

# Parameterized complexity of modification problems on edge-coloured and signed graph homomorphisms

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# Overview

- 1 Modification Problems
- 2 *H*-Colouring
- 3 Vertex Deletion and Edge Deletion
- 4 Switching *H*-colouring

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## Definition

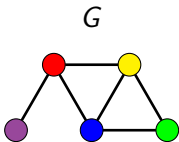
A **homomorphism**  $\varphi$  of  $G$  to  $H$  is a mapping from the vertices of  $G$  to the vertices of  $H$  such that for every edge  $xy \in E(G)$ ,  $\varphi(x)\varphi(y) \in E(H)$ .

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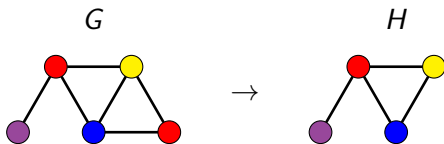
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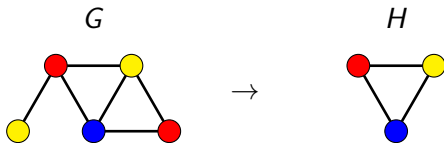
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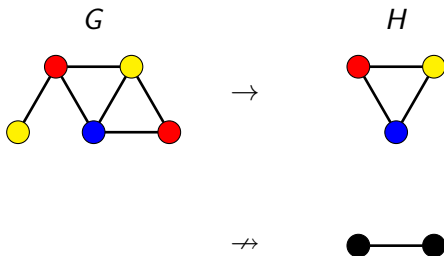




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### Theorem (Hell and Nešetřil, 1990)

*H*-COLOURING is polynomial if  $H$  has a loop or is bipartite. It is NP-Complete otherwise.

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## Definition (core)

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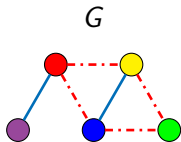
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*A homomorphism  $\varphi$  of a  $t$ -edge-coloured graph  $G$  to an edge-coloured graph  $H$  is a mapping from the vertices of  $G$  to the vertices of  $H$  such that for every edge  $xy \in E(G)$ ,  $\varphi(x)\varphi(y)$  is an edge of  $H$  with the same color as  $xy$ .*

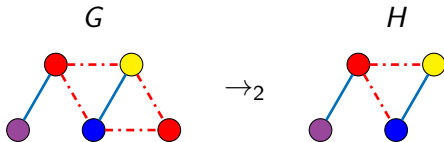
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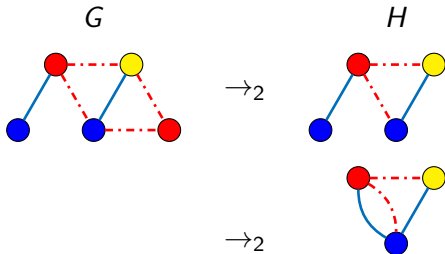
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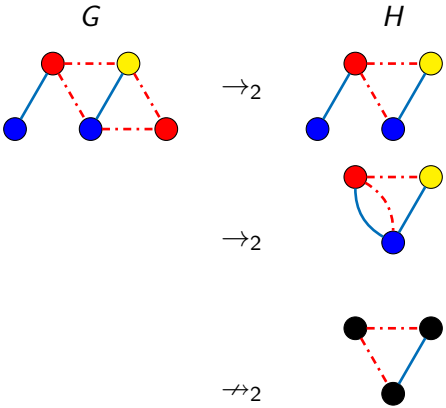
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Theorem (R. C. Brewster, 1994)

$H$ -COLOURING $_t$  is polynomial when  $H$  is a path.

Theorem (R. C. Brewster, R. Dedić, F. Huard and J. Queen, 2005)

$H$ -COLOURING $_t$  is polynomial in the following cases:

- $H$  has order 2,
- $H$  has order 3, is loop-free and contains no monochromatic triangle.

Theorem (R. C. Brewster and P. Hell, 2009)

$H$ -COLOURING $_t$  is polynomial for some cycles.

