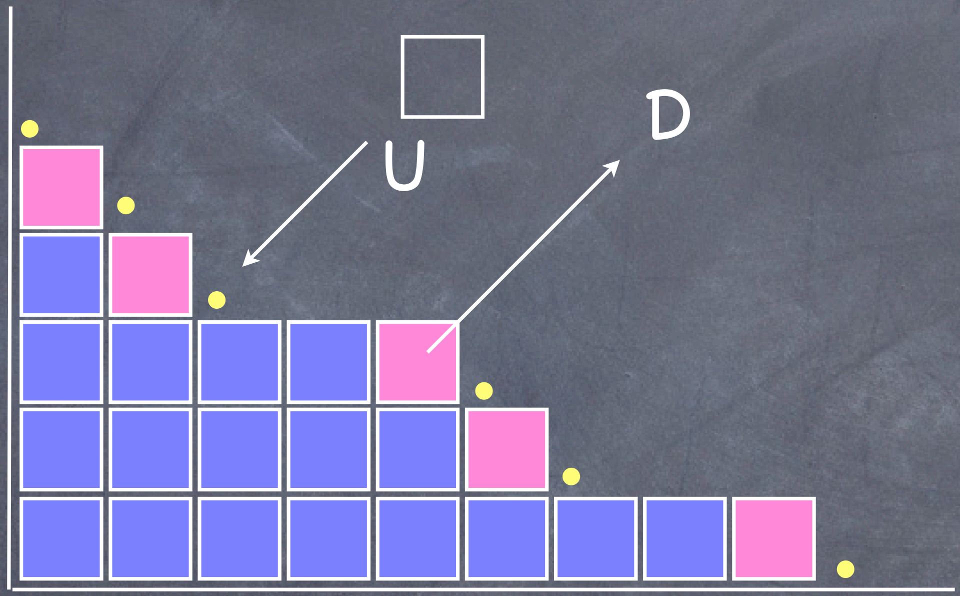
Une approche algébrique à RSK

(d'après S. Fomin et R. Stanley)

xavier viennot

GT Combinatoire, LaBRI, 3 Fév 2006

Opérateurs U et D



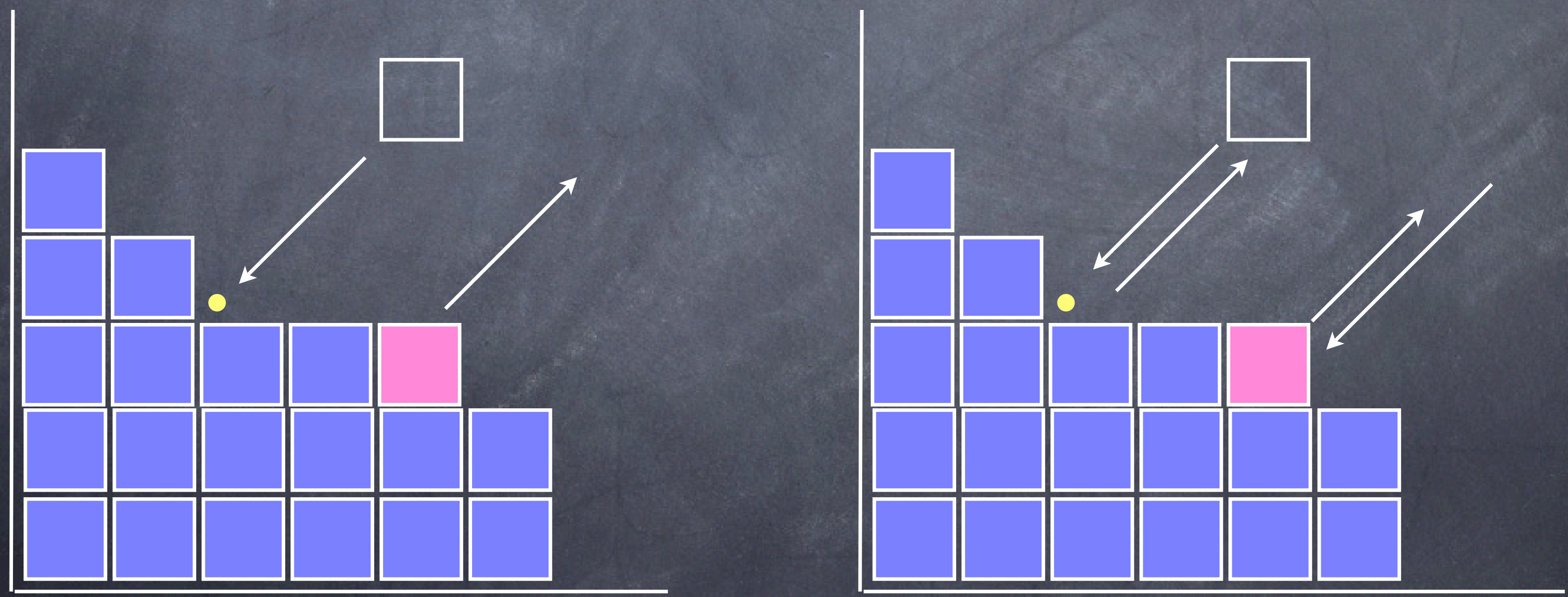
Treillis de Young

relation de commutation de Heisenberg

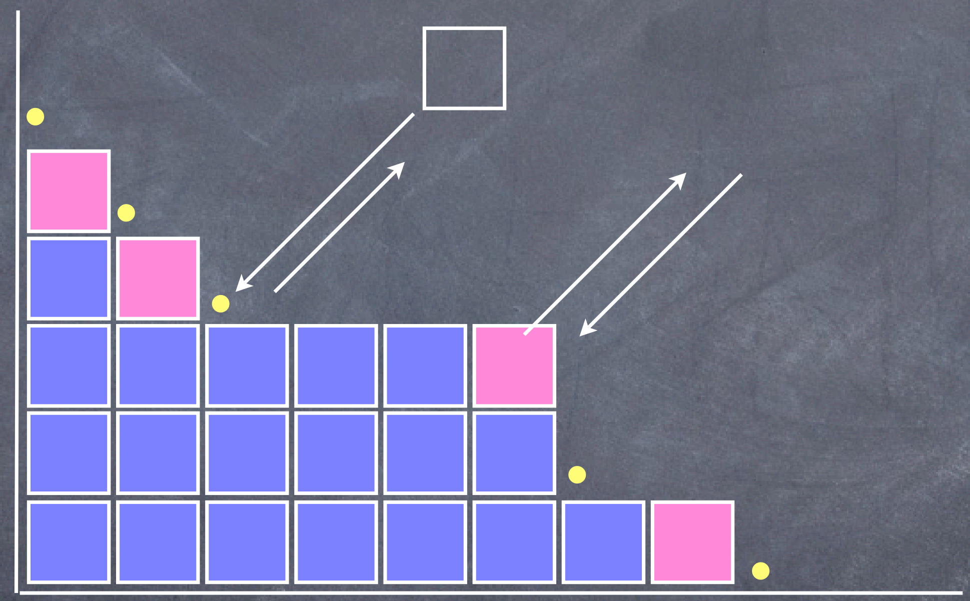
$$UD = DU + I$$

relation de commutation de Heisenberg

$$UD = DU + I$$



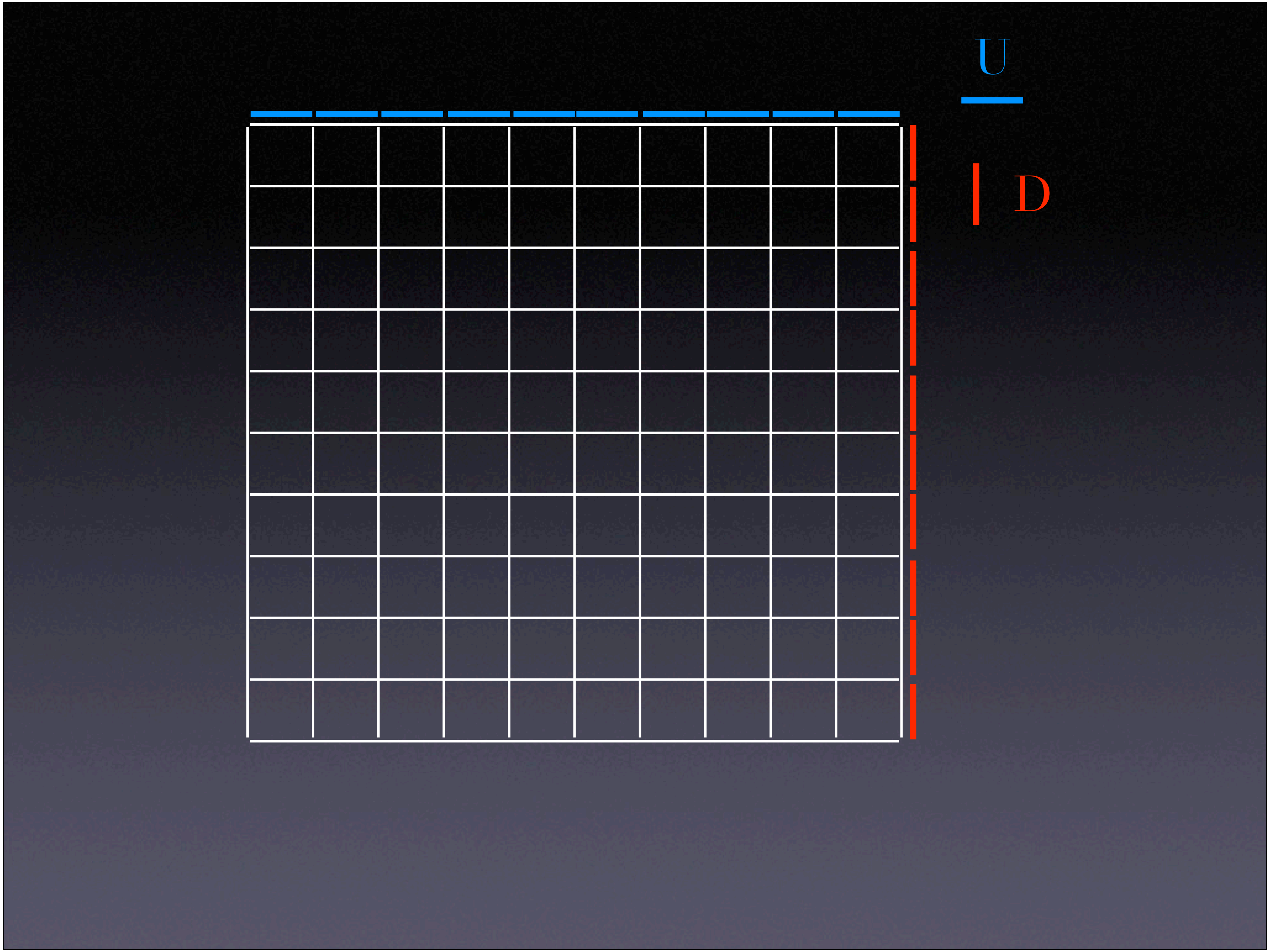
$$UD = DU + I$$



$$U^n D^n = ?$$

$$\langle \emptyset | U^n D^n = (? \; n!) \; \emptyset$$

