

Power domination in product graphs*

Paul Dorbec[†] Michel Mollard[†] Sandi Klavžar[‡] Simon Špacapan[§]

Abstract

The power system monitoring problem asks for as few as possible measurement devices to be put in an electric power system. The problem has a graph theory model involving power dominating sets in graphs. The power domination number $\gamma_P(G)$ of G is the minimum cardinality of a power dominating set. Dorfling and Henning [?] determined the power domination number of the Cartesian product of paths. In this paper the power domination number is determined for all direct products of paths except for the odd component of the direct product of two odd paths. For instance, if n is even and C a connected component of $P_m \times P_n$, where m is odd or $m \geq n$, then $\gamma_P(C) = \lceil n/4 \rceil$. For the strong product we prove that $\gamma_P(P_n \boxtimes P_m) = \max\{\lceil n/3 \rceil, \lceil (n+m-2)/4 \rceil\}$, unless $3m - n - 6 \equiv 4 \pmod{8}$. The power domination number is also determined for an arbitrary lexicographic product.

Key words: domination, power domination, electric power monitoring, graph products, paths

2000 Mathematics Subject Classification: 05C69

*Supported in part by the Proteus project BI-FR/04-002, by the Ministry of Science of Slovenia under the grant P1-0297, by the ENS Lyon, and by the CNRS.

[†]ERTÉ Maths à Modeler, GéoD research group, Leibniz laboratory, 46 av. Félix Viallet, 38031 Grenoble CEDEX, France; e-mail: paul.dorbec@imag.fr (UJF); michel.mollard@imag.fr (CNRS)

[‡]Department of Mathematics and Computer Science, PeF, University of Maribor, Koroška cesta 160, 2000 Maribor, Slovenia, e-mail: sandi.klavzar@uni-mb.si. Also with the Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana.

[§]University of Maribor, FME, Smetanova 17, 2000 Maribor, Slovenia, e-mail: simon.spacapan@uni-mb.si. Also with the Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana.