

# Power domination in triangular grids

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School of Computer Science, Carleton University

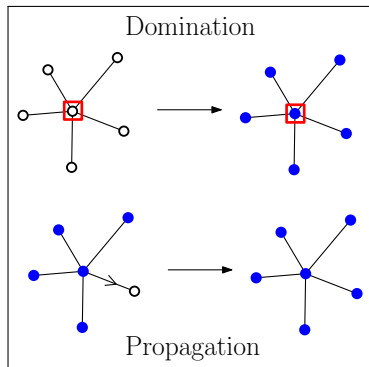
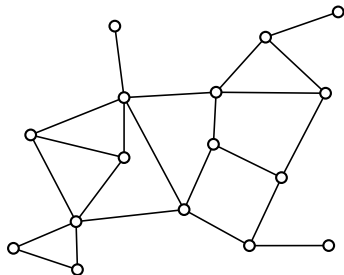
LaBRI, University of Bordeaux

CCCG, July 26, 2017

# POWER DOMINATION [HAYNES ET AL. '02]

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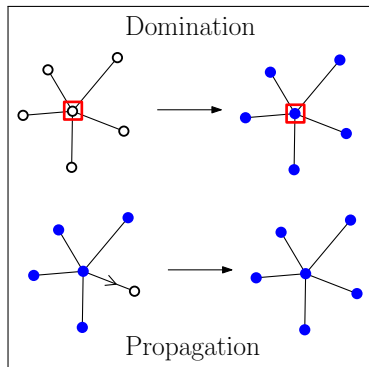
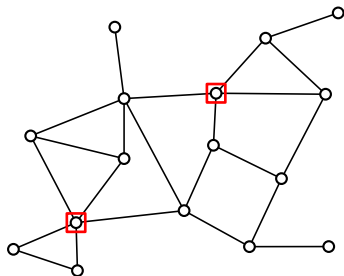
$S$  = selected,  $M$  = monitored