Réécritures sur une grille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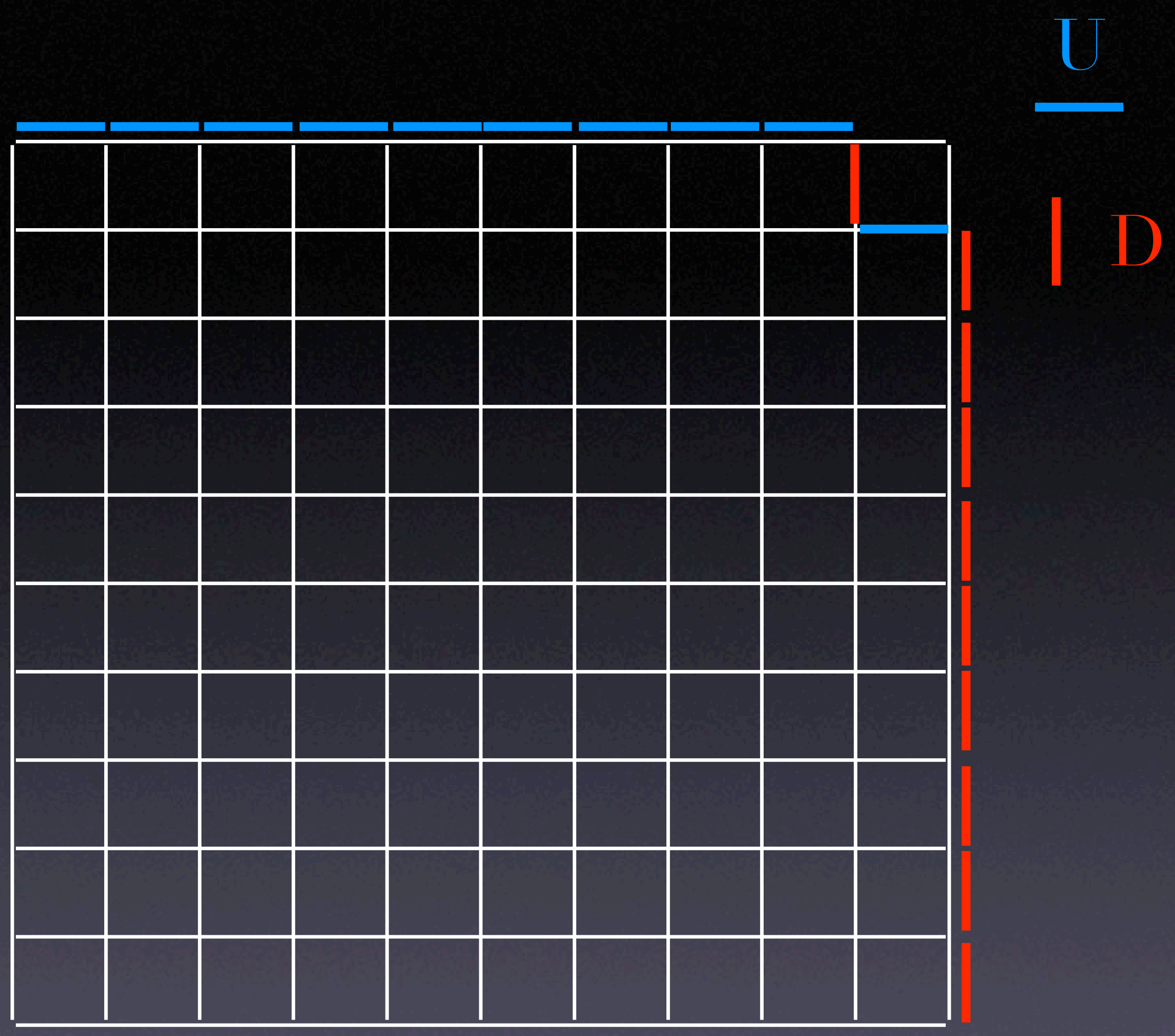
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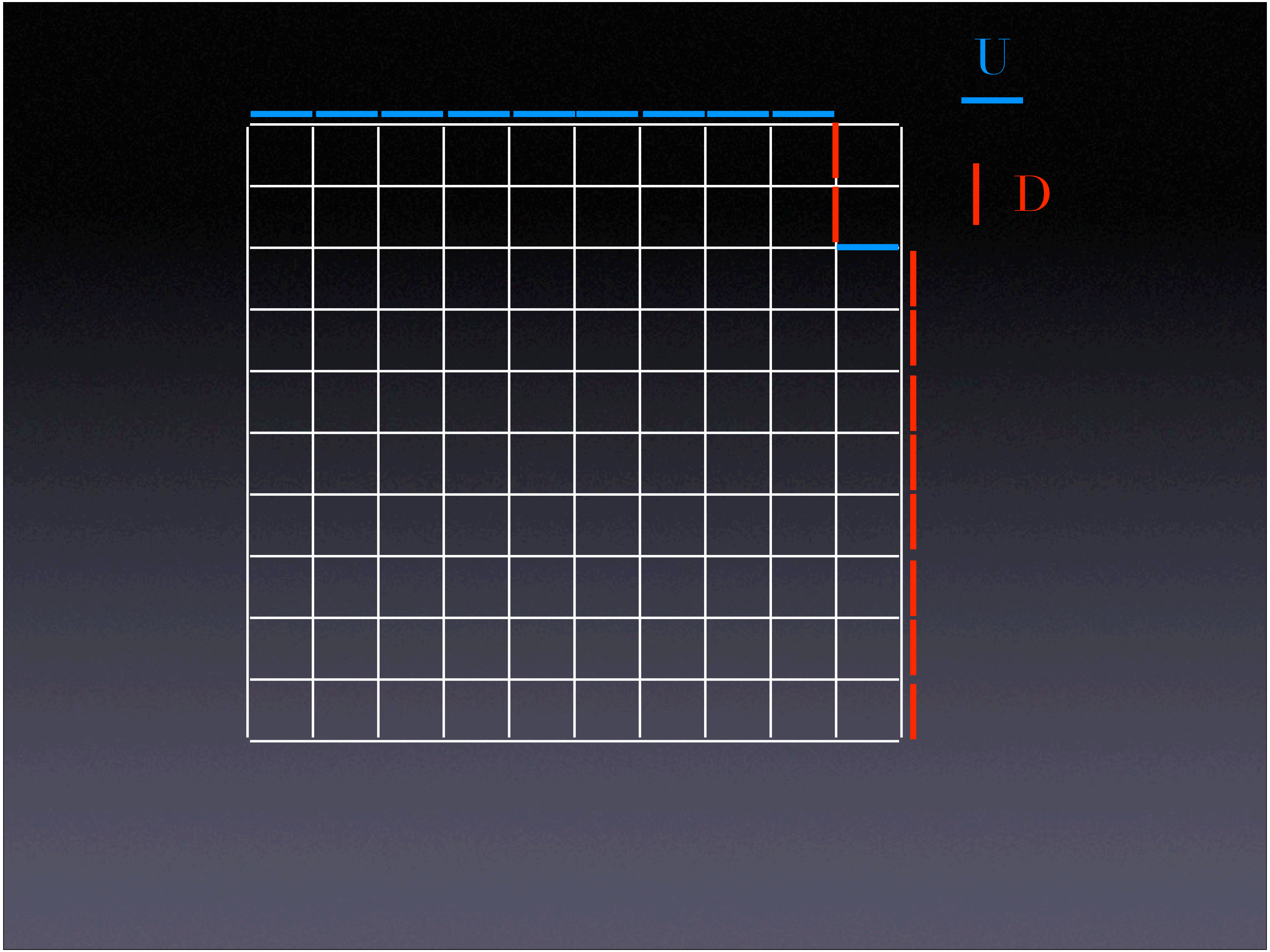
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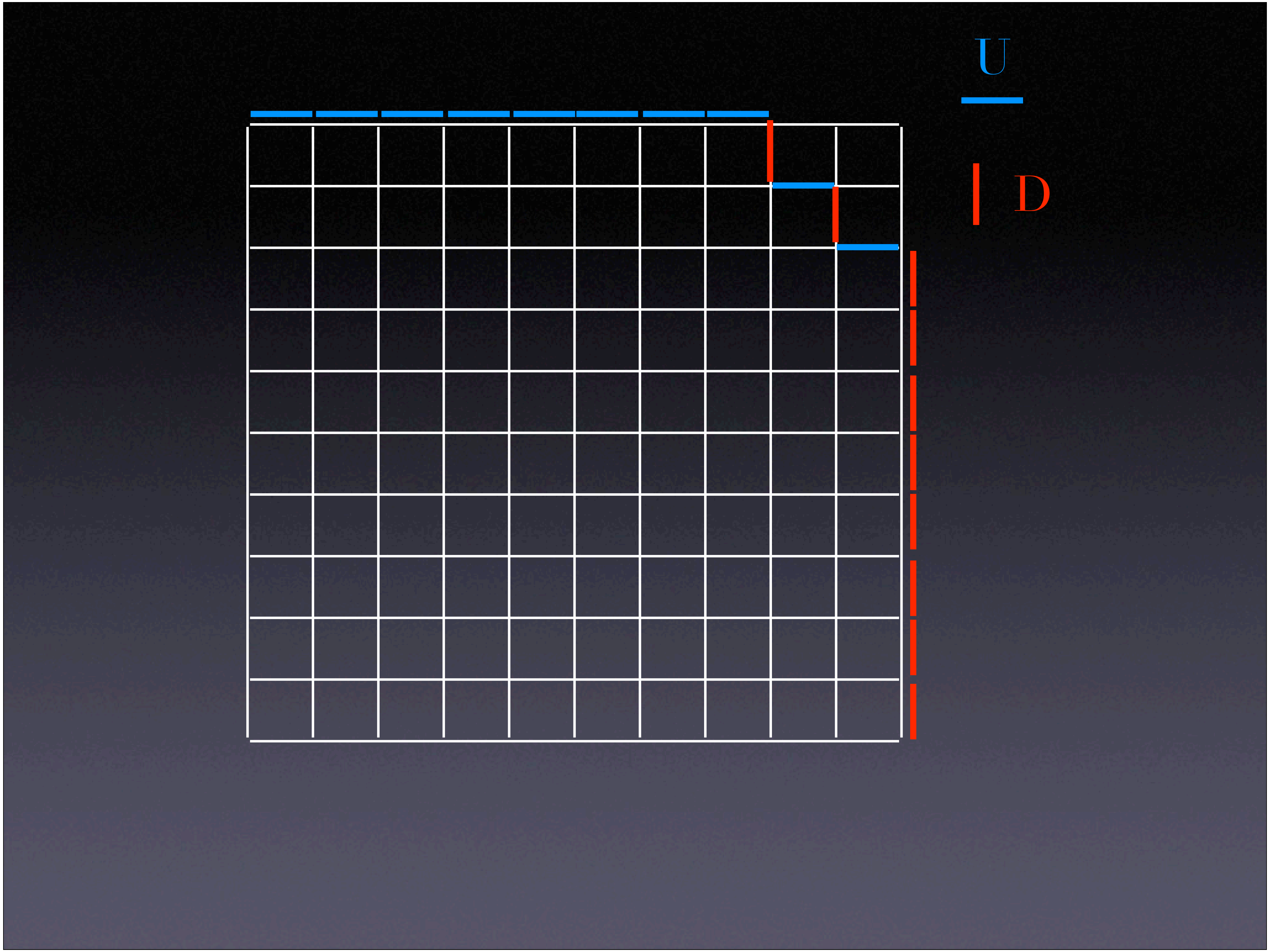
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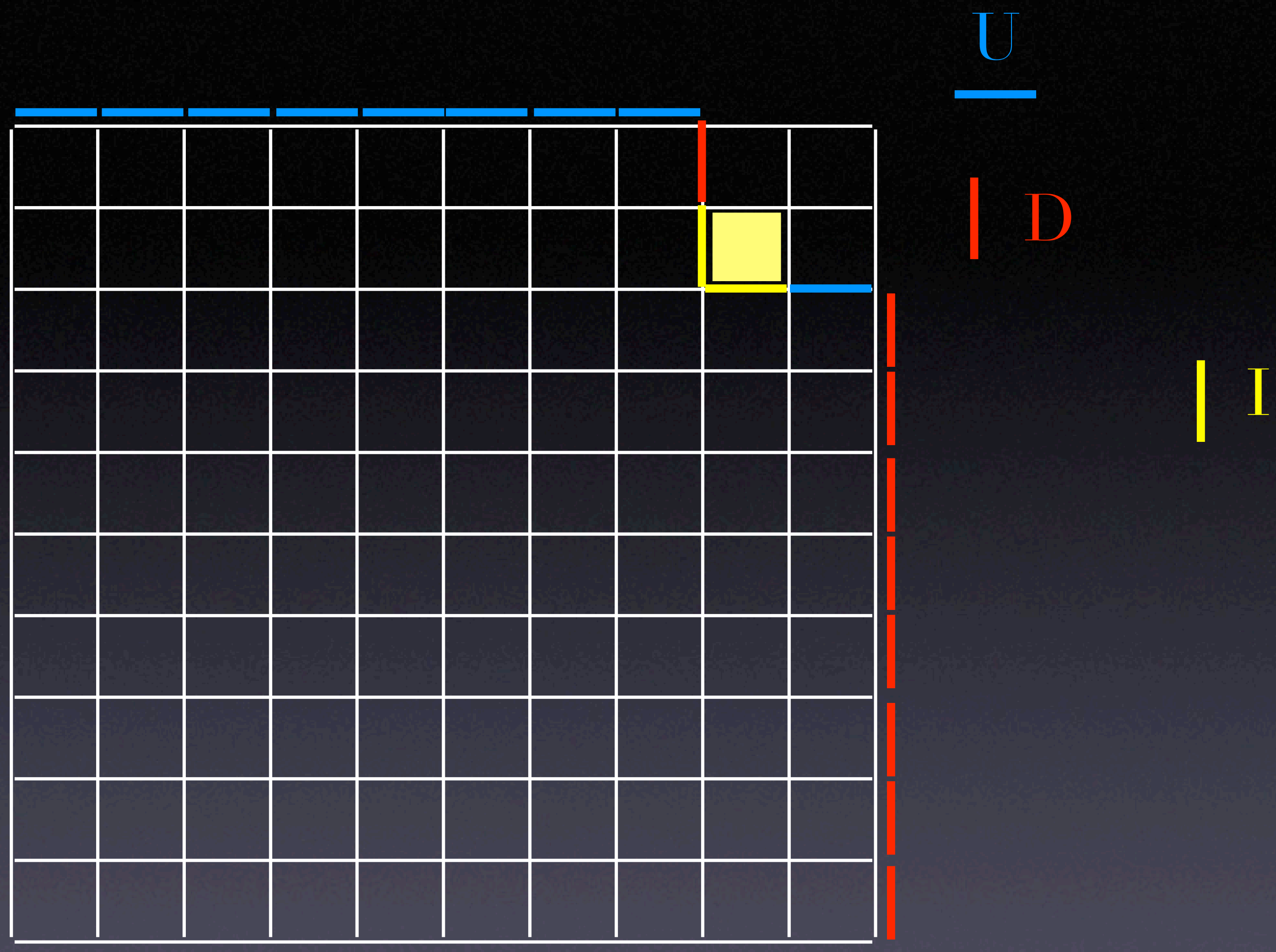

