

Reasoning about data repetitions

with counter systems

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reasoning on data words

data
word $a \ b \ a \ b \ a \ b \ b \ b \ b \ a \ b \ b \ \in (\{a, b\} \times \mathbb{N})^*$
 $2 \ 4 \ 7 \ 6 \ 9 \ 3 \ 9 \ 7 \ 6 \ 2 \ 2 \ 9$

reasoning on data words

data
word

<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
2	4	7	6	9	3	9	7	6	2	2	9

$\in (\{a, b\} \times \mathbb{N})^*$



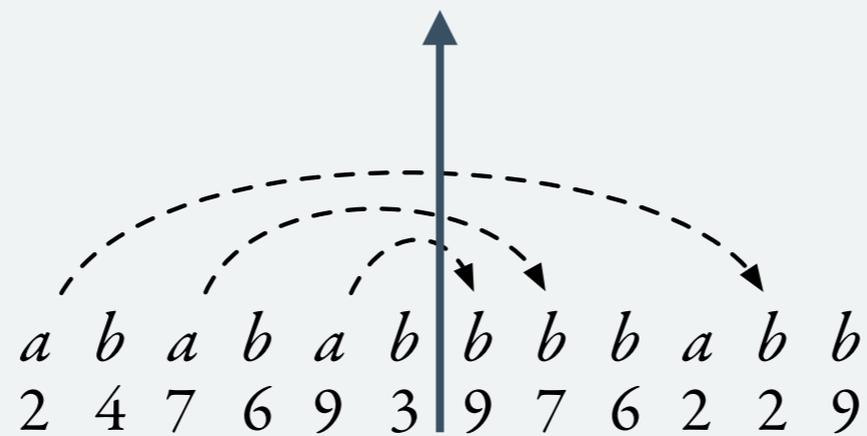
“for every a there is a b with same data value to its right”

reasoning with
infinite alphabets = counting

<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
2	4	7	6	9	3	9	7	6	2	2	9

*“for every *a* there is a *b* with same data value to its right”*

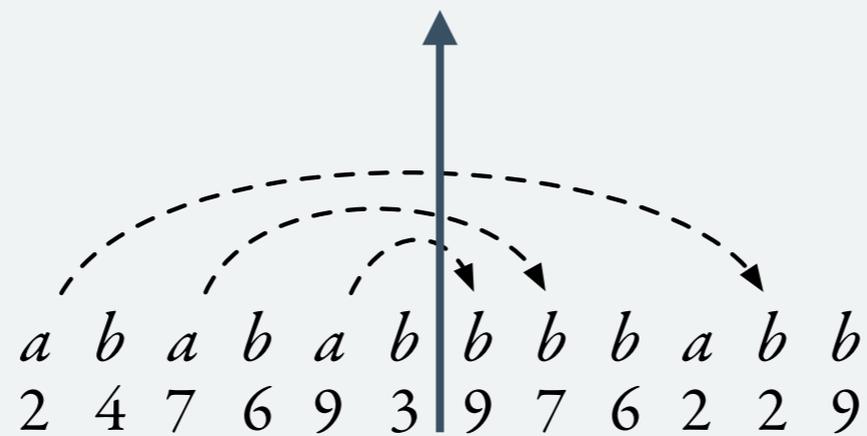
reasoning with *infinite* alphabets = counting



*“for every *a* there is a *b* with same data value to its right”*

reasoning with *infinite* alphabets = counting

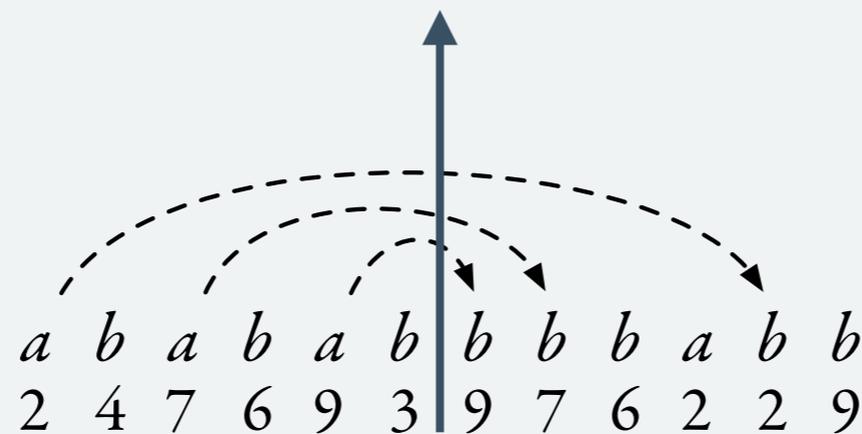
*there must be 3 distinct
data values to the right*



*“for every *a* there is a *b* with same data value to its right”*

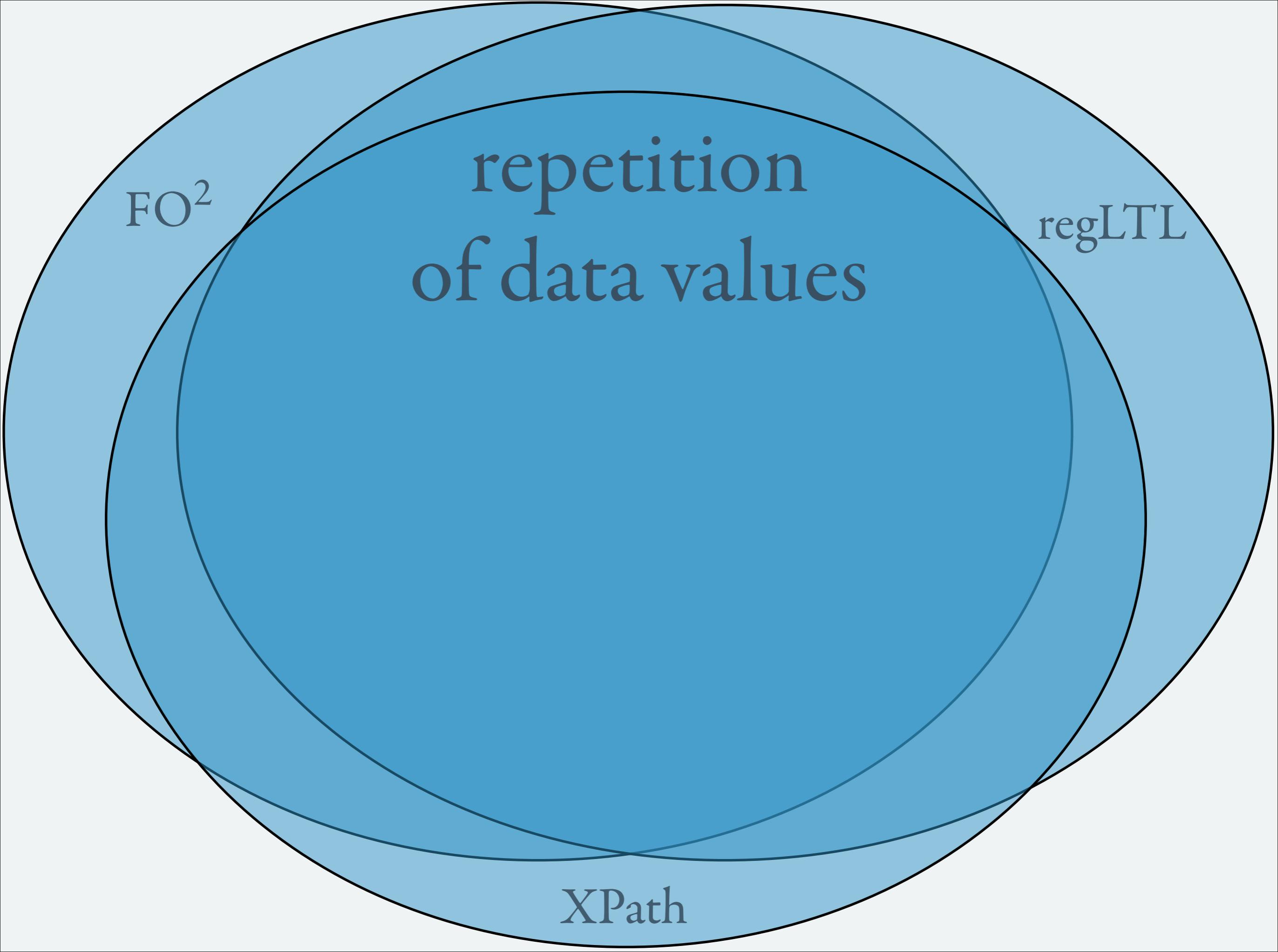
reasoning with *infinite* alphabets = counting