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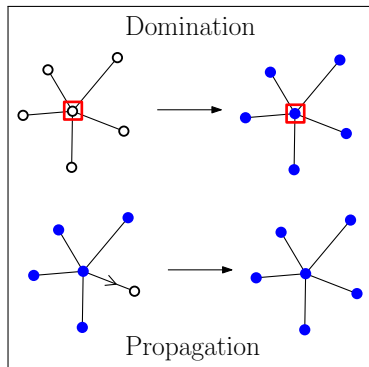
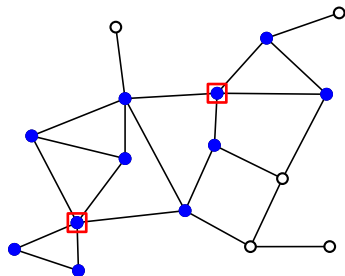
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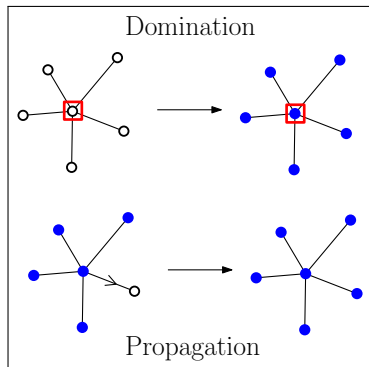
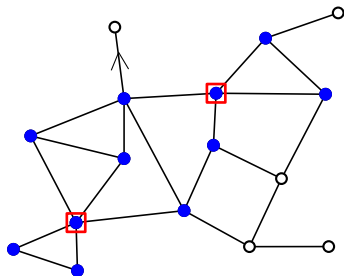
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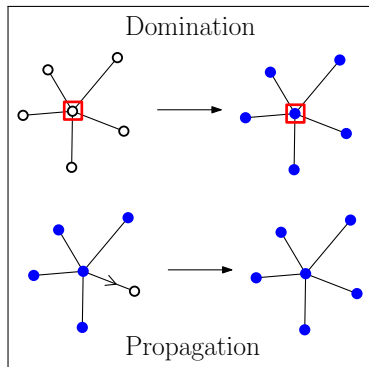
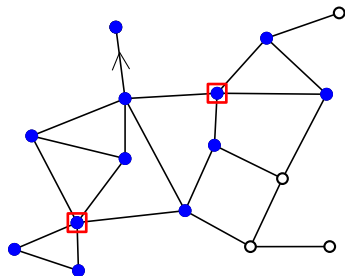
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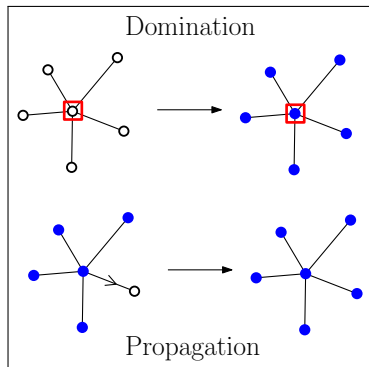
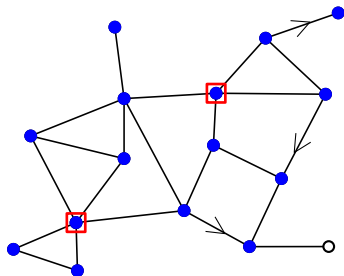
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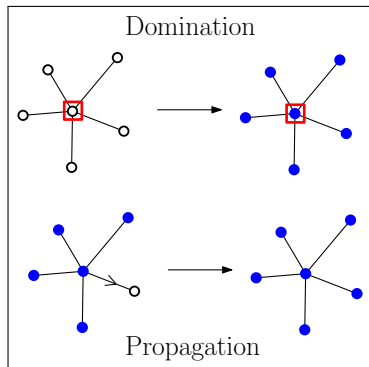
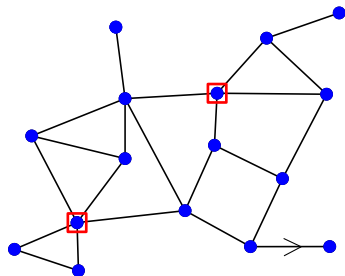
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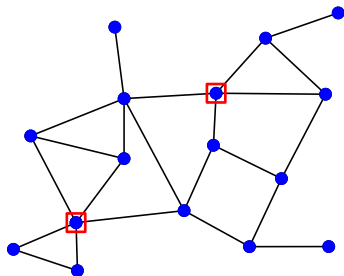




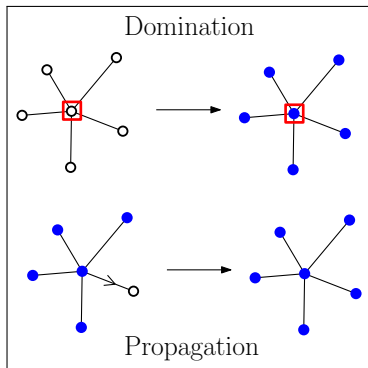
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$$\gamma_P(G) = 2$$



The **power domination number**  $\gamma_P(G)$  of  $G$  is the minimum size of  $S$  s.t.  $M = V(G)$  at the end.

## POWER DOMINATION: SOME RESULTS

Background story: Placing a minimal number of "sensors" to monitor an electrical system

POWER DOMINATING SET

**Input:**  $G, k \geq 0$ .

**Question:** Is there a power dominating set  $S \subseteq V(G)$  with  $|S| \leq k$ ?

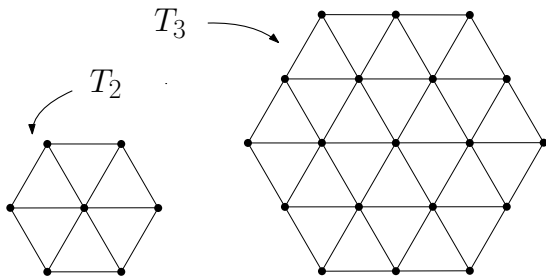
is **NP-complete** even for planar graphs, bipartite graphs, chordal graphs...  
[Guo et al. '05]

Well-studied classes: regular graphs and grids

Hexagonal grids [Ferrero et al. '11], some products of paths [Dorfling and Henning '06, Dorbec et al. '08]...

