

Dicots, and a taxonomic ranking for misère games

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December 19, 2013

Abstract

We study combinatorial games in misère version. In a general context, little can be said about misère games. For this reason, several universes were earlier considered for their study, which can be ranked according to their inclusion ordering. We study in particular a special universe of games called dicots, which turns out to be the known universe of lowest rank modulo which equivalence in misère version implies equivalence in normal version. We also prove that modulo the dicot universe, we can define a canonical form as the reduced form of a game that can be obtained by getting rid of dominated options and most reversible options. We finally count the number of dicot equivalence classes of dicot games born by day 3.

1 Introduction

In this paper, we study combinatorial games in misère version, and in particular a special family of games called dicots. We first recall basic definitions, following [1, 3, 4].

A combinatorial game is a finite two-player game with no chance and perfect information. The players, called Left and Right, alternate moves until one player has no available move. Under the normal convention, the last player to move wins the game while under the misère convention, that player loses the game. In this paper, we are mostly interested in games under the misère convention.

A game can be defined recursively by its sets of options $G = \{G^L|G^R\}$, where G^L is the set of games reachable in one move by Left (called Left options), and G^R the set of games reachable in one move by Right (called Right options). We note G^L for the typical option of G^L , and G^R for the typical option of G^R . The zero game $0 = \{\cdot|\cdot\}$, is the game with no options. The birthday of a game is defined recursively as one plus the maximum birthday of its options, with 0 being the only game with birthday 0. We say a game G is born *on day* n if its birthday is n and that it is born *by day* n if its birthday is at most n . The games born on day 1 are $\{0|\cdot\} = 1$, $\{\cdot|0\} = \bar{1}$ and $\{0|0\} = *$.

A game can also be depicted by its game tree, where the game trees of its options are linked to the root by downward edges, left-slanted for Left options and right-slanted for Right options. For instance, the game trees of games born by day 1 are depicted on Figure 1.

Given two games $G = \{G^L|G^R\}$ and $H = \{H^L|H^R\}$, the (disjunctive) *sum* of G and H is recursively defined as $G + H = \{G^L + H, G + H^L|G^R + H, G + H^R\}$ (where $G^L + H$ is the set of sums of H with an element of G^L), i.e. the game where each player chooses on their turn which

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