there must be 3 distinct data values to the right



“for every a there is a b with same data value to its right”

	DA / FO ² SAT	↔	reachability of VASS	[Bojańczyk & al.]
	ARA-1 / 1-regLTL	↔	reachability of ICA	[Demri & Lazić]
<i>on data trees</i>	XPath SAT	↔	reachability of ITCA	[Jurdziński & Lazić]
	FO ² SAT	↔	reachability of BVASS ⁺	[Dimino & al.]



FO^2

repetition
of data values

regLTL

XPath

FO²

repetition
of data values

regLTL

local $a\ b\ a\ b\ a\ b\ b\ b\ b\ a\ b\ b$
2 4 7 6 9 3 9 7 6 2 2 9

XPath

FO²

repetition
of data values

regLTL

local

a b a b a b b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9

global

a b a b a b b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9

XPath

logic of repeating values

repeating values
on data words



reachability of
counter systems

different flavours of
repeating values



different **precisions**
of counting

a b a b a b b b a b b
2 4 7 6 9 3 9 7 6 2 2 9

data₁
word

	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
\vdots												

$$\in (\{a, b\} \times \mathbb{N}\{x_1, x_2, \dots\})^*$$

*data
word*

LOGIC OF REPEATING VALUES

LTL + $\left\{ \begin{array}{l} \textit{global} \\ \textit{local} \end{array} \right.$

	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
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$\in (\{a, b\} \times \mathbb{N}\{x_1, x_2, \dots\})^*$

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LTL + $\left\{ \begin{array}{l} \textit{global} \\ \textit{local} \end{array} \right.$ $x_1 \sim \diamond x_3$

	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
x_1	2	4	7	6	9	3	9	7	6	2	2	9
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\vdots												

$\in (\{a, b\} \times \mathbb{N}\{x_1, x_2, \dots\})^*$

*data
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LOGIC OF REPEATING VALUES

LTL + $\left\{ \begin{array}{l} \textit{global} \\ \textit{local} \end{array} \right.$ $x_1 \sim \diamond x_3$ $x_1 \sim \langle \phi? \rangle x_3$

	ϕ											
	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
x_1	2	4	7	6	9	3	9	7	6	2	2	9
x_2	5	2	7	5	1	2	2	8	5	2	8	9
x_3	2	3	5	6	9	8	9	7	3	1	2	2
\vdots												

$\in (\{a, b\} \times \mathbb{N}\{x_1, x_2, \dots\})^*$

*data
word*

LOGIC OF REPEATING VALUES

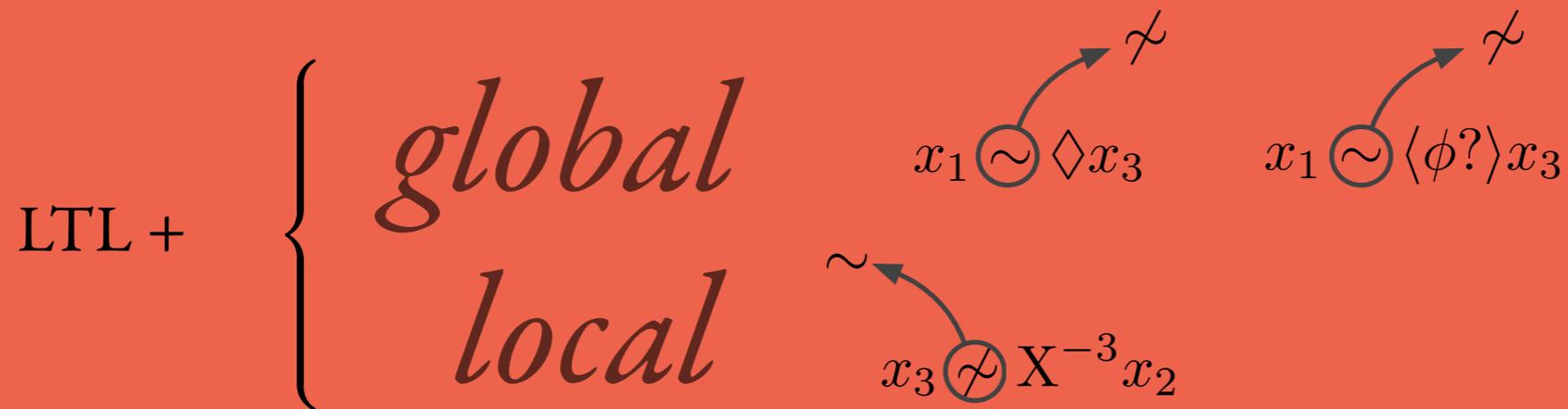
$$\text{LTL} + \begin{cases} \textit{global} & x_1 \sim \diamond x_3 & x_1 \sim \langle \phi? \rangle x_3 \\ \textit{local} & x_3 \not\sim X^{-3} x_2 \end{cases}$$

	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
<i>x</i> ₁	2	4	7	6	9	3	9	7	6	2	2	9
<i>x</i> ₂	5	2	7	5	1	2	2	8	5	2	8	9
<i>x</i> ₃	2	3	5	6	9	8	9	7	3	1	2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