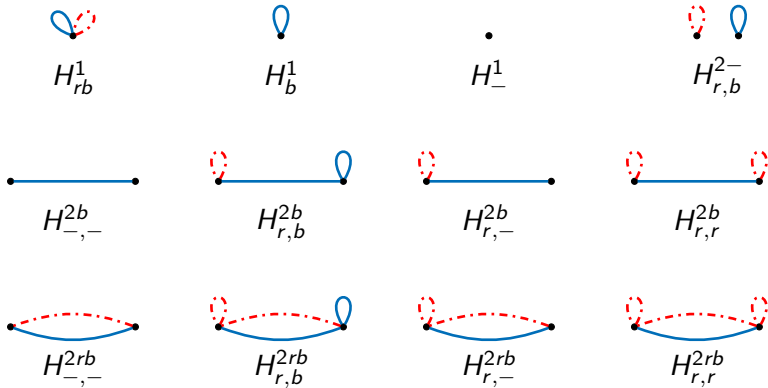


Figure: The twelve 2-edge-coloured graphs of order at most 2.

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## Theorem

VERTEX DELETION  $H$ -COLOURING <sub>$t$</sub>  for a  $t$ -edge-coloured core  $H$  is polynomial if  $H$  is a vertex having all  $t$  coloured loops and NP-complete otherwise.

# Vertex deletion: P vs NP-complete



P



NP-complete



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# Edge deletion: P vs NP-complete

## Theorem

*Let  $H$  be an edge-coloured core of order at most 2. If each colour of  $H$  induces a set of loops or contains all three possible edges, then  $\text{EDGE DELETION } H\text{-COLOURING}_t$  lies in  $P$ , otherwise it is NP-complete.*

Alternate formulation, if  $H$  contains one of these two patterns, then it is NP-complete:



# Edge deletion: P vs NP-complete



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P



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# Vertex deletion: P vs FPT vs W-hard



P



FPT



FPT



FPT



FPT



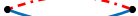
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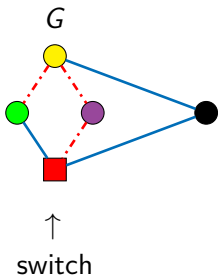


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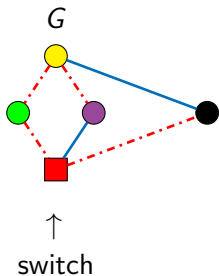
## Definition

A signed graph  $G$  is a graph where each edge can be either positive or negative. Moreover we can switch at any vertex  $v$ . Switching at  $v$  consists in inverting the signs of all edges incident with  $v$ .