# POWER DOMINATION OF TRIANGULAR GRIDS

**Triangular grid  $T_k$ :** regular triangular grid with border of hexagonal shape ( $k$  vertices on each side)



# POWER DOMINATION OF TRIANGULAR GRIDS

Theorem [Bose, P., Verdonschot 17+]

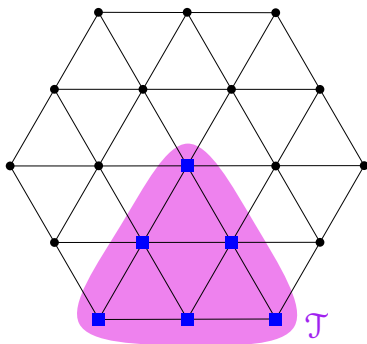
$$\gamma_P(T_k) = \left\lceil \frac{k}{3} \right\rceil.$$

Upper bound: simple construction depending on the value of  $k$  modulo 3.

Lower bound: bound an invariant (the number of vertices potentially able to propagate at each step)

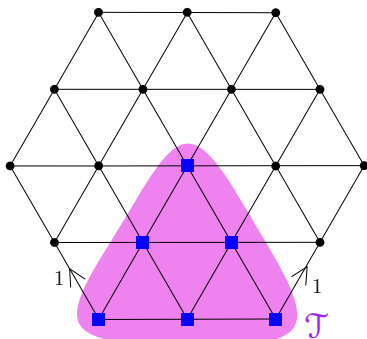
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Monitoring set  $\mathcal{T}$  is sufficient



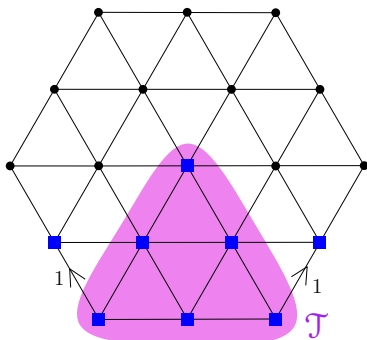
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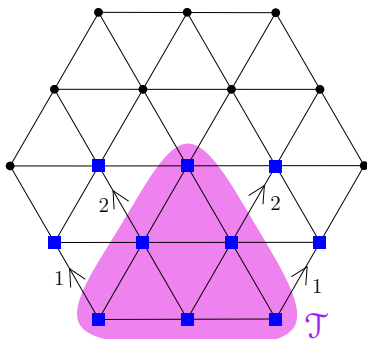
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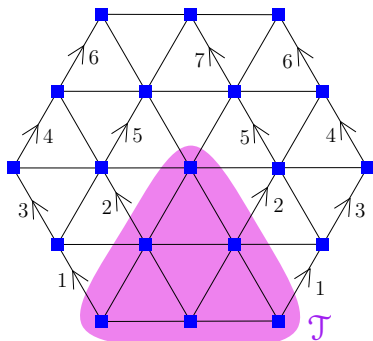
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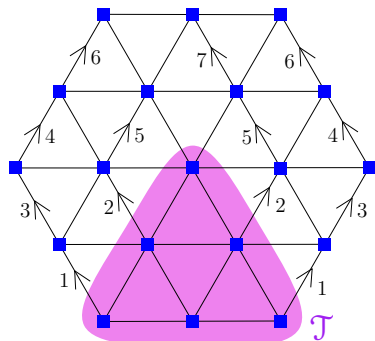
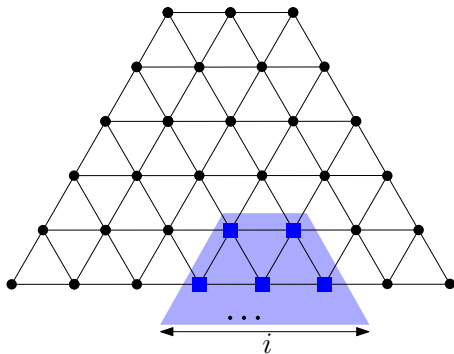
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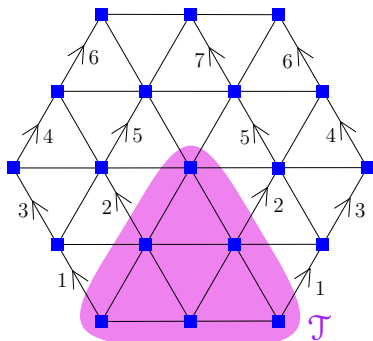
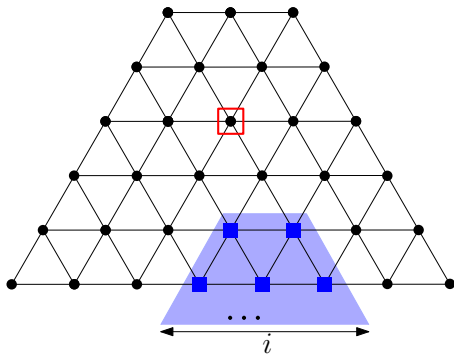
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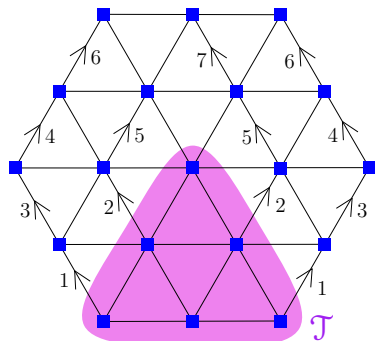
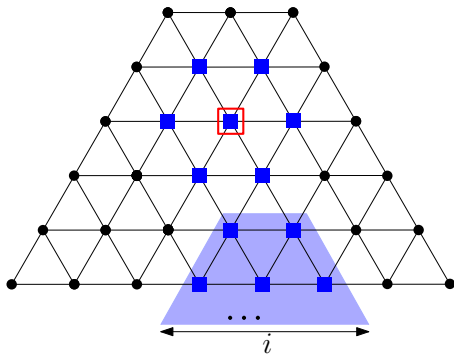
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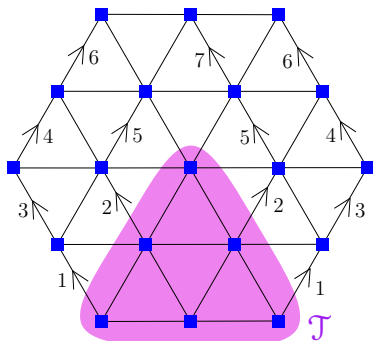
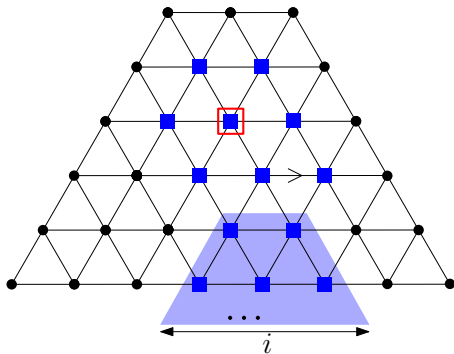
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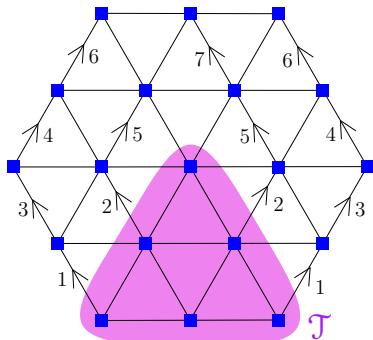
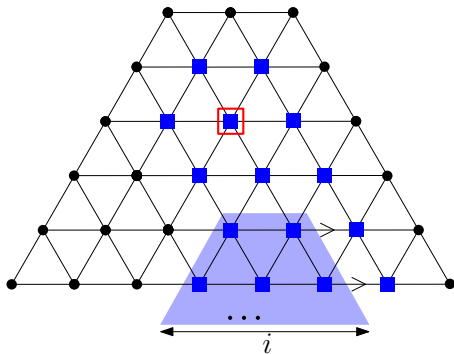
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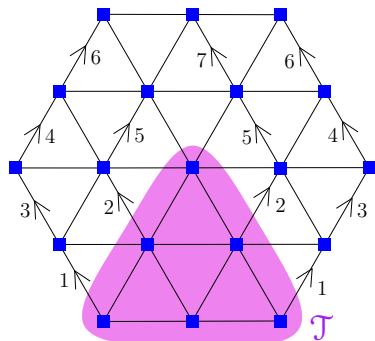
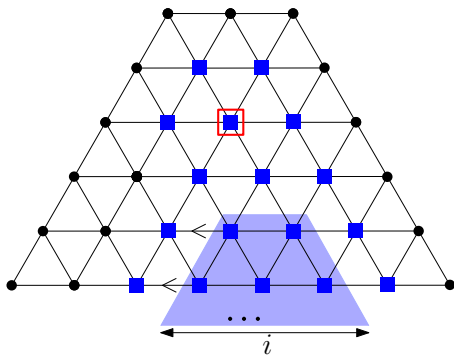
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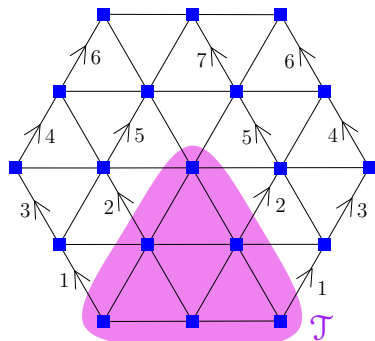
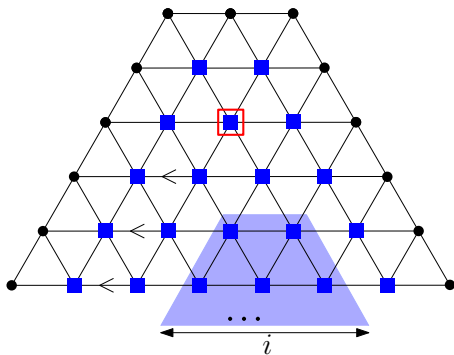
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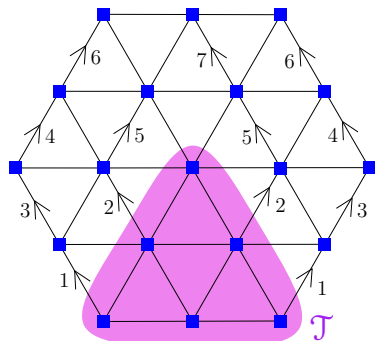
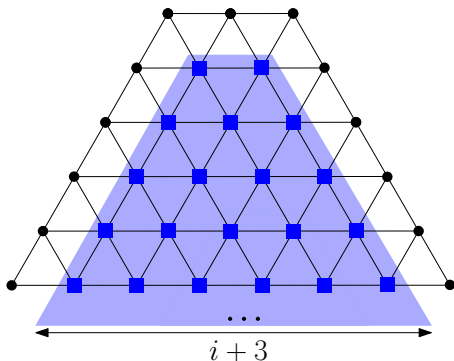


