

Paired versus Double Domination in $K_{1,r}$ -Free Graphs

¹Paul Dorbec, ²Bert Hartnell, and ³Michael A. Henning

¹Université de Bordeaux – CNRS
LaBRI, 351 cours de la Libération
33405 Talence Cedex, France
Email: paul.dorbec@labri.fr

²Department of Mathematics and Computing Science
Saint Mary's University
Halifax, Nova Scotia, B3H 3C3, Canada
Email: bert.hartnell@smu.ca

³Department of Mathematics
University of Johannesburg
Auckland Park 2006, South Africa
Email: mahenning@uj.ac.za

Abstract

A vertex in G is said to dominate itself and its neighbors. A subset S of vertices is a dominating set if S dominates every vertex of G . A paired-dominating set is a dominating set whose induced subgraph contains a perfect matching. The paired-domination number of a graph G , denoted by $\gamma_{\text{pr}}(G)$, is the minimum cardinality of a paired-dominating set in G . A subset $S \subseteq V(G)$ is a double dominating set of G if S dominates every vertex of G at least twice. The minimum cardinality of a double dominating set of G is the double domination number $\gamma_{\times 2}(G)$. A claw-free graph is a graph that does not contain $K_{1,3}$ as an induced subgraph. Chellali and Haynes [Utilitas Math. 67 (2005), 161–171] showed that for every claw-free graph G , we have $\gamma_{\text{pr}}(G) \leq \gamma_{\times 2}(G)$. In this paper we extend this result by showing that for $r \geq 2$, if G is a connected graph that does not contain $K_{1,r}$ as an induced subgraph, then $\gamma_{\text{pr}}(G) \leq \left(\frac{2r^2-6r+6}{r(r-1)}\right) \gamma_{\times 2}(G)$.

Keywords: Paired-domination; double domination; claw-free; $K_{1,r}$ -free.

AMS subject classification: 05C69