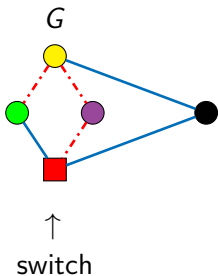
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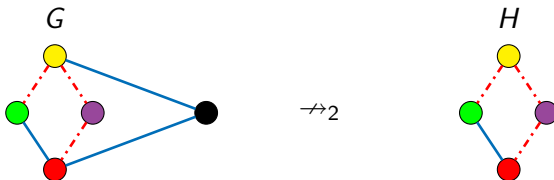
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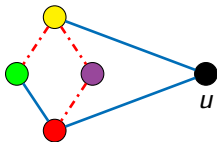
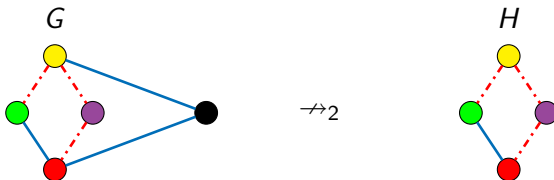
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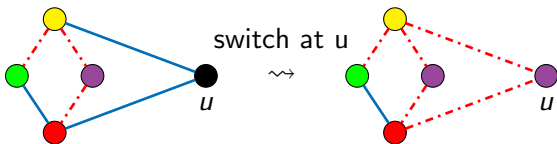
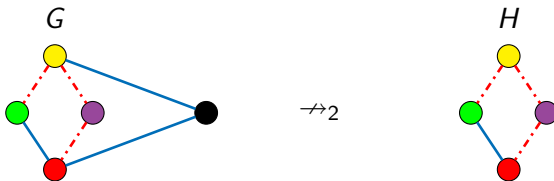
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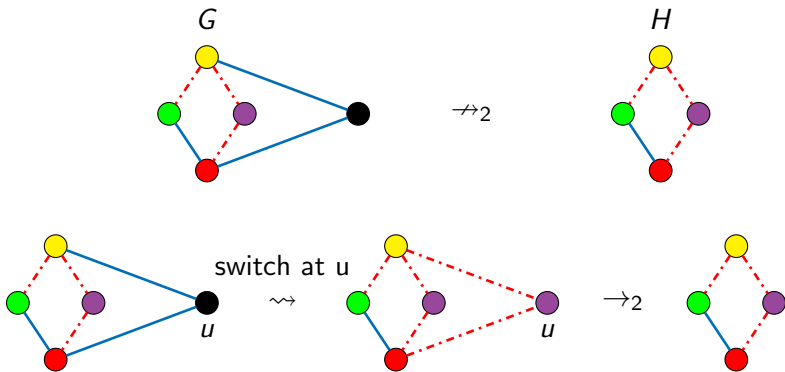
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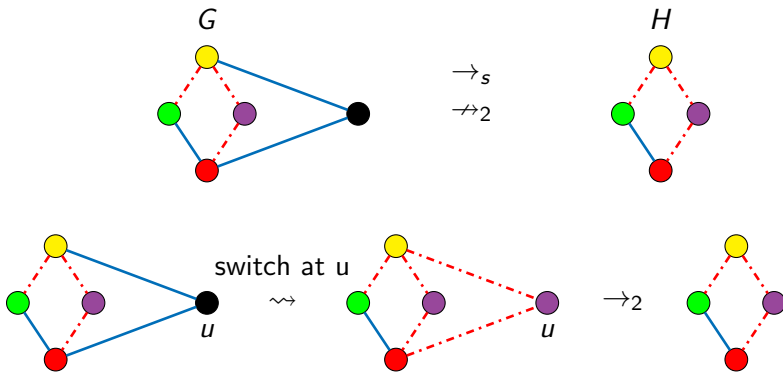
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We consider two types of homomorphisms:

- $G \rightarrow_s H$ : We can switch on  $G$  an **arbitrary number of times** to get  $G'$  and  $G' \rightarrow_2 H$ ,
- $G \rightarrow_{\leq k} H$ : We can switch on  $G$  **at most  $k$  times** to get  $G'$  and  $G' \rightarrow_2 H$ .

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### Problem (SIGNED *H*-COLOURING)

*Input*:  $G$ .

*Question*: Does  $G \rightarrow_s H$  ?

### Problem (SWITCHING *H*-COLOURING)

*Input* :  $G$  and  $k$ .

*Parameter*:  $k$ .

*Question*: Does  $G \rightarrow_s^{\leq k} H$  ?

# Switching: P vs NP-complete



P



P



P



P



P



?



?



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P



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# Switching: P vs NP-complete

Theorem (Brewster, Foucaud, Hell, Naserasr, 2017 and Brewster, Siggers, 2018)

*Let  $H$  be a signed graph. SIGNED  $H$ -COLOURING is polynomial if the switching core of  $H$  has at most two edges, and NP-complete otherwise.*

# Switching: P vs NP-complete



P



P



P



P



P



NP-Complete



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P



NP-Complete



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NP-Complete



NP-Complete



NP-Complete

# Switching: P vs FPT vs W-hard



P



P



P



P



P



FPT



FPT



P



P



W[1]-hard



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Thank you for your attention!

Theorem (R. C. Brewster, R. Dedić, F. Huard and J. Queen)

*The  $H$ -COLOURING<sub>t</sub> problem is polynomial when  $H$  has order 2 by reduction to 2-SAT.*

$E_i(H)$	Clause
$\emptyset$	$\perp$
$\{00\}$	$(\overline{x_u})(\overline{x_v})$
$\{01\}$	$(x_u + x_v)(\overline{x_u} + \overline{x_v})$
$\{11\}$	$(x_u)(x_v)$
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$\{00, 01, 11\}$	$\top$

### Problem (VARIABLE DELETION ALMOST 2-SAT)

*Input: A 2-CNF formula  $F$ , an integer  $k$ .*

*Parameter:  $k$ .*

*Question: Is there a set of  $k$  variables that can be deleted from  $F$  (together with the clauses containing them) so that the resulting formula is satisfiable?*

