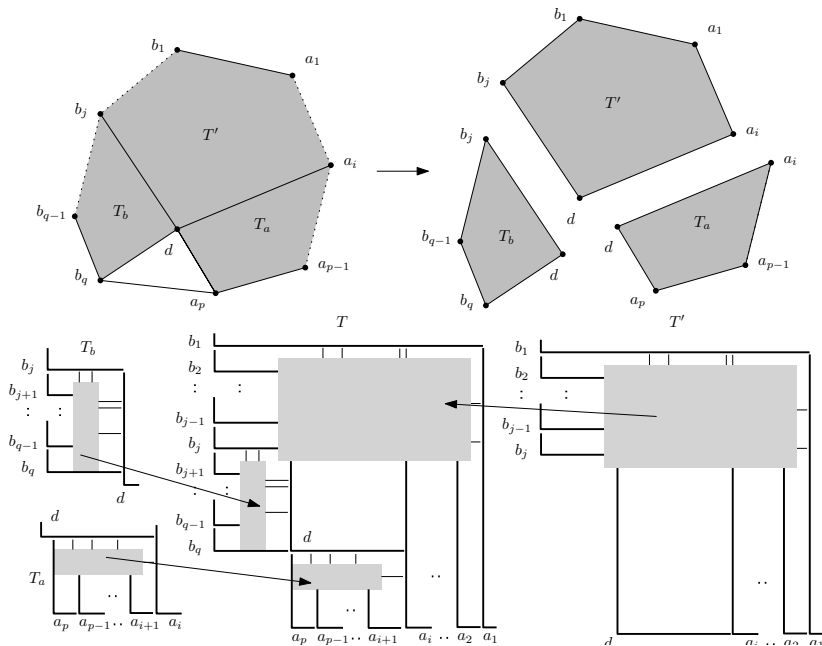


Proof: by induction on the number of vertices

THICK L-CONTACT REPRESENTATION



THICK L-CONTACT REPRESENTATION

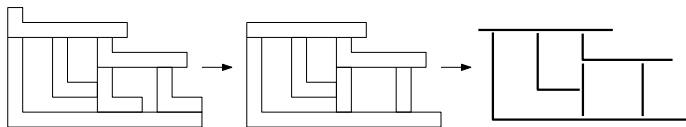
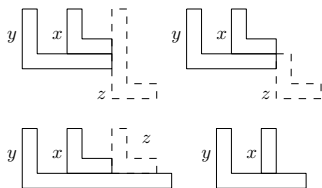


WRAPPING IT UP

Theorem

Every \triangle -free planar graph G has a $\{\perp, |, -\}$ -contact representation.

- 4-connected triangulation T containing G as an induced subgraph
- Draw the thick \perp CR of T
- Remove vertices of $T \setminus G$
- If segment-contact x/y : "reduce" x \longrightarrow
- Replace thick \perp -shapes by thin ones

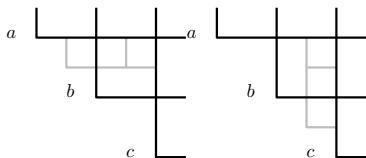


INTERSECTION REPRESENTATION

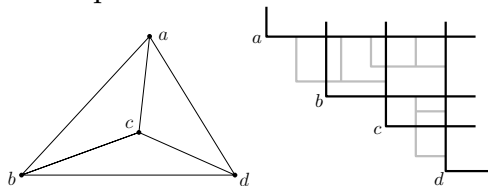
Theorem

Every planar graph has a $\{\perp, |\}$ -intersection representation.

Placing an **anchor** for each inner face of the drawing. Two types:



Full \perp intersection representation (FLIR): intersection representation of the graph with \perp -shapes + anchors

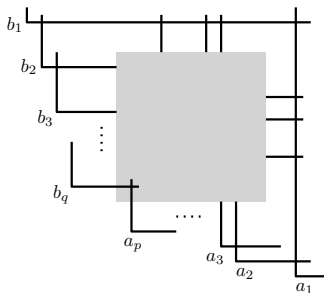


INTERSECTION REPRESENTATION

Lemma

Every 2-sided near-triangulation has a FLIR such that:

- From left to right: $b_1, b_2, \dots, b_q, \dots, a_1$
- From bottom to top: $a_1, a_2, \dots, a_p, \dots, b_1$

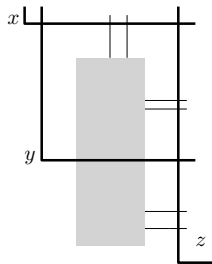


Proof: similar to contact representations

INTERSECTION REPRESENTATION

Lemma

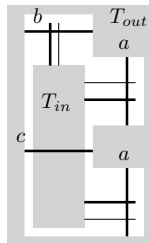
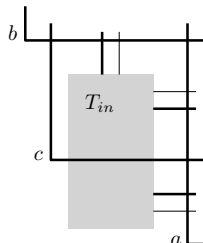
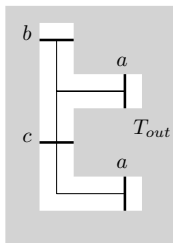
Every triangulation T with outer vertices x, y, z has an FLIR with this configuration.



Induction on the number of vertices

If T is 4-connected: apply preceding Lemma.

If T has a separating triangle $\rightarrow T_{in}$ and T_{out}



CONCLUSION

- Δ -free planar graphs: B_1 -VPG, with a $\{\perp, |, -\}$ -**contact representation**
- planar graphs: B_1 -VPG with $\{\perp, |\}$ -**intersection representation**
→ segment-intersection representation

Open questions:

- Do Δ -free planar graphs have a $\{\perp, |\}$ -contact representation?
- Extending the approach of 2-sided near-triangulations to area-prescribed variants (with thick shapes)?

- ▶ **Stefan Felsner** (TU Berlin) :
Les Schnyder woods et leurs applications.
- ▶ **András Sebő** (G-SCOP Grenoble) :
Nouvelles techniques pour TSP.
- ▶ **Uno Takeaki** (NII Tokyo) :
Algos d'énumération "Output Sensitive".
- ▶ **Petr Golovach** (Univ. Bergen) :
Algos d'énumération "Input Sensitive".



Thank you!