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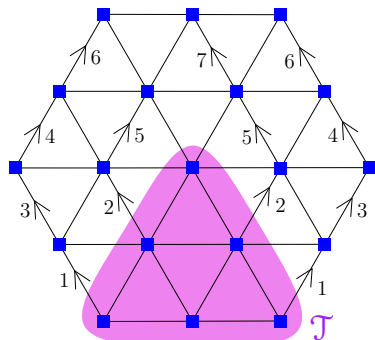
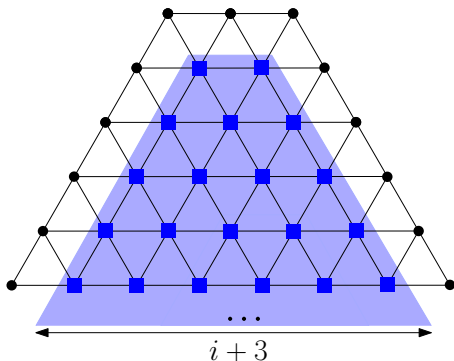
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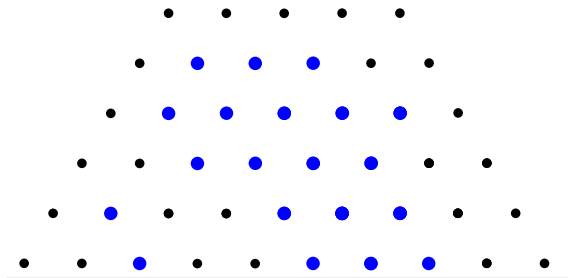
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$$\rightarrow \gamma_P(T_k) \leq \left\lceil \frac{k}{3} \right\rceil$$