∈ ({a, b} × ℕ_{x₁, x₂, ...})^{*}

*data
word*

LOGIC OF REPEATING VALUES

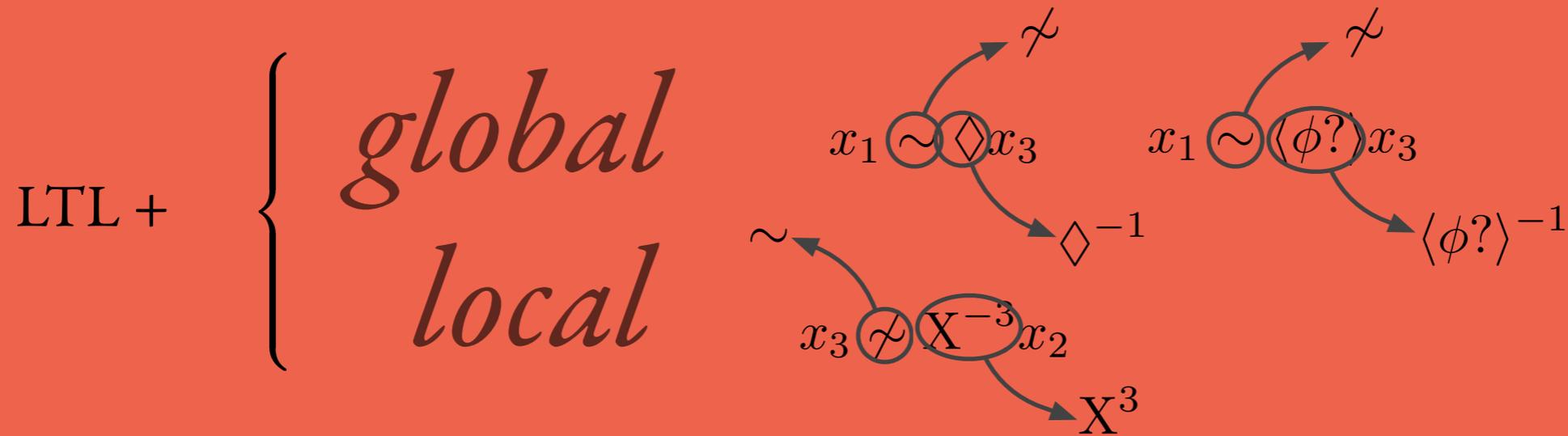


	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
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⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

∈ ({*a*, *b*} × ℕ_{*x*₁, *x*₂, ...})*

data
word

LOGIC OF REPEATING VALUES



	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>b</i>
x_1	2	4	7	6	9	3	9	7	6	2	2	9
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\vdots												

$\in (\{a, b\} \times \mathbb{N}\{x_1, x_2, \dots\})^*$

*data
word*

VASS

Vector
Addition
Systems with
States

finite state automaton A with n counters containing natural numbers

transitions can $\left\{ \begin{array}{l} \text{increment} \\ \text{decrement (if } > 0) \end{array} \right\}$ a counter

run starts with $\left\{ \begin{array}{l} \text{initial state} \\ \text{all counters in zero.} \end{array} \right.$

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control-state reachability problem

whether A can reach a given state

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control-state reachability problem

whether A can reach a given state

reachability problem

whether A can reach a given state and counter values

results

results

having or not $\langle \phi? \rangle$ is the same

results

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with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

[Demri, D'Souza & Gascon]

results

having or not $\langle \phi? \rangle$ is the same

with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

[Demri, D'Souza & Gascon]

without \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

results

having or not $\langle \phi? \rangle$ is the same

with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

[Demri, D'Souza & Gascon]

without \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ control state reach. of 2^n -VASS

results

having or not $\langle \phi? \rangle$ is the same

with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS [Demri, D'Souza & Gascon]

without \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ control state reach. of 2^n -VASS

2ExpSpace -complete

results

having or not $\langle \phi? \rangle$ is the same

with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

[Demri, D'Souza & Gascon]

without \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ control state reach. of VASS

SAT $\xleftrightarrow{\text{PTime}}$ control state reach. of 2^n -VASS

2ExpSpace -complete

the power of $\langle \phi? \rangle$:

if we bound the number of variables: $\left\{ \begin{array}{l} \text{SAT without } \langle \phi? \rangle : \text{PSpace-complete} \\ \text{SAT with } \langle \phi? \rangle \sim \text{reachability of VASS} \end{array} \right.$

results

having or not $\langle \phi? \rangle$ is the same

with \diamond^{-1} :

SAT $\xleftrightarrow{\text{ExpTime}}$ reachability of VASS

[Demri, D'Souza & Gascon]

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f -VASS

VASS:

Counters $C_0 \dots C_{n-1}$

Transition from q to q' :

- 1 choose $s \in \{0, \dots, n-1\}$
- 2 $C_s := C_s \pm 1$, or nop
- 3 go to q'

f -VASS

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configuration:

$q,$

		∞
		\vdots
	C_0	$\rightarrow 6$
	C_3	$\rightarrow 5$
		4
		3
C_4	C_1	$\rightarrow 2$
		1
	C_2	$\rightarrow 0$

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use a counter s

configuration:

Transition from q to q' :

