

Upper Total Domination versus Upper Paired-Domination

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Abstract

Let G be a graph with no isolated vertices. A set S of vertices in G is a total dominating set of G if every vertex of G is adjacent to some vertex in S , while a paired-dominating set of G is a dominating set of vertices whose induced subgraph has a perfect matching. The maximum cardinality of a minimal total dominating set and a minimal paired-dominating set of G is the upper total domination number and upper paired-domination number of G , respectively, denoted by $\Gamma_t(G)$ and $\Gamma_{pr}(G)$. If G does not contain $K_{1,3}$ as an induced subgraph, then G is said to be claw-free. In this paper, we investigate the relationship between the upper total domination and upper paired-domination numbers of a graph. We show that for every graph G with no isolated vertex $(\Gamma_{pr}(G) + 2)/2 \leq \Gamma_{pr}(G)$. We establish that although there is no general relationship between the upper total and upper paired-domination numbers of a graph, for the family of trees T on at least two vertices, $\Gamma_t(T) \leq \Gamma_{pr}(T)$.

Keywords: upper total domination, upper paired-domination

AMS subject classification: 05C69

*Research supported in part by the South African National Research Foundation and the University of KwaZulu-Natal.