### Problem (CLAUSE DELETION ALMOST 2-SAT)

*Input: A 2-CNF formula  $F$ , an integer  $k$ .*

*Parameter:  $k$ .*

*Question: Is there a set of  $k$  clauses that can be deleted from  $F$  so that the resulting formula is satisfiable?*

CLAUSE DELETION ALMOST 2-SAT and VARIABLE DELETION ALMOST 2-SAT were proved to be FPT (I. Razgon and B. O'Sullivan for the former and M. Cygan *et al.* for the later).

$E_i(H)$	Clause before	Clause after modification
$\emptyset$	$\perp$	$(x_u + x_v)(\bar{x}_u + x_v)(x_u + \bar{x}_v)(\bar{x}_u + \bar{x}_v)$
$\{00\}$	$(\bar{x}_u)(\bar{x}_v)$	$(\bar{x}_u + x_v)(\bar{x}_u + \bar{x}_v)(\bar{x}_v + x_u)$
$\{01\}$	$(x_u + x_v)(\bar{x}_u + \bar{x}_v)$	$(x_u + x_v)(\bar{x}_u + \bar{x}_v)$
$\{11\}$	$(x_u)(x_v)$	$(x_u + x_v)(x_u + \bar{x}_v)(x_v + \bar{x}_u)$
$\{00, 01\}$	$(\bar{x}_u + \bar{x}_v)$	$(\bar{x}_u + \bar{x}_v)$
$\{01, 11\}$	$(x_u + x_v)$	$(x_u + x_v)$
$\{00, 11\}$	$(x_u + \bar{x}_v)(\bar{x}_u + x_v)$	$(x_u + \bar{x}_v)(\bar{x}_u + x_v)$
$\{00, 01, 11\}$	$\top$	$\top$

## Theorem

*If  $H$  has order 2 then there exists an FPT algorithm for VERTEX DELETION  $H$ -COLOURING $_t$ .*

## Problem (GROUP DELETION ALMOST 2-SAT)

*Input: A 2-CNF formula  $F$ , an integer  $k$ , and a partition of the clauses of  $F$  into groups such that each group has a variable appears in all of its clauses.*

*Parameter:  $k$ .*

*Question: Is there a set of  $k$  groups of clauses that can be deleted from  $F$  so that the resulting formula is satisfiable?*

## Theorem

*GROUP DELETION ALMOST 2-SAT is solvable in FPT time.*

## Corollary

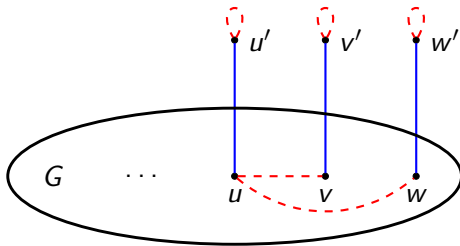
*If  $H$  has order 2 then there exists an FPT algorithm for EDGE DELETION  $H$ -COLOURING<sub>t</sub>.*



For  $H = \text{---} \overset{\circ}{\text{---}} \text{---} \text{---} \overset{\circ}{\text{---}} \text{---}$ ,  $G \rightarrow_2 H$  iff  $G$  has no cycles with an odd number of positive edges (Bawar, Brewster and Marcotte).

$\Rightarrow$  The parity of the number of positive edges of a cycle does not change by switching.

For  $H = \text{---}$ , reduction from VERTEX COVER.



For  $H = \text{---} \circ \text{---} \circ \text{---}$ ,  $G \rightarrow_2 H$  iff  $\text{---} \circ \text{---} \circ \text{---} \circ \text{---} \rightarrow_2 G$  (Bawar, Brewster and Marcotte).

$\Rightarrow$  We have an FPT branching algorithm.

For  $H = \text{---}$ ,  $G \rightarrow_2 H$  iff  $\overset{\text{odd}}{\text{---}} \rightarrow_2 G$

(Bawar, Brewster and Marcotte).

The only way to remove such a path is to switch at one of  $v_0, v_1, v_{2p}, v_{2p+1}$ .

Thus we can have an FPT branching algorithm.