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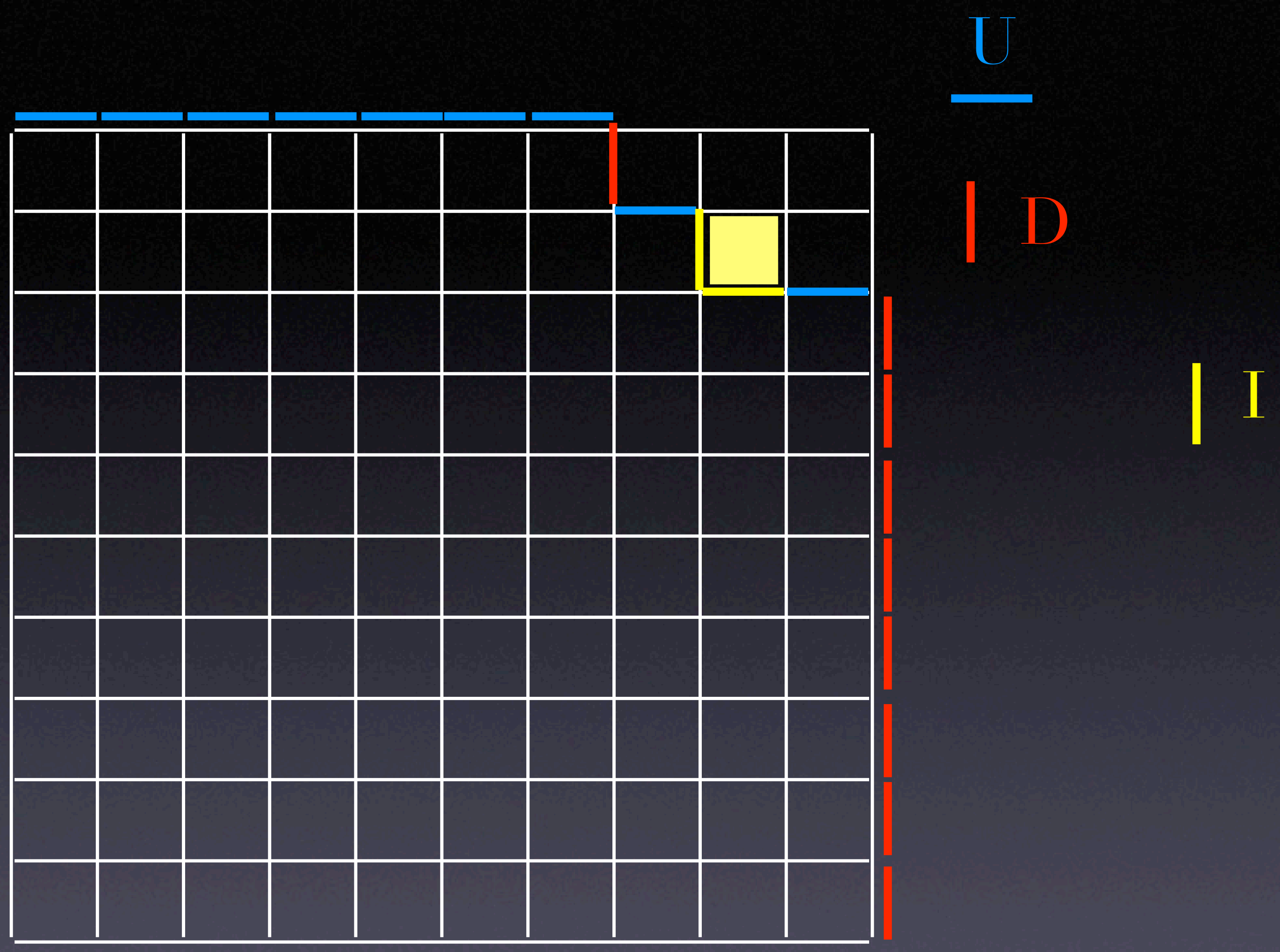