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f -VASS

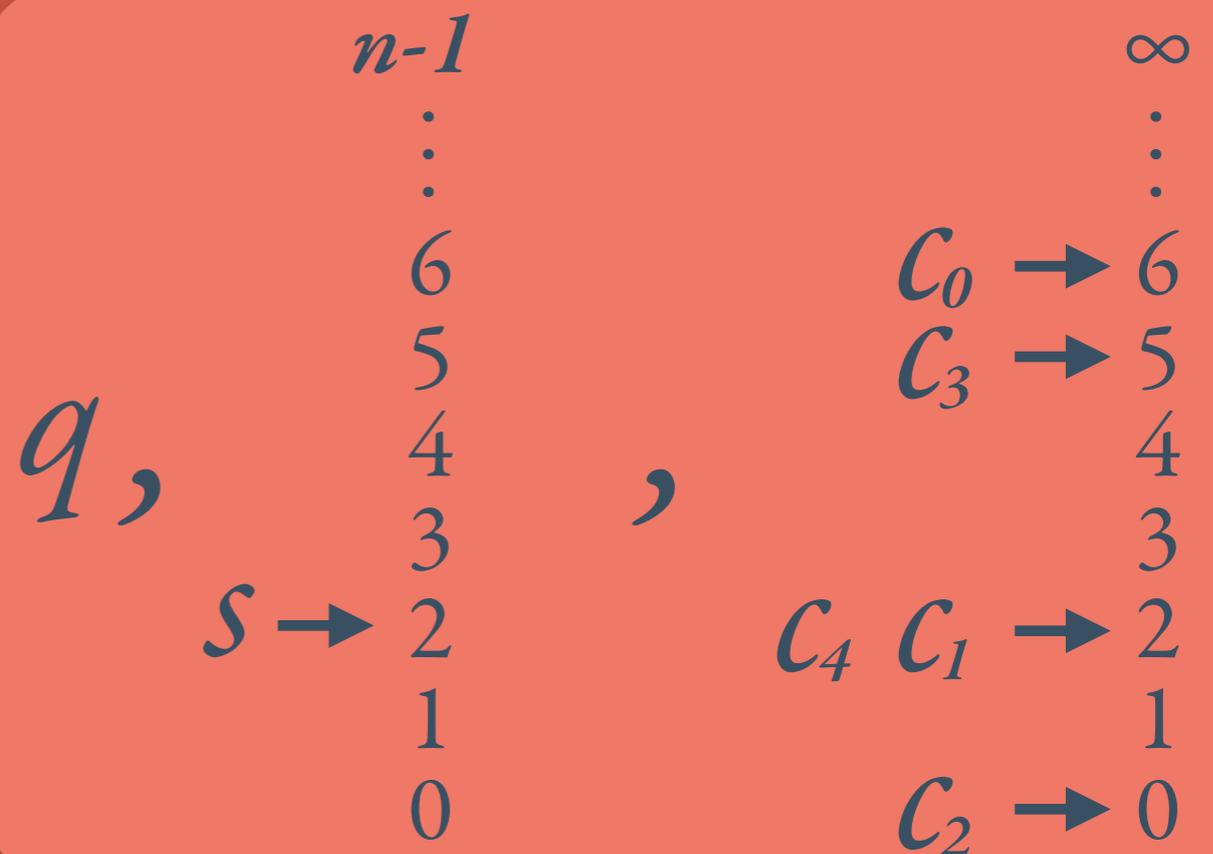
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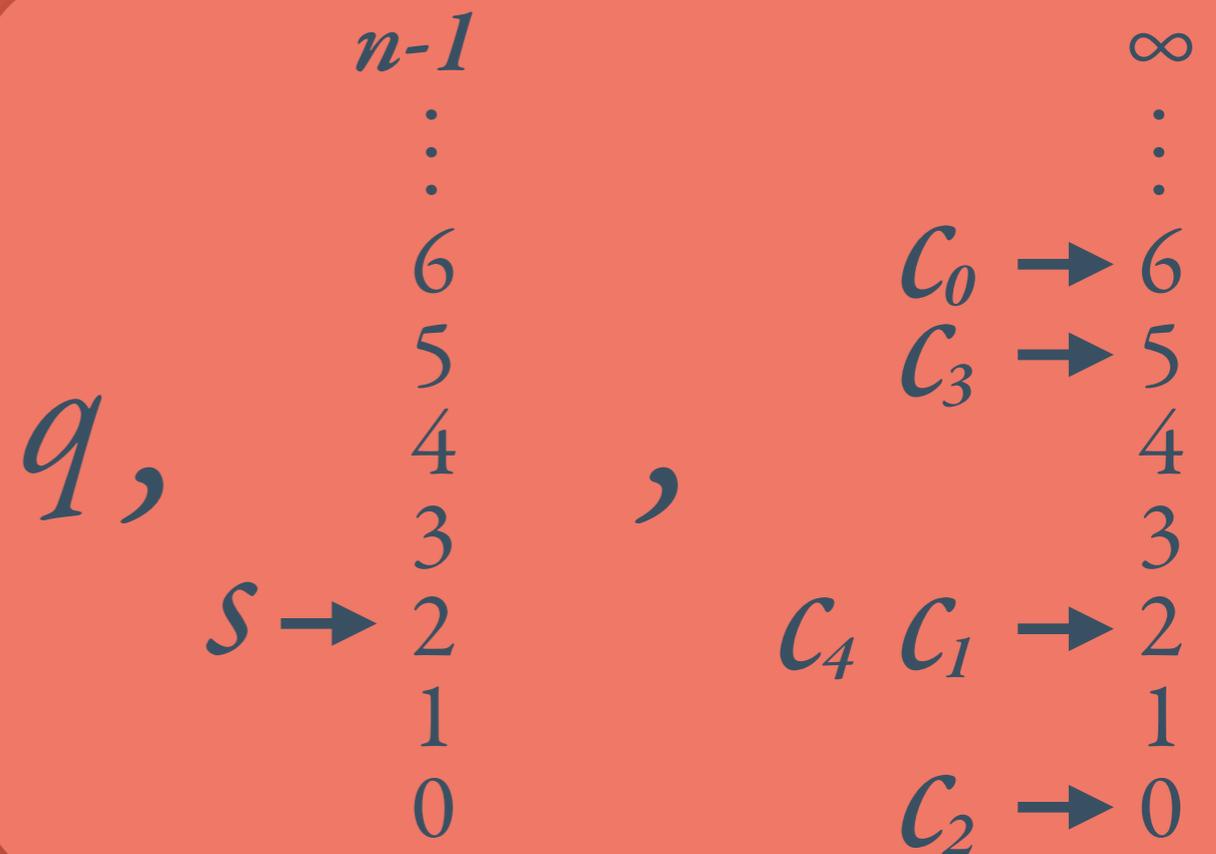
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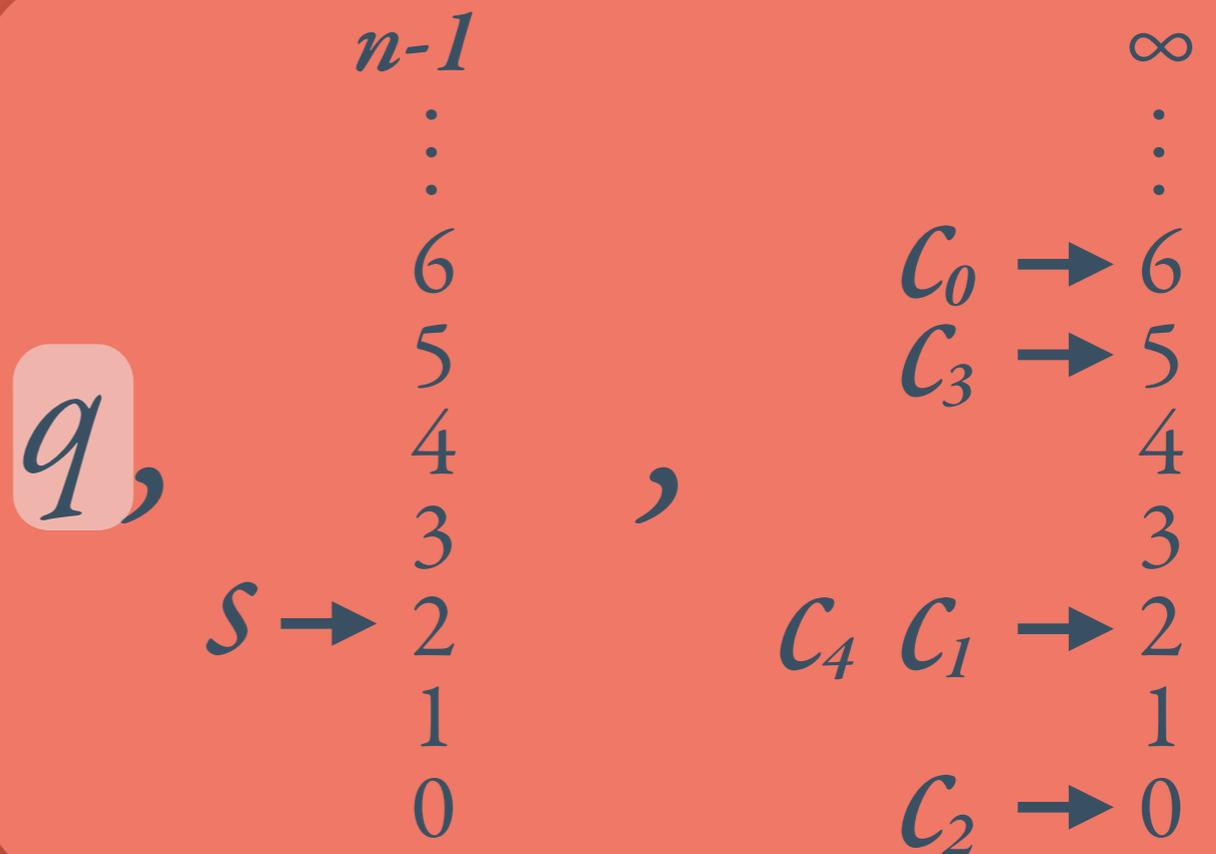
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$(q, s := s + 1, C_s := C_s - 1, q')$