## LOWER BOUND

General idea: study the size of the **border** of the set of monitored vertices (= vertices that could potentially propagate in the future)

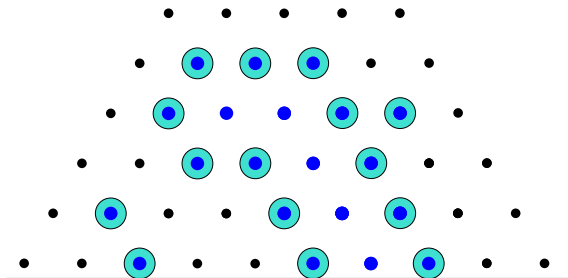
Border  $B_A$  of a set  $A$ : vertices of  $A$  such that  $N(v) \setminus A \neq \emptyset$



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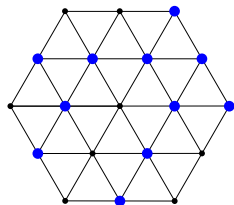
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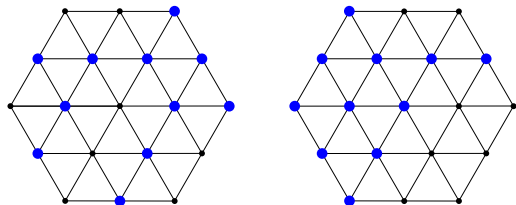
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**Shifting operation** in one direction  $A$  to  $A'$  (here direction  $\leftarrow$ )



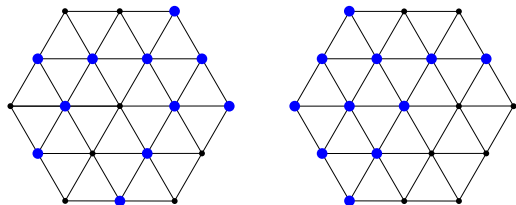
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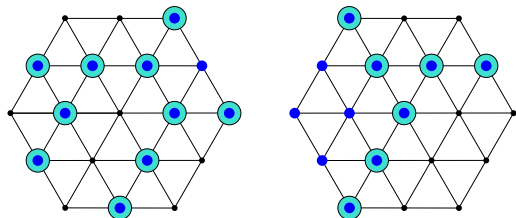
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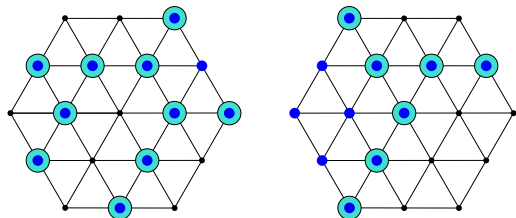


