

Upper Paired-Domination in Claw-Free Graphs

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Abstract

A set S of vertices in a graph G is a paired-dominating set of G if every vertex of G is adjacent to some vertex in S and if the subgraph induced by S contains a perfect matching. The maximum cardinality of a minimal paired-dominating set of G is the upper paired-domination number of G , denoted by $\Gamma_{\text{pr}}(G)$. We establish bounds on $\Gamma_{\text{pr}}(G)$ for connected claw-free graphs G in terms of the number n of vertices in G with given minimum degree δ . We show that $\Gamma_{\text{pr}}(G) \leq 4n/5$ if $\delta = 1$ and $n \geq 3$, $\Gamma_{\text{pr}}(G) \leq 3n/4$ if $\delta = 2$ and $n \geq 6$, and $\Gamma_{\text{pr}}(G) \leq 2n/3$ if $\delta \geq 3$. All these bounds are sharp. Further, if $n \geq 6$ the graphs G achieving the bound $\Gamma_{\text{pr}}(G) = 4n/5$ are characterized, while for $n \geq 9$ the graphs G with $\delta = 2$ achieving the bound $\Gamma_{\text{pr}}(G) = 3n/4$ are characterized.

Keywords: claw-free graphs, minimum degree, upper paired-domination

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