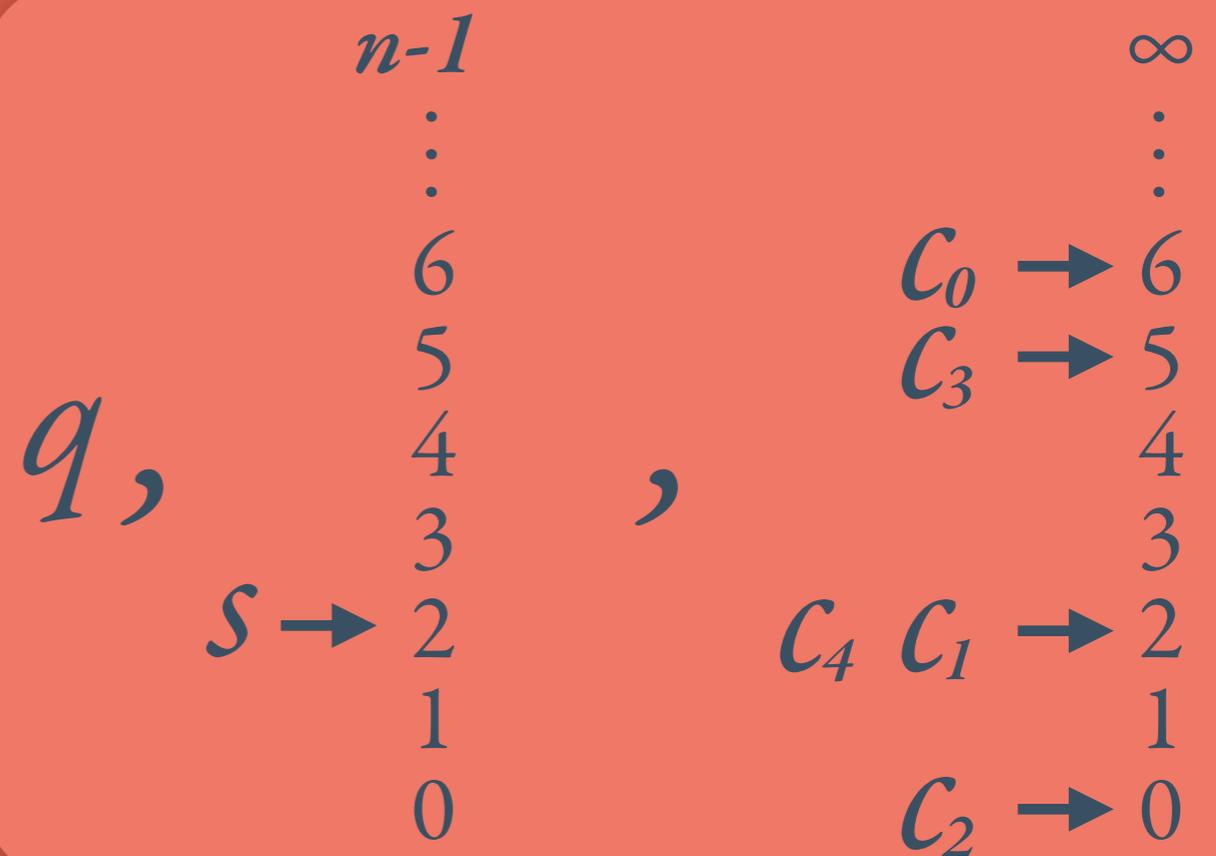
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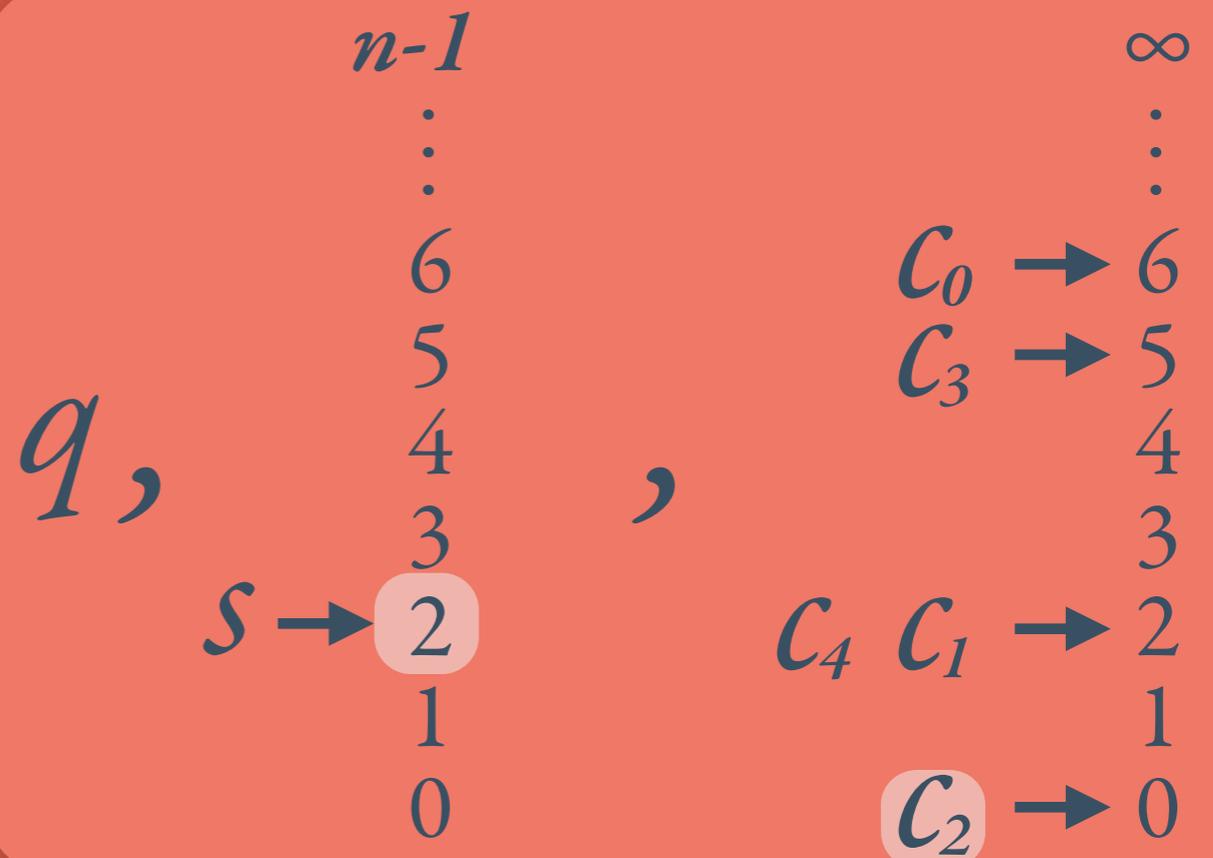
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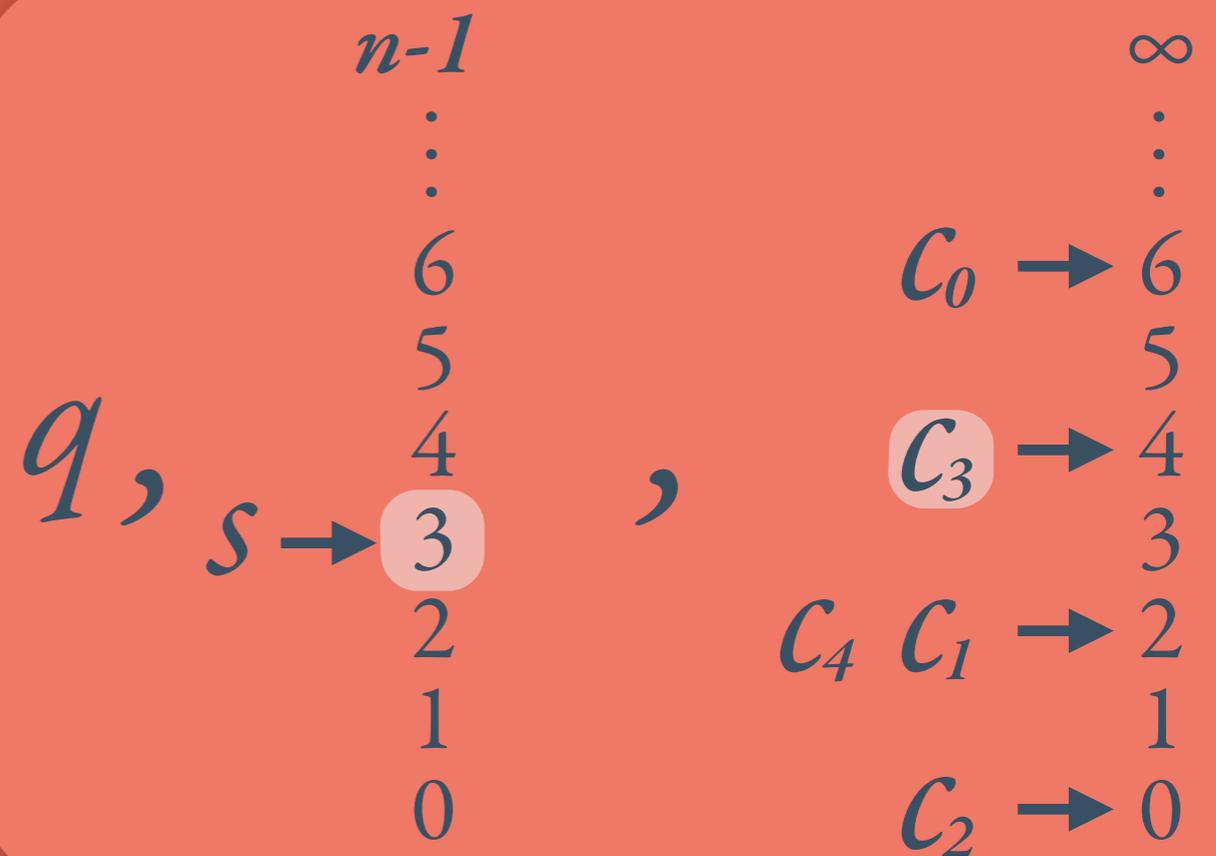
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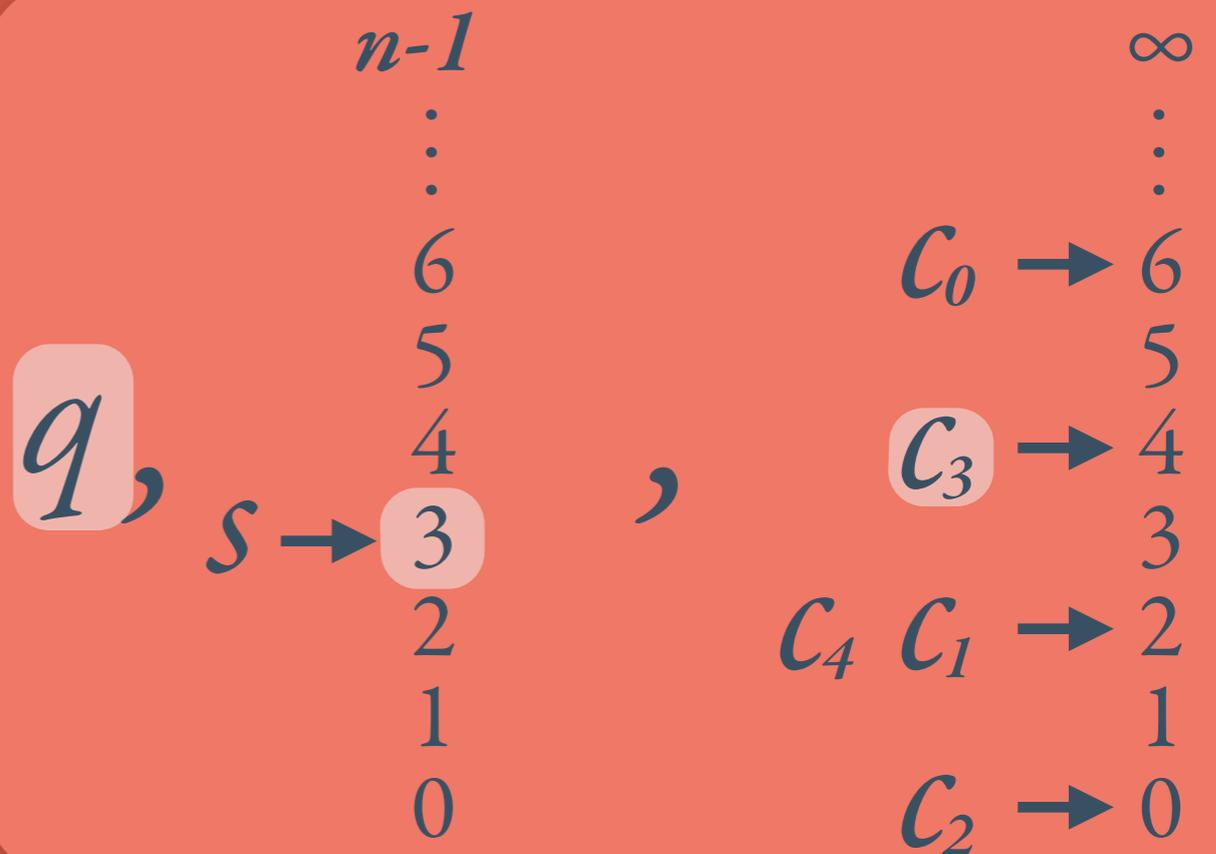
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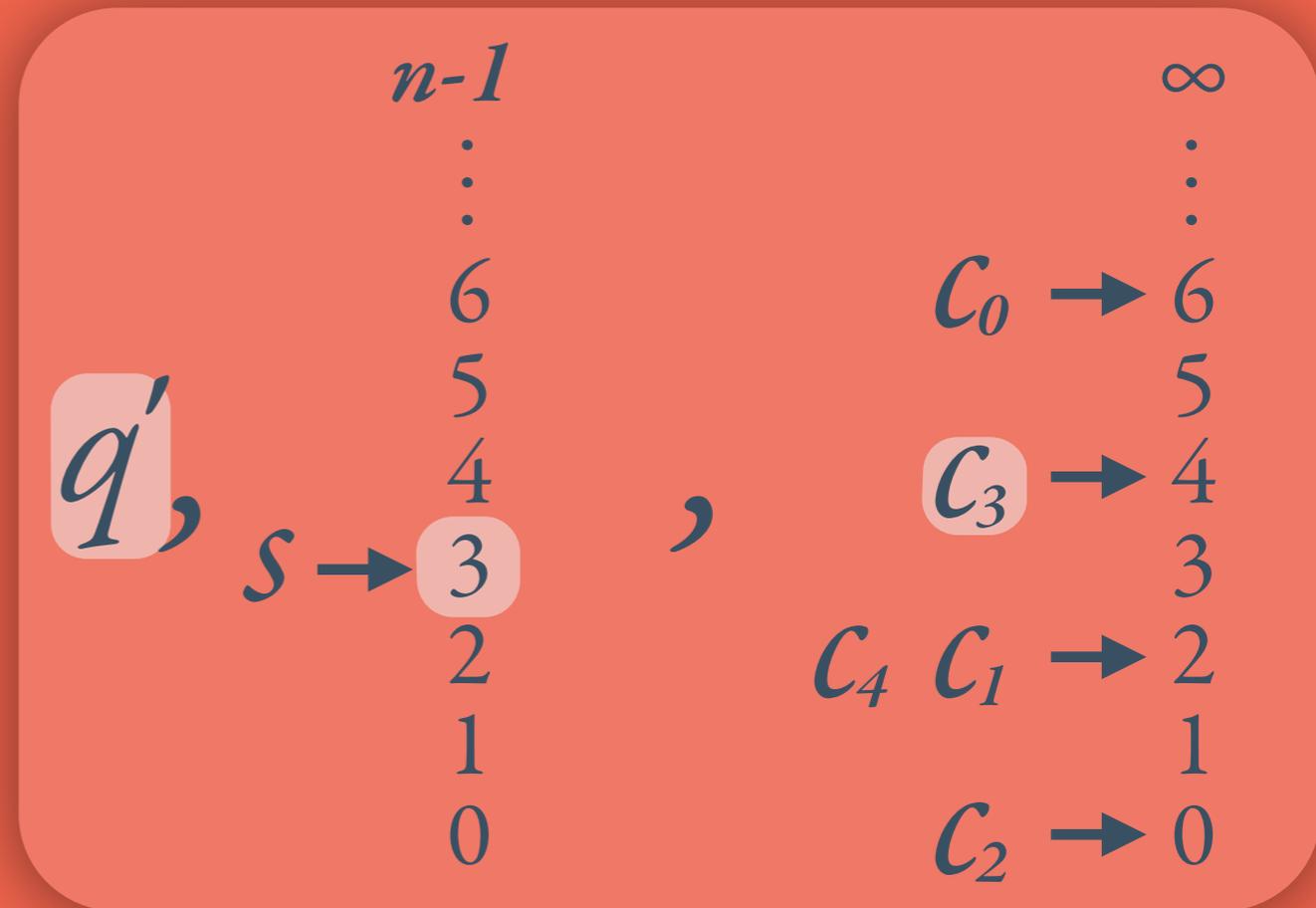
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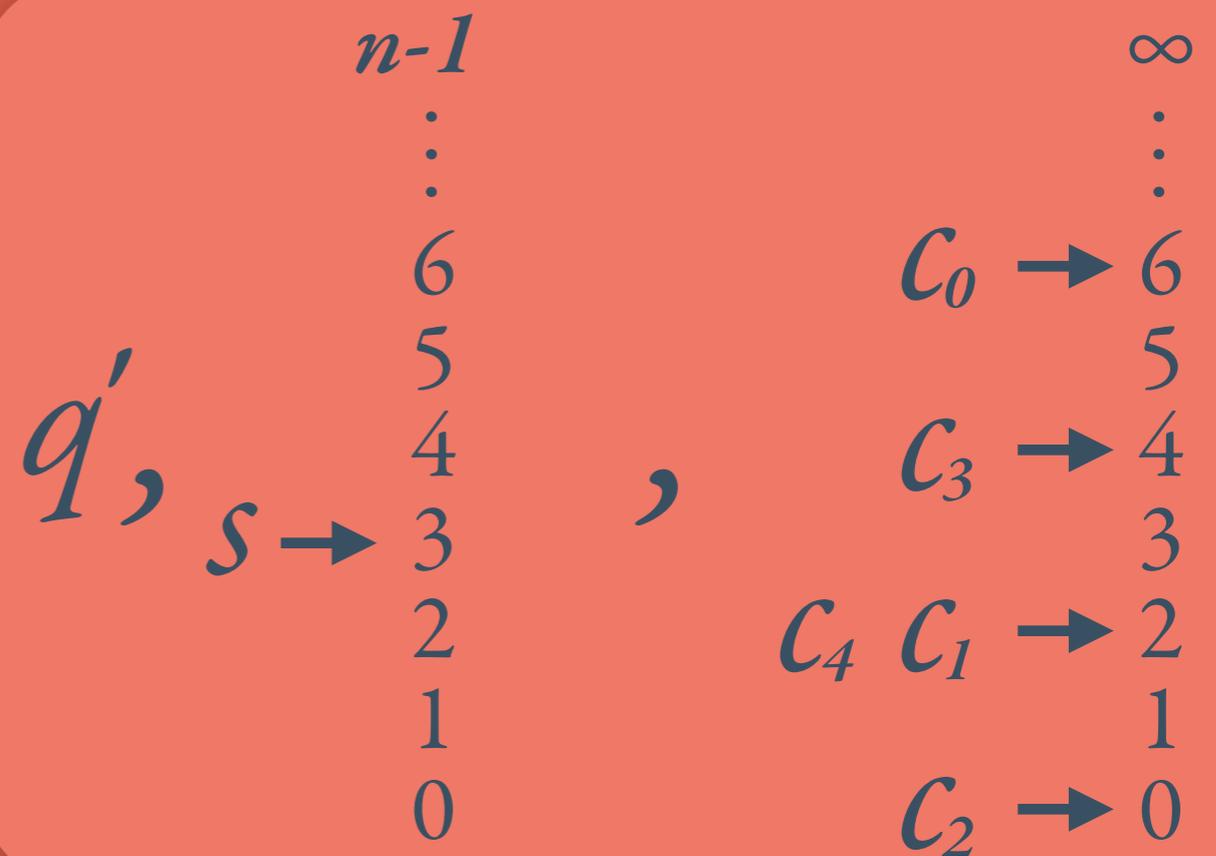
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Input: n in unary, transitions, initial/final states