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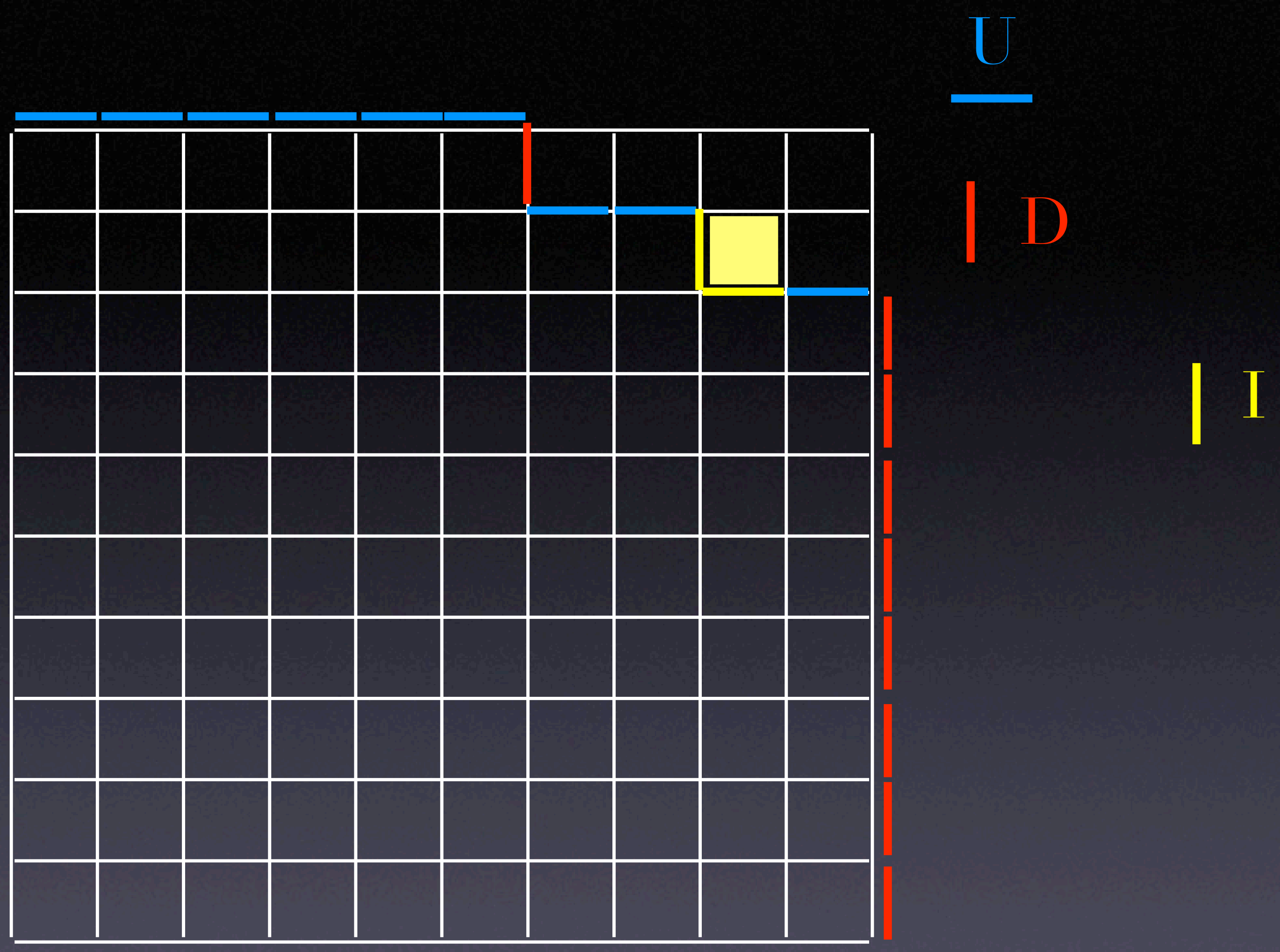
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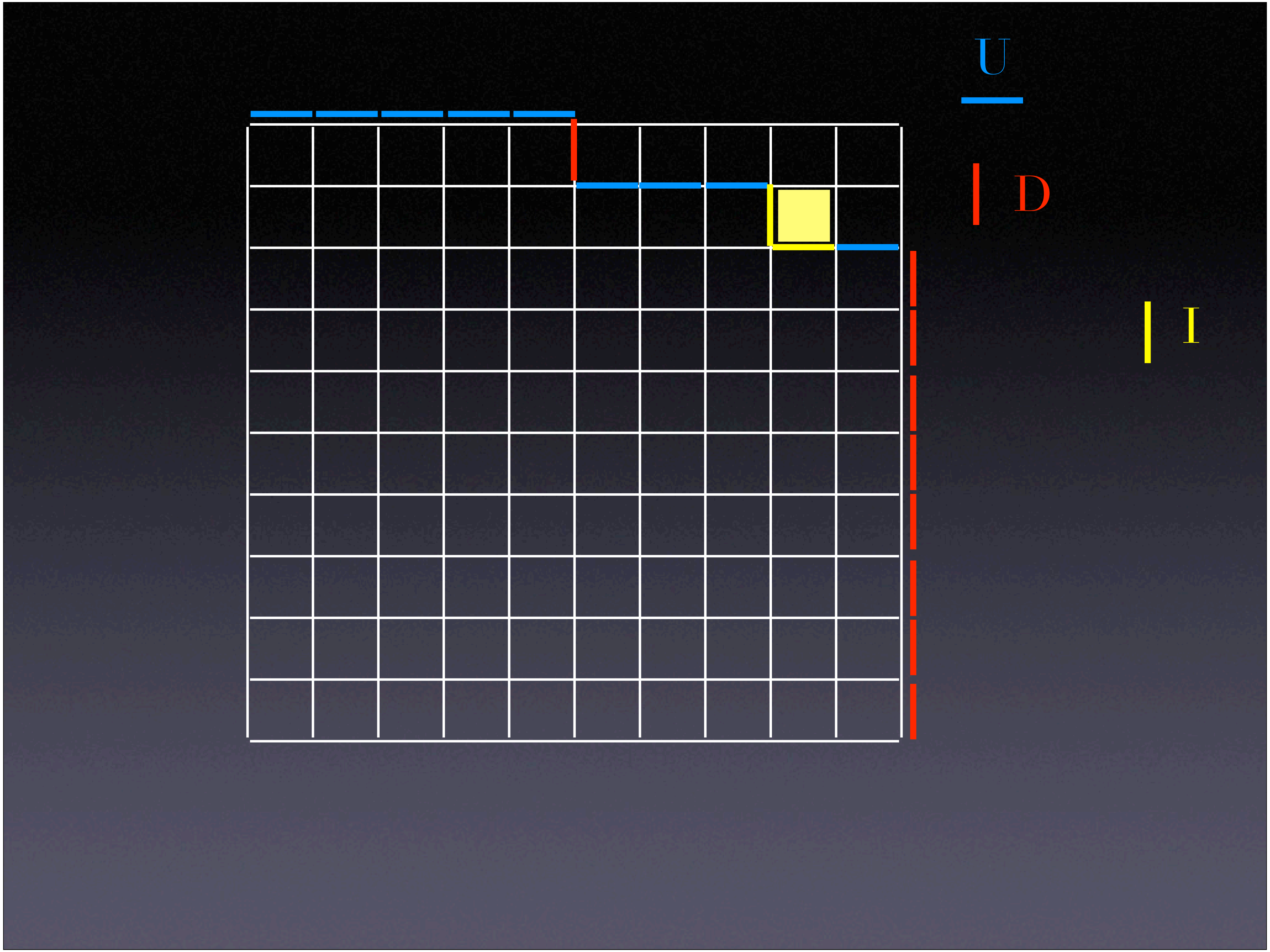
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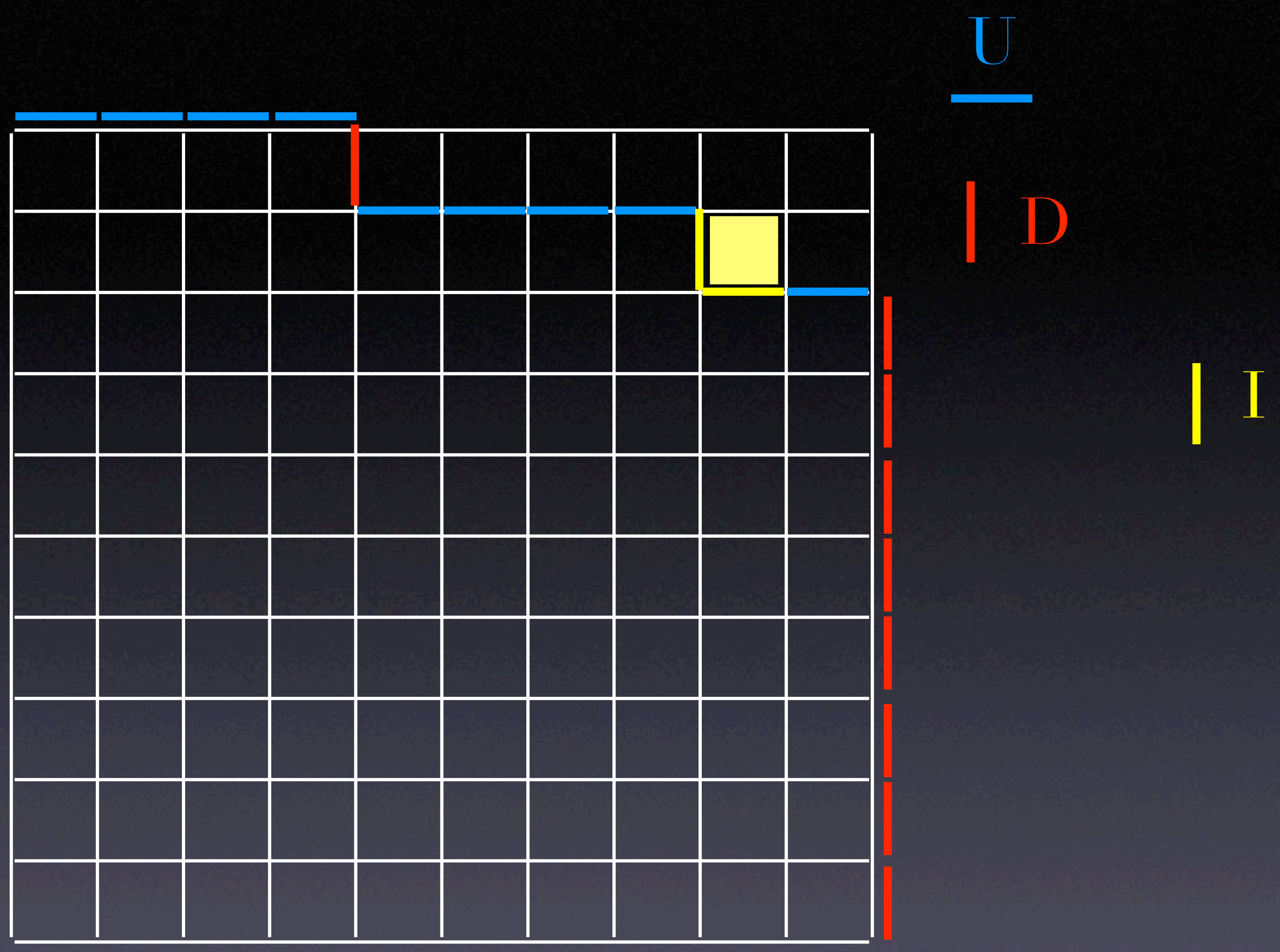
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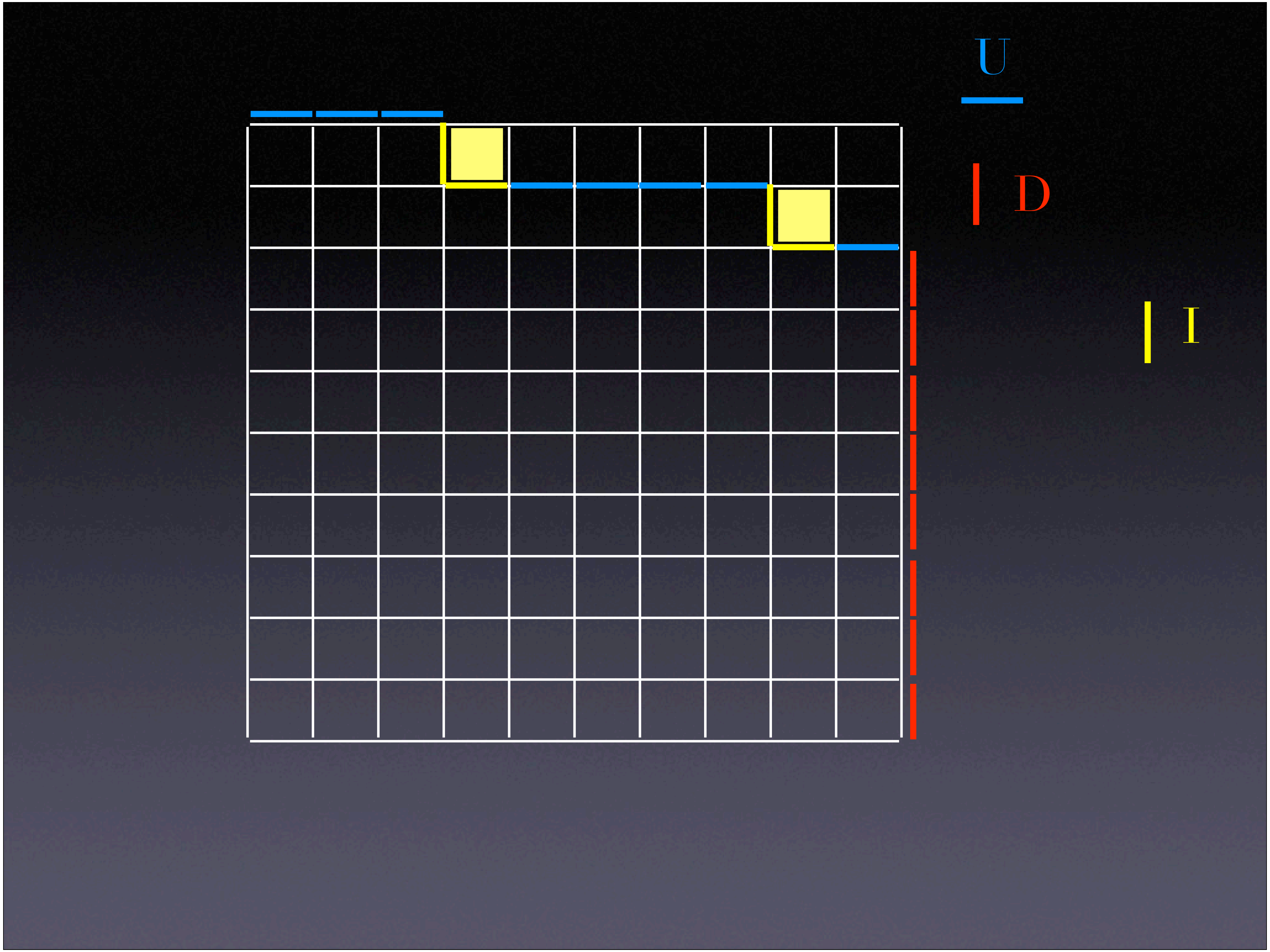

