

# On the upper total domination number of Cartesian products of graphs

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## Abstract

In this paper we continue the investigation of total domination in Cartesian products of graphs first studied in *Graphs Combin.* **21** (2005), 63–69. A set  $S$  of vertices in a graph  $G$  is a total dominating set of  $G$  if every vertex in  $G$  is adjacent to some vertex in  $S$ . The maximum cardinality of a minimal total dominating set of  $G$  is the upper total domination number of  $G$ , denoted by  $\Gamma_t(G)$ . We prove that the product of the upper total domination numbers of any graphs  $G$  and  $H$  without isolated vertices is at most twice the upper total domination number of their Cartesian product; that is,  $\Gamma_t(G)\Gamma_t(H) \leq 2\Gamma_t(G \square H)$ .

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