$(q, s := s + 1, C_s := C_s - 1, q')$

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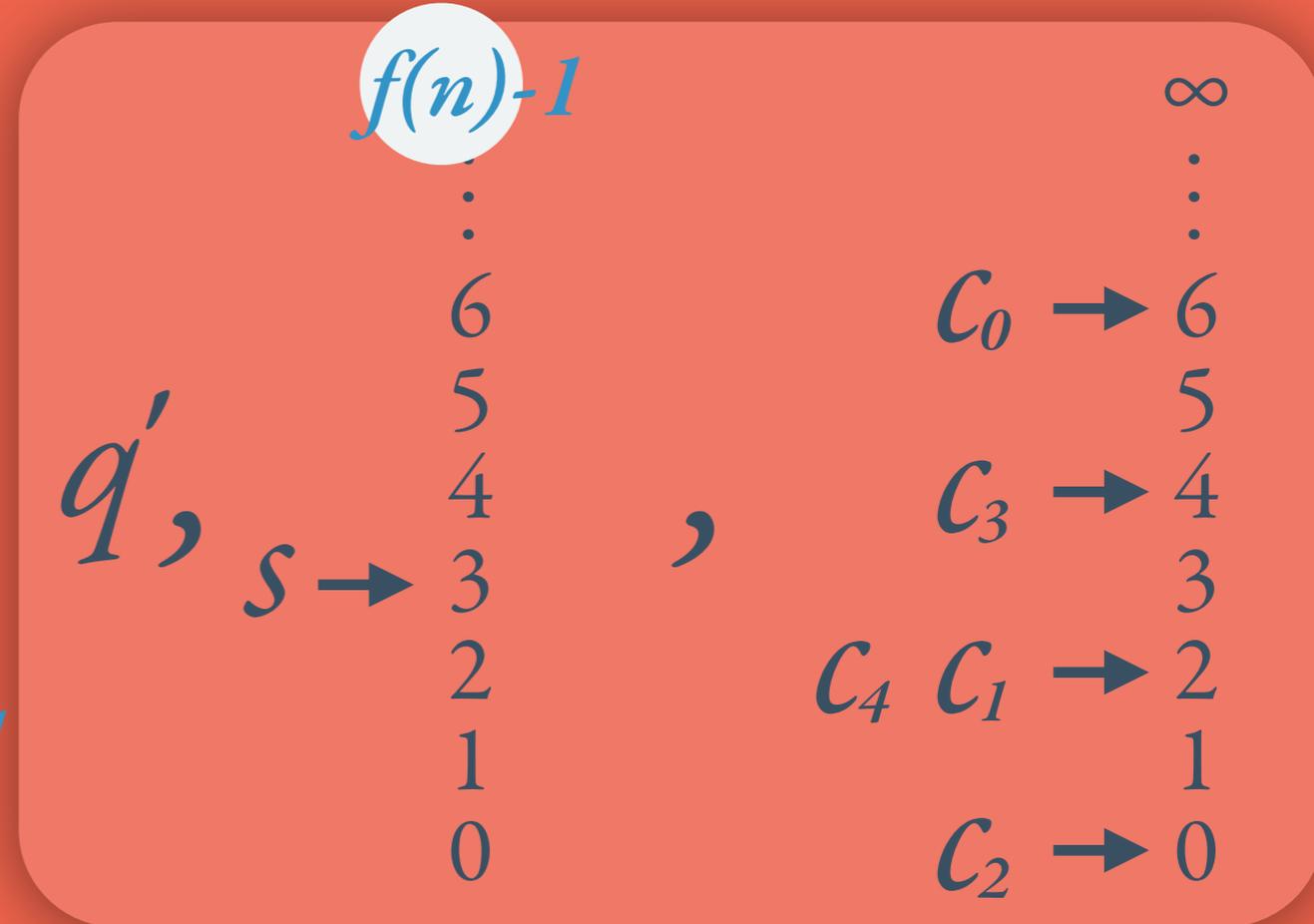
VASS:

Counters $C_0 \dots C_{n-1}$

Transition from q to q' :

- 1 $s := s \pm 1, s = 0$, or $s \stackrel{?}{=} f(n) - 1$
- 2 $C_s := C_s \pm 1$, or nop
- 3 go to q'

Input: n in unary, transitions, initial/final states



$(q, s := s + 1, C_s := C_s - 1, q')$

f -VASS : VASS with $f(n)$ counters

results

[Lipton, Rackoff]

if $f(n)=n$: f -VASS = VASS / c.s. reach.: ExpSpace-c

if $f(n)=2^n$: csreach(f -VASS) : 2ExpSpace-c

if $f(n)= \left. 2^{\dots 2^n} \right\}^k$: csreach(f -VASS) : $(k+1)$ ExpSpace-c

f -VASS : VASS with $f(n)$ counters

Open Problem

SAT on data trees \longleftrightarrow ? \longleftrightarrow reachability for branching VASS