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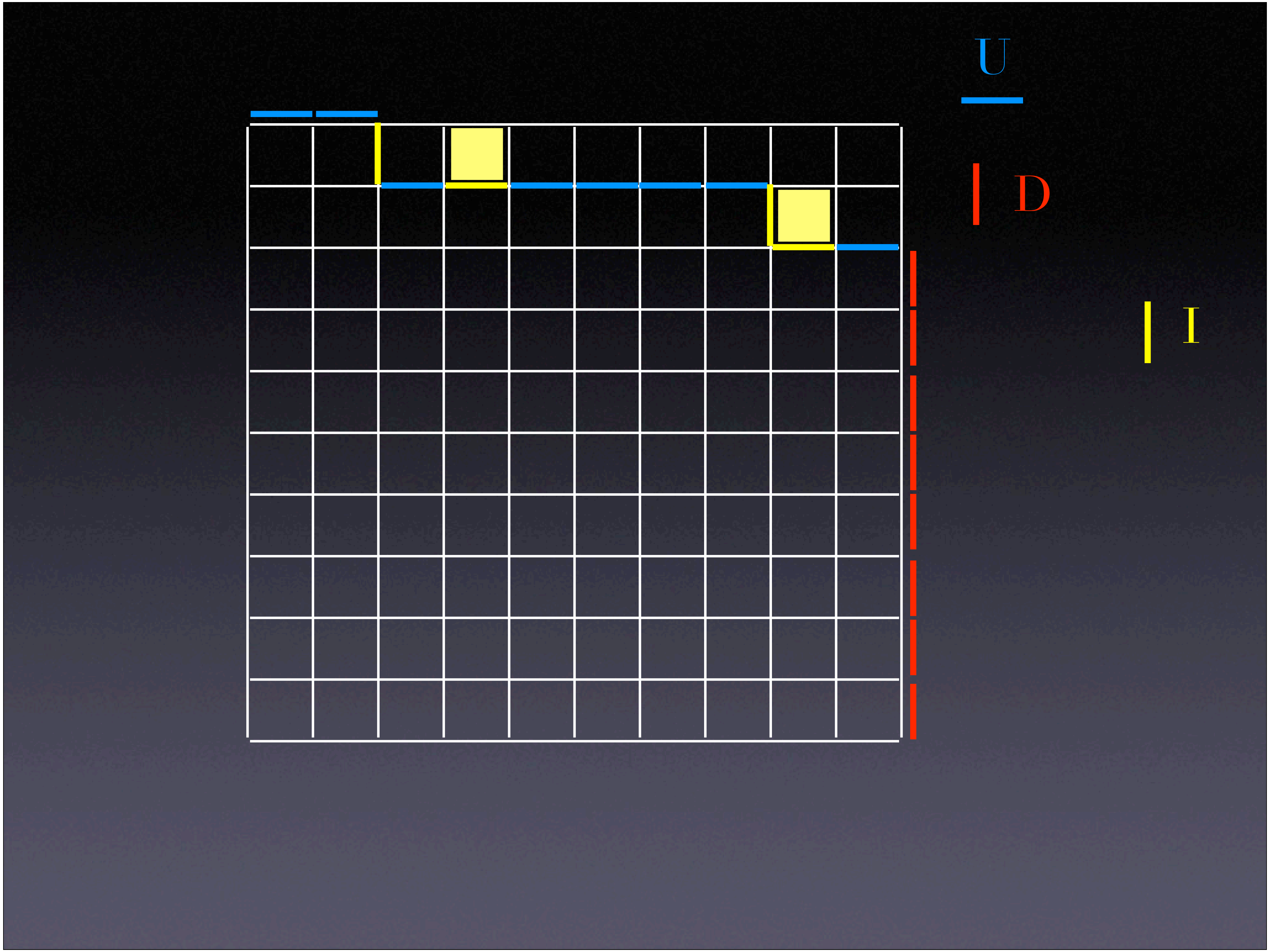
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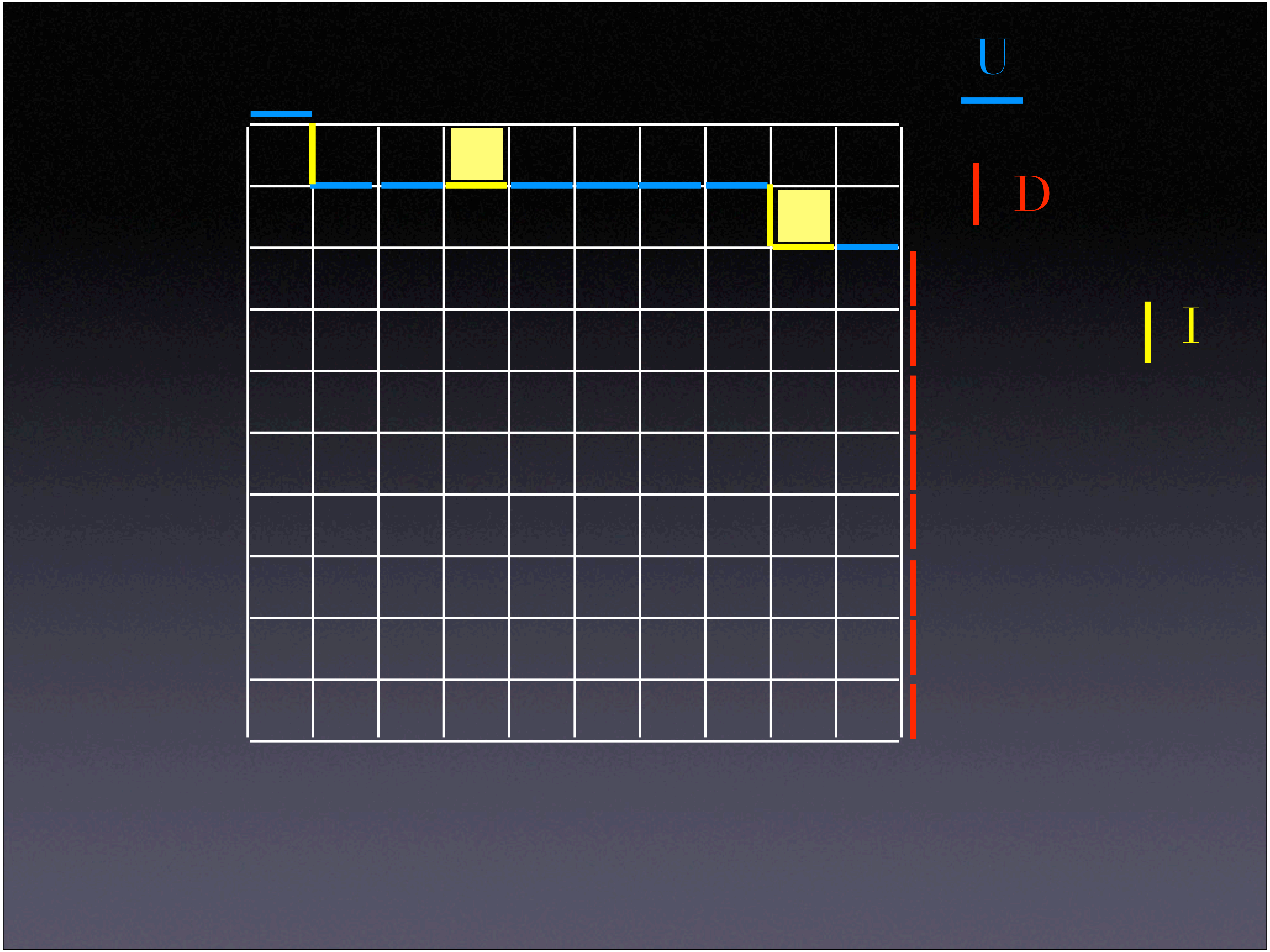

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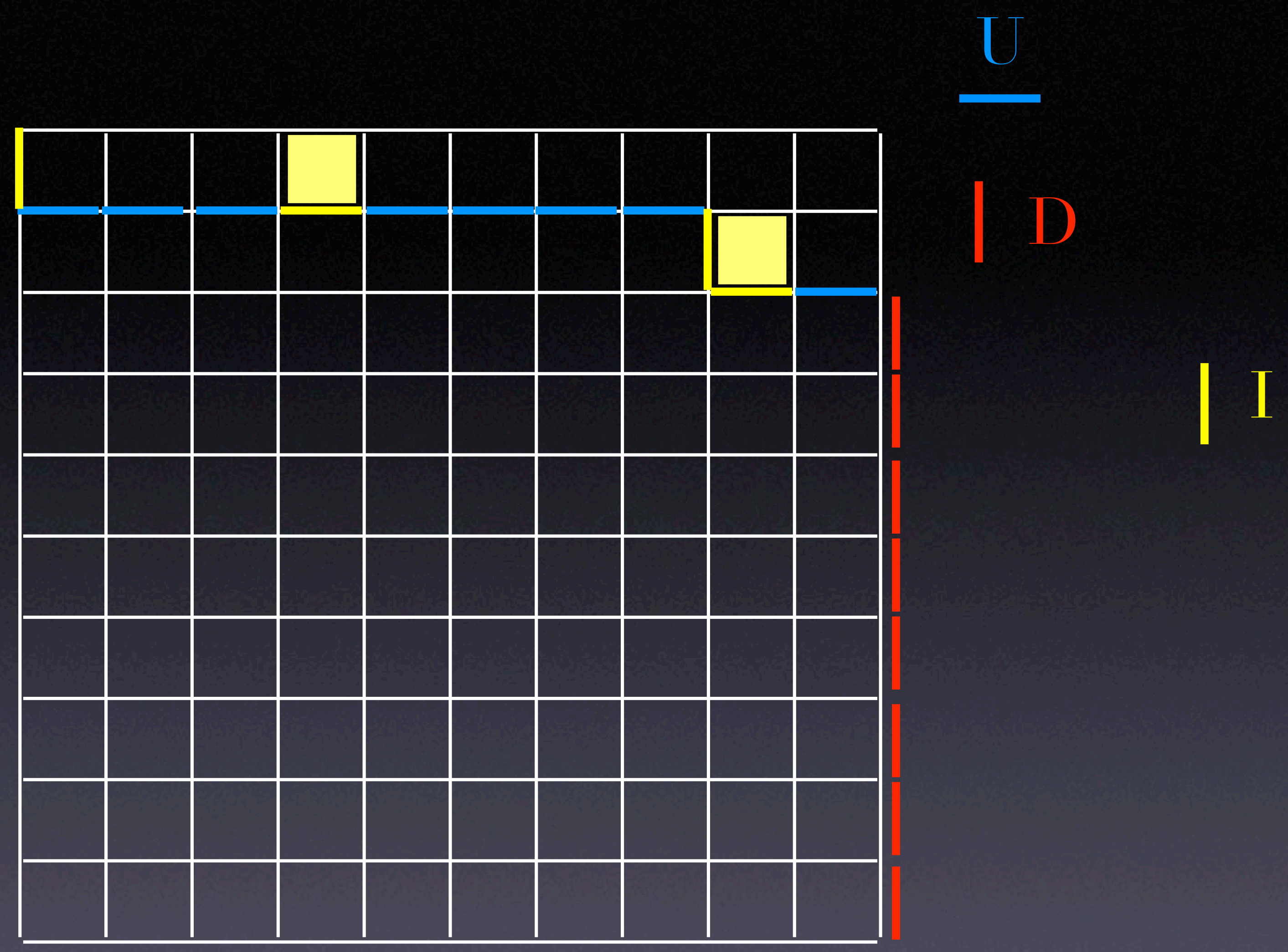
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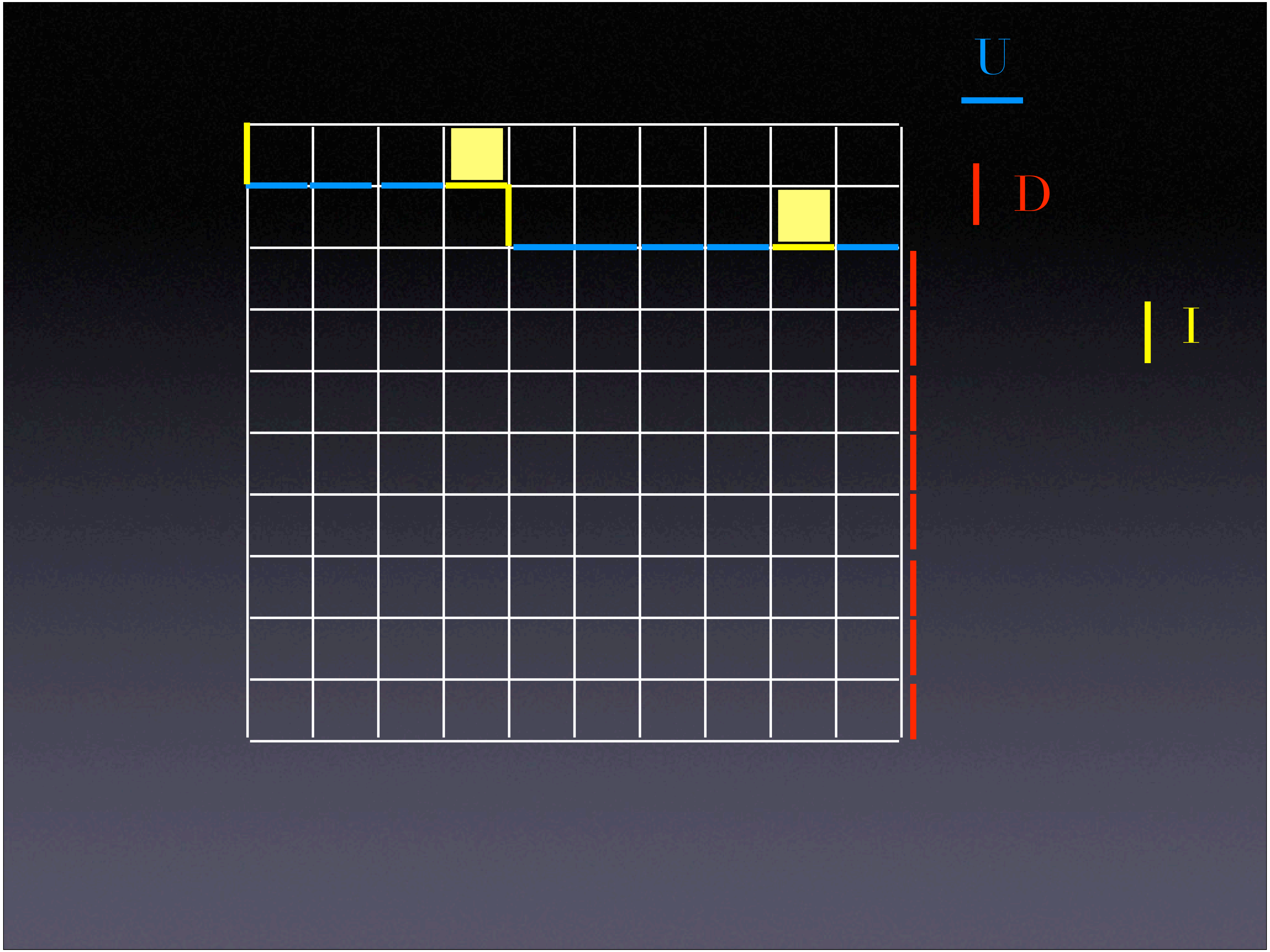

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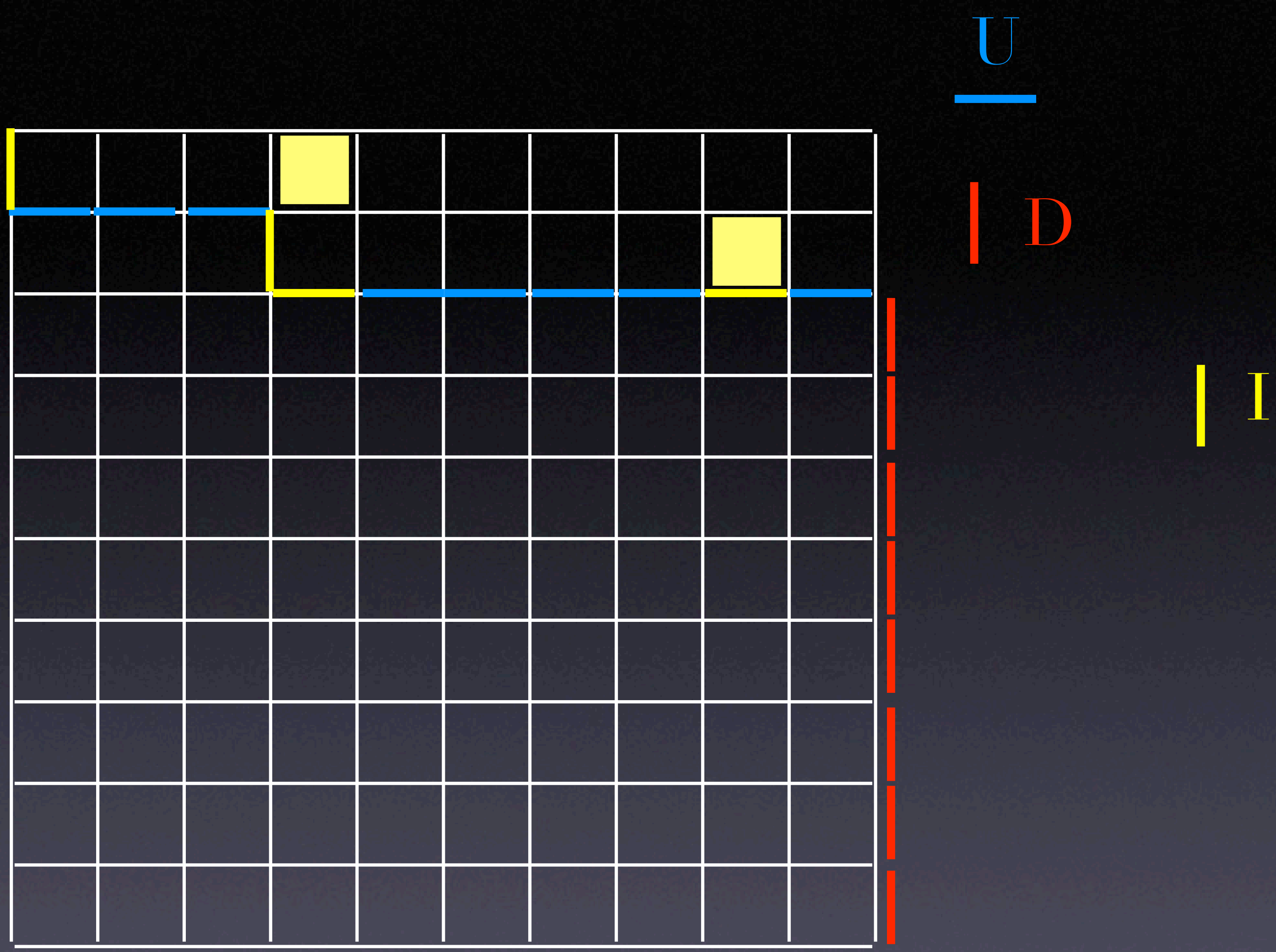
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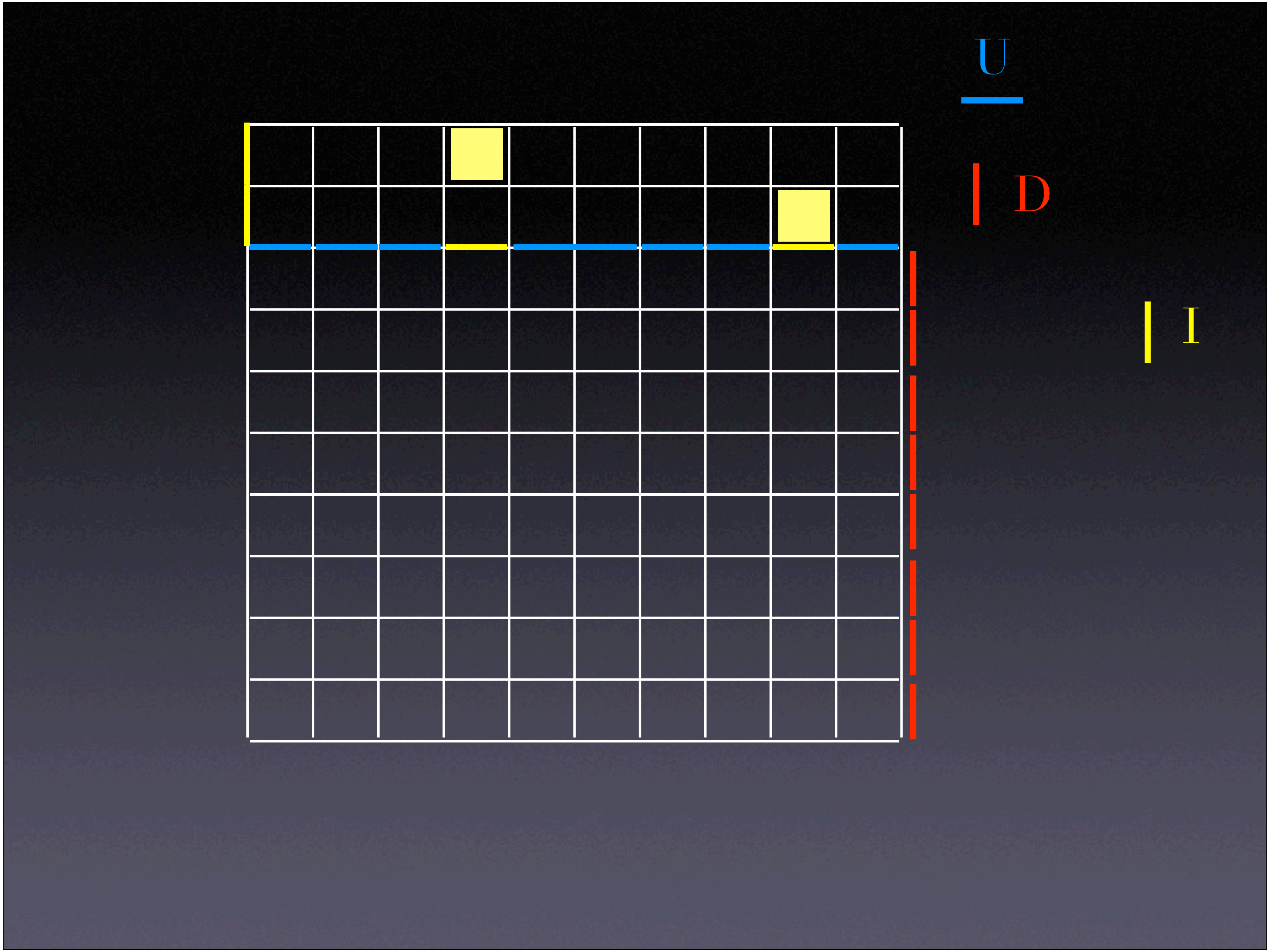
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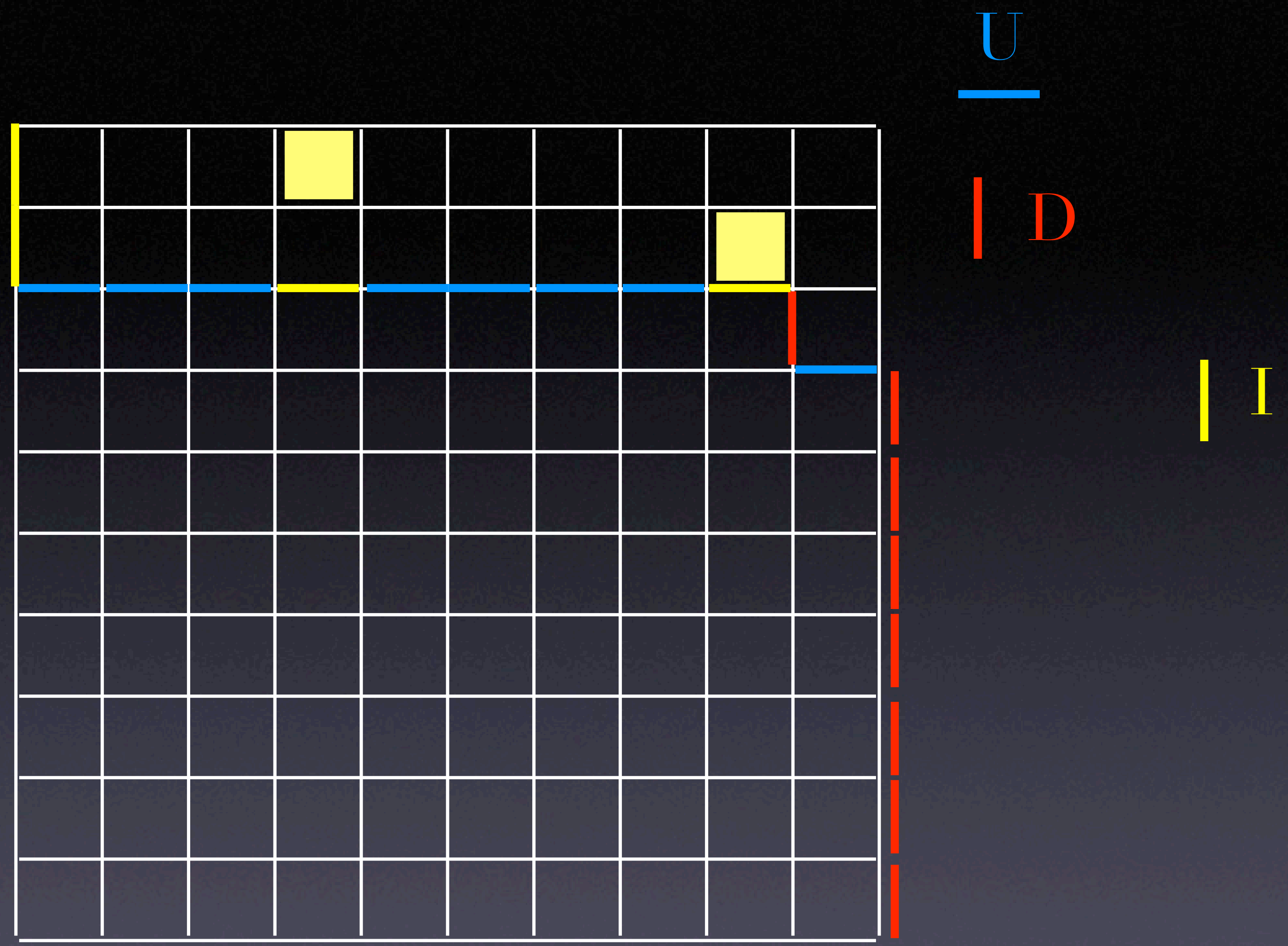
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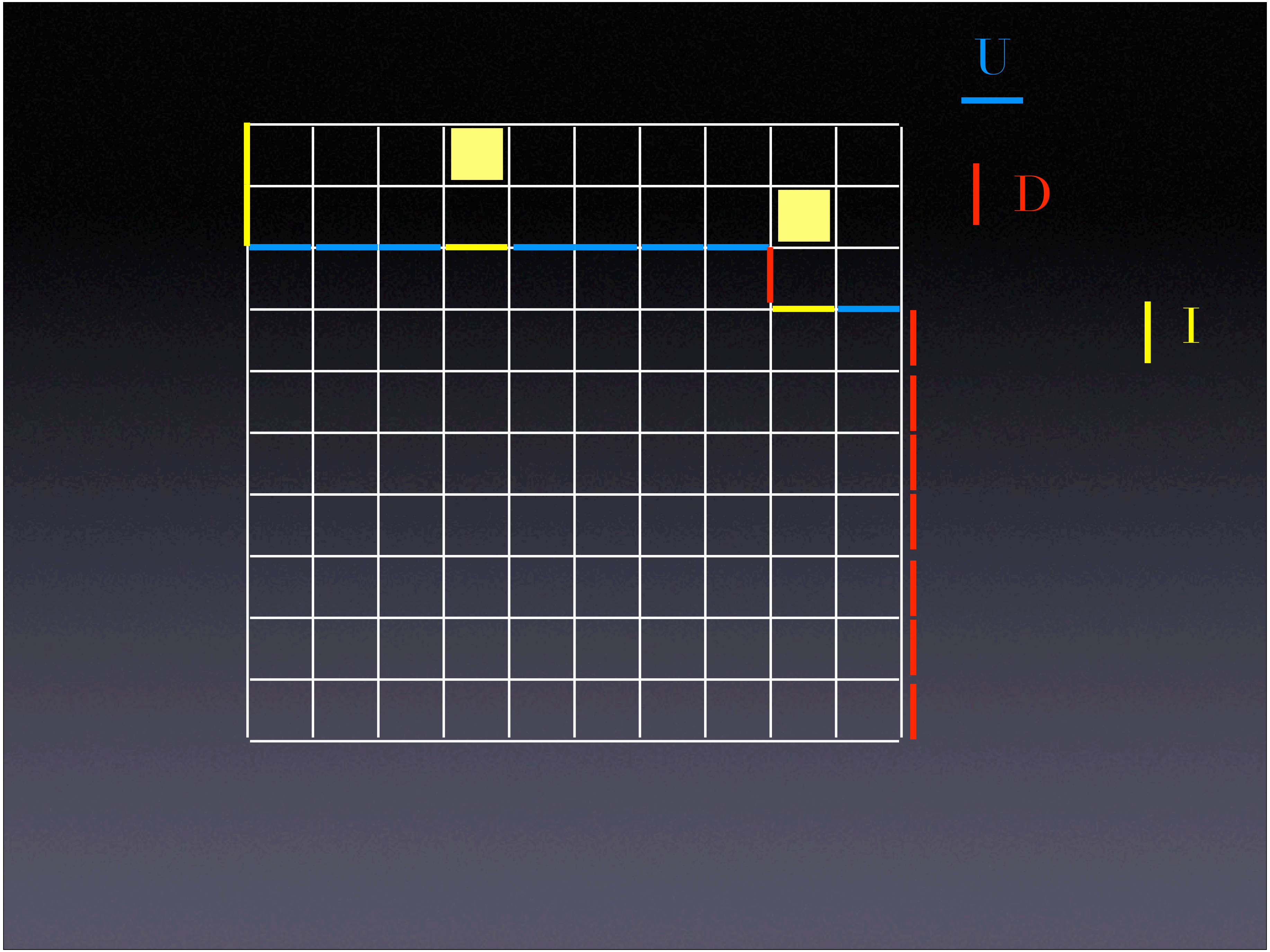
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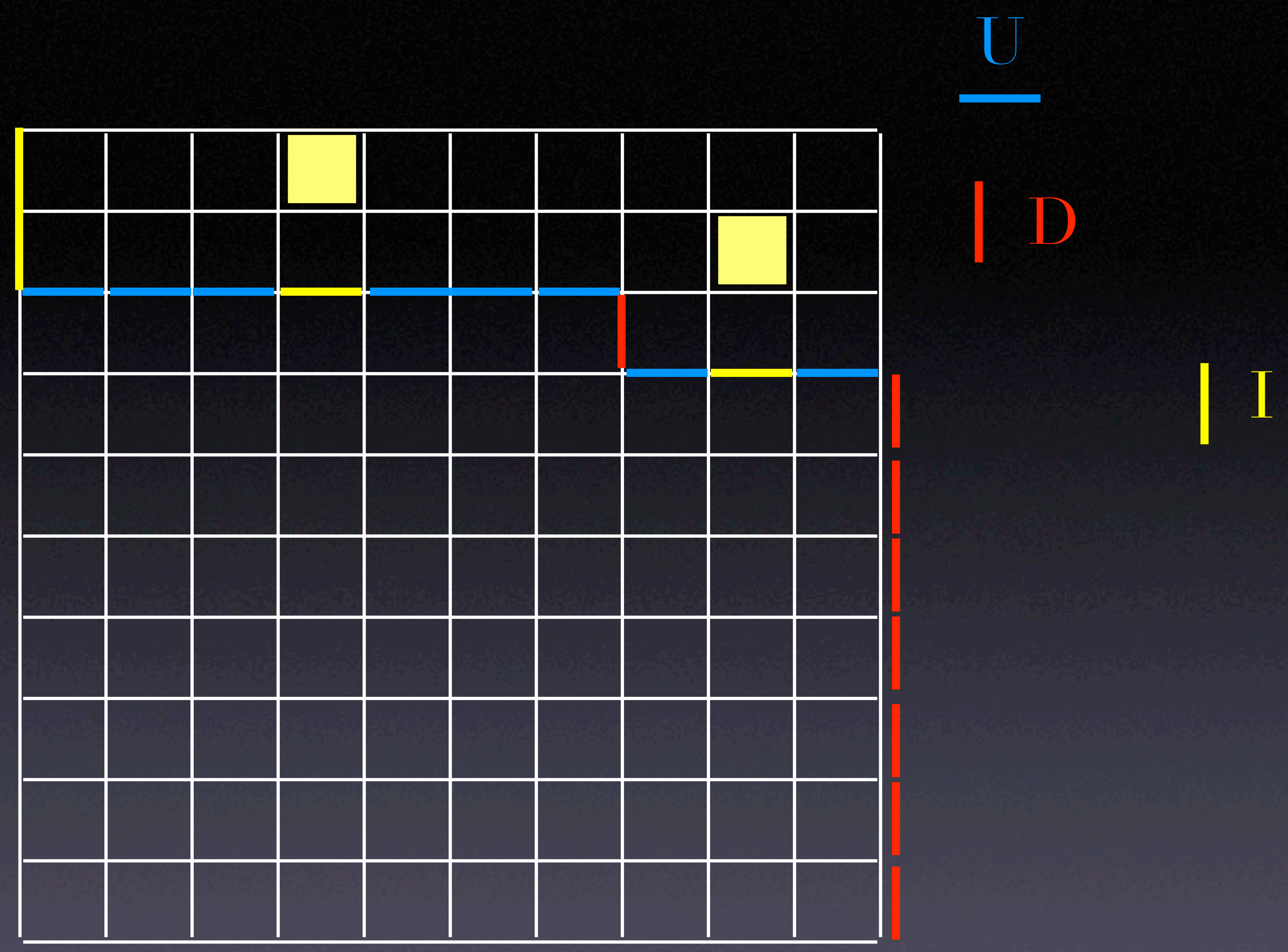
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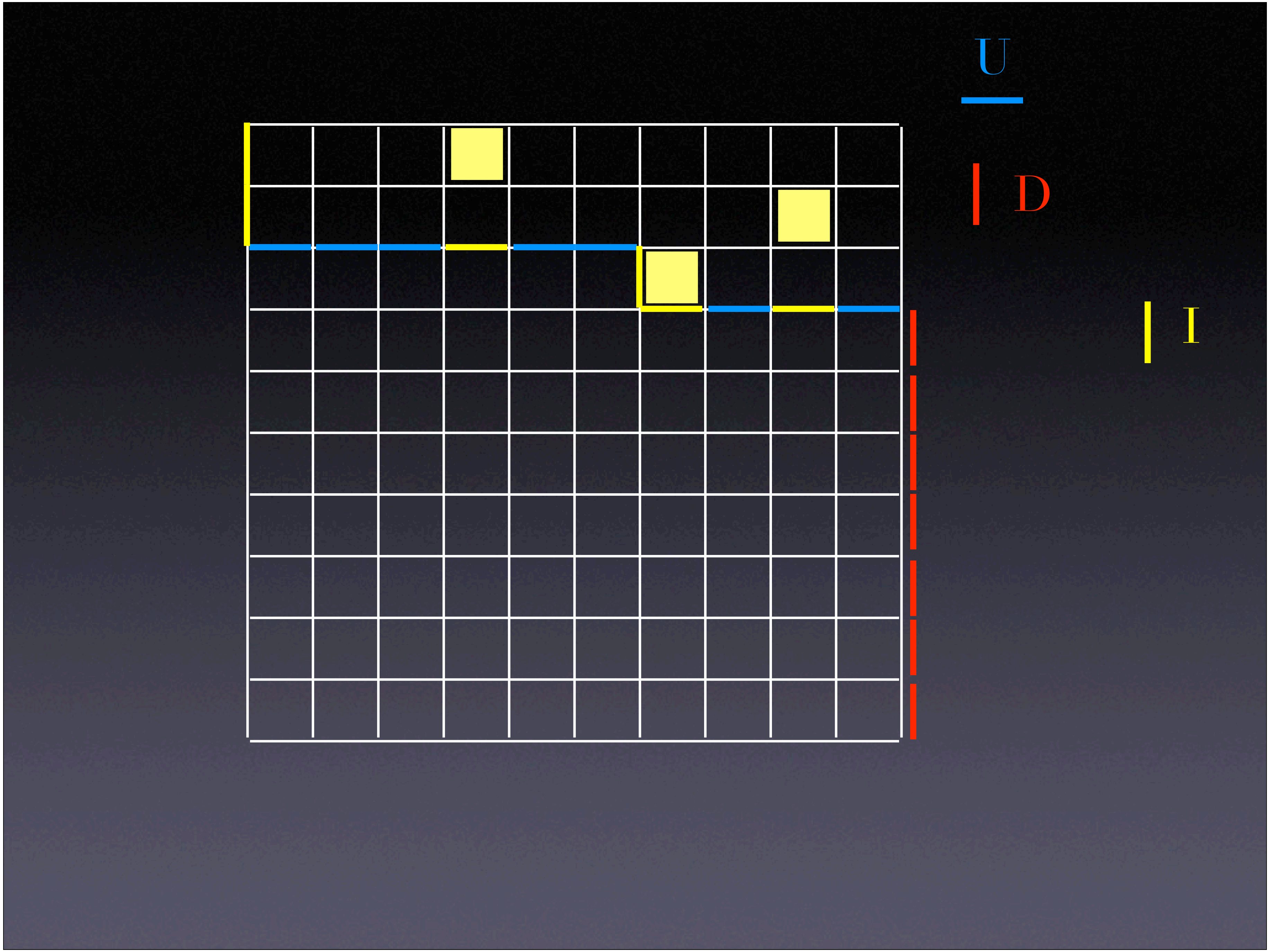
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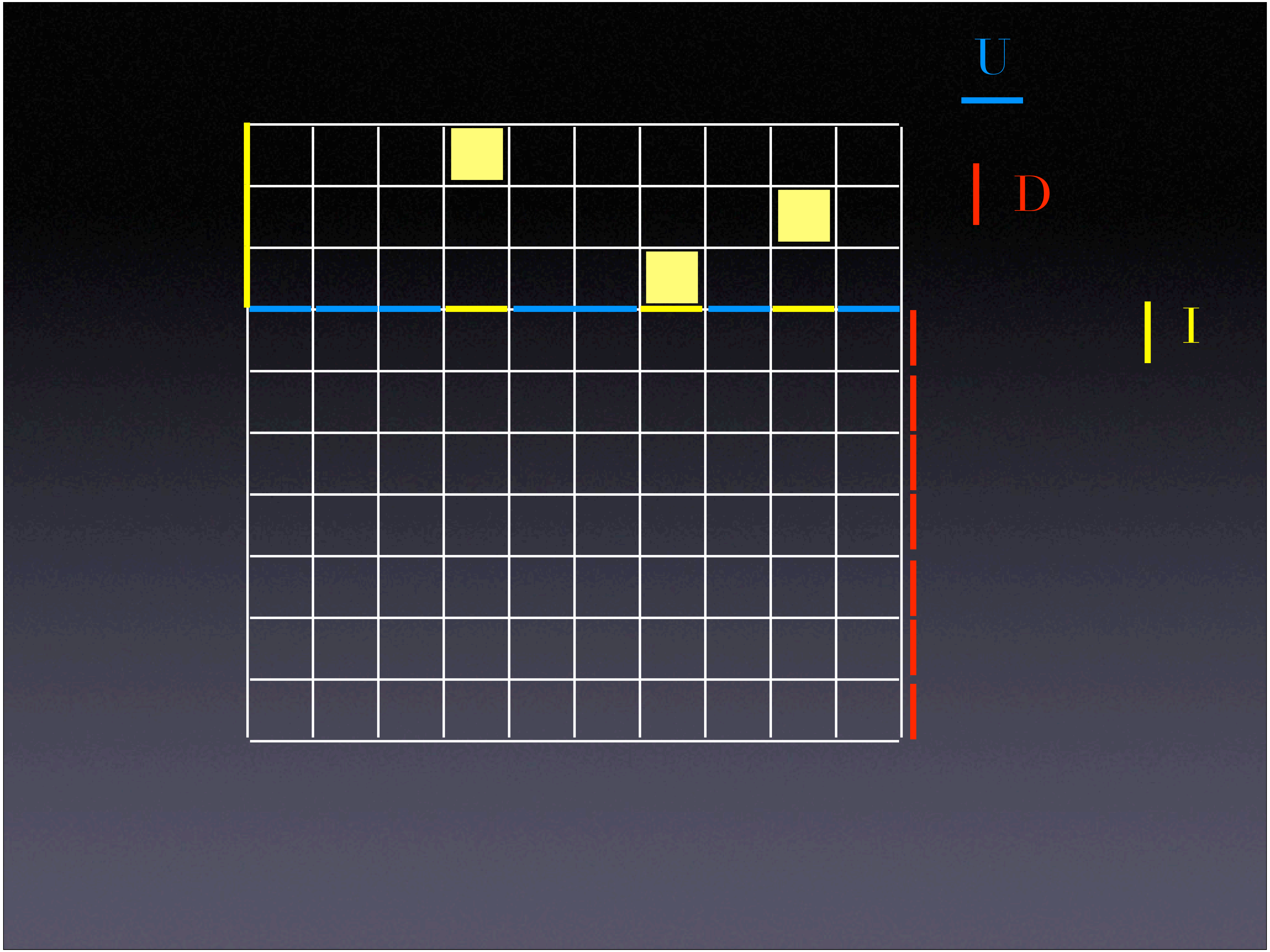
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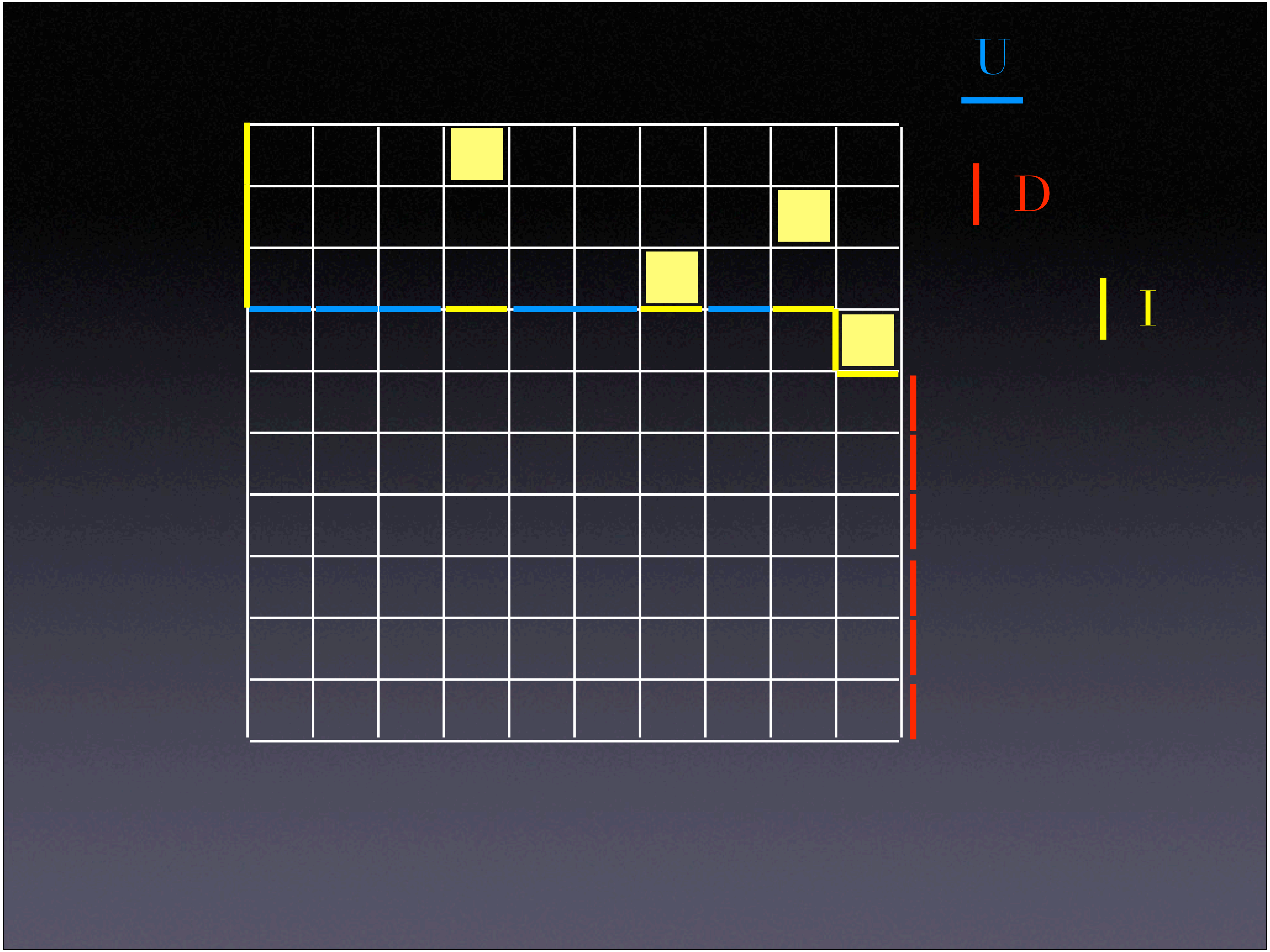
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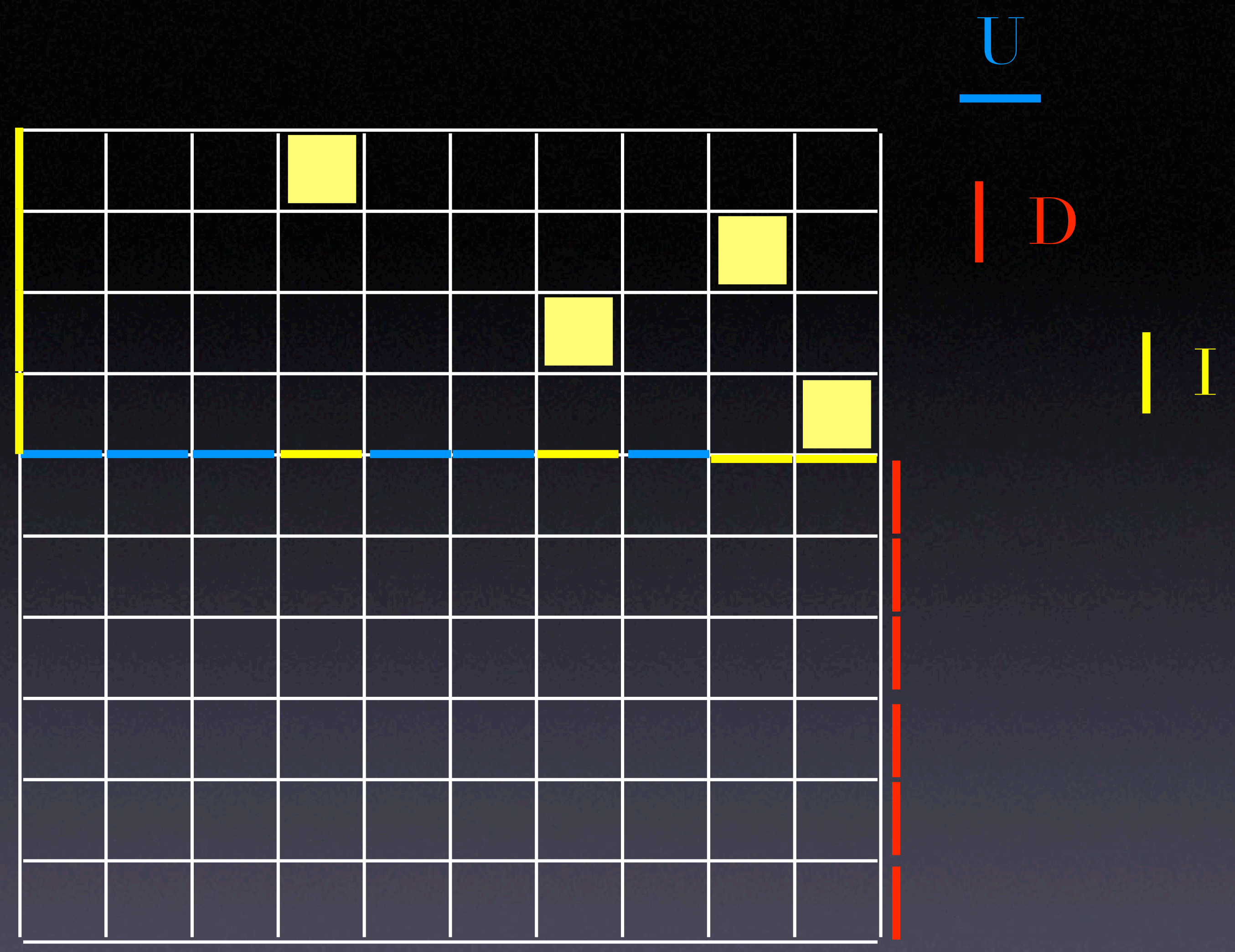
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