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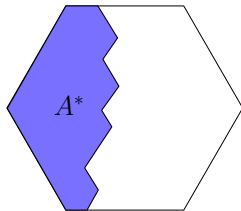
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After shifting in directions  $\{\leftarrow, \swarrow, \searrow\}$  until stabilization,  $A^*$  has a **staircase shape**



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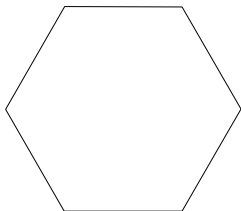
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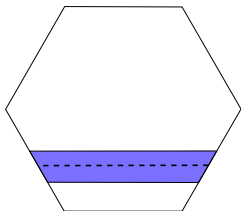


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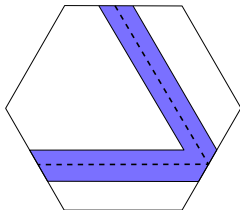


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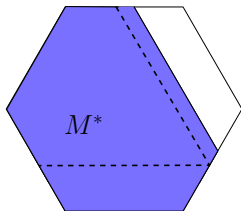


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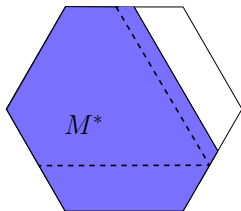


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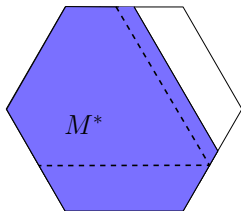
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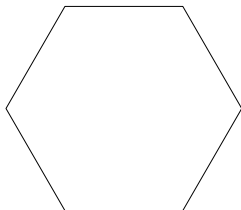
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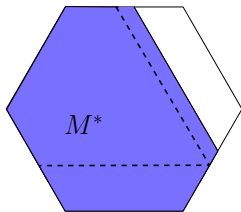


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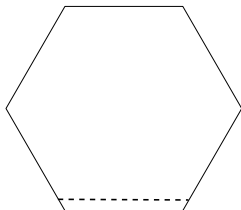
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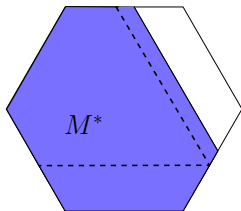


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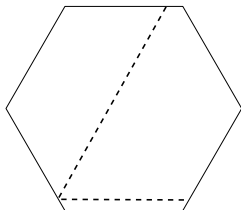
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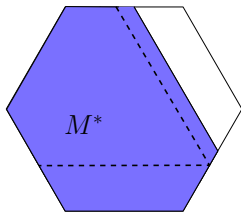


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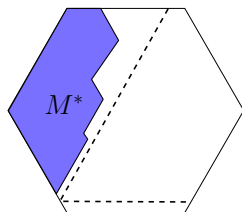
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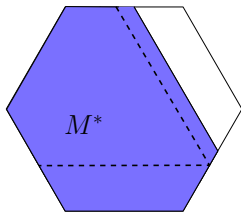


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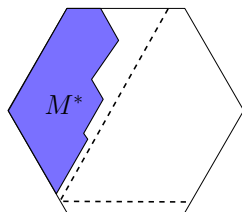
At any time,  $|B_M| \leq 6|S|$  (by induction on the propagation steps)

Consider  $M$  once it has  $\frac{n}{2}$  vertices. Shift  $M \rightarrow M^*$

Each horizontal line has at least one vertex in  $B_{M^*}$



$$|M^*| > \frac{n}{2}$$



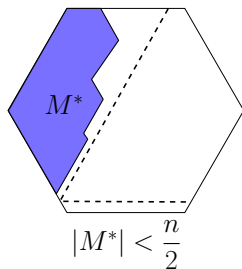
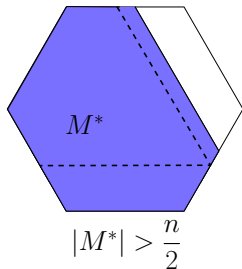
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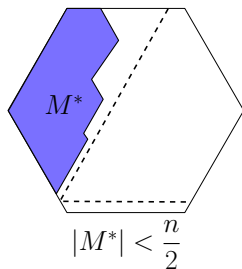
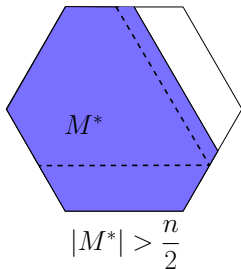
$\rightarrow |B_{M^*}| \geq 2k - 1$  and  $|B_M| \geq |B_{M^*}|$

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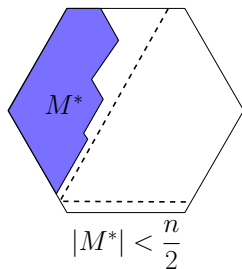
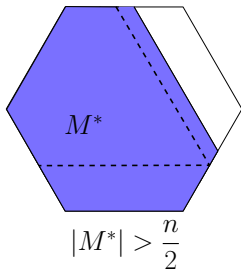


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$$2k - 1 \leq |B_M| \leq 6|S| \rightarrow |S| \geq \frac{2k - 1}{6}$$

## RESULT

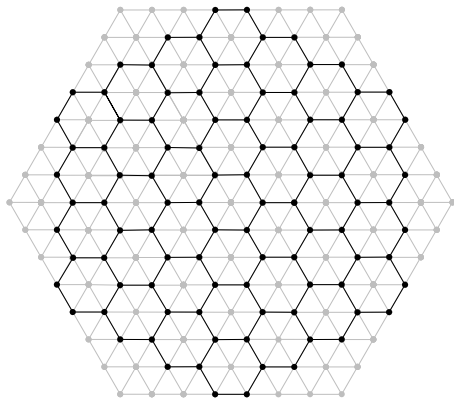
$$\frac{2k-1}{6} \leq \gamma_P(T_k) \leq \lceil \frac{k}{3} \rceil.$$

$\gamma_P(T_k)$  is an integer + there are no integers between  $\frac{2k-1}{6}$  and  $\lceil \frac{k}{3} \rceil$ .

$$\rightarrow \boxed{\gamma_P(T_k) = \lceil \frac{k}{3} \rceil}.$$

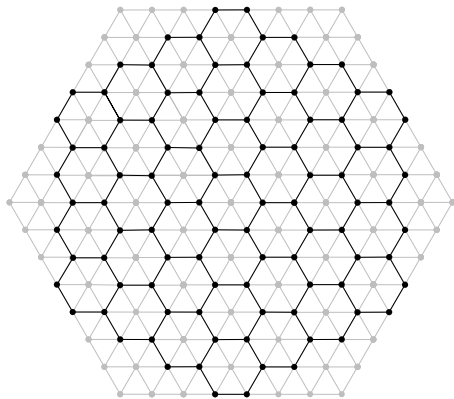
## RELATION WITH THE HEXAGONAL GRID

**Hexagonal grid**  $H_\ell$  with dimension  $\ell$  (number of hexagons per side) is an induced subgraph of  $T_{2\ell}$ ...



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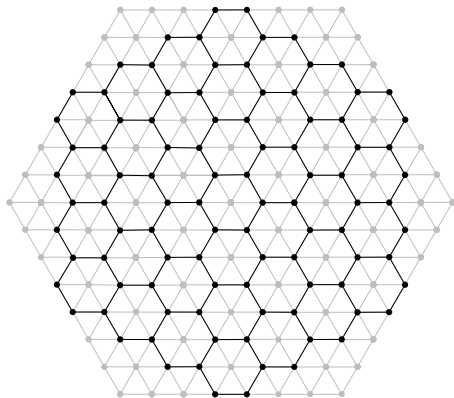
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When/why does this happen? Look at induced regular grids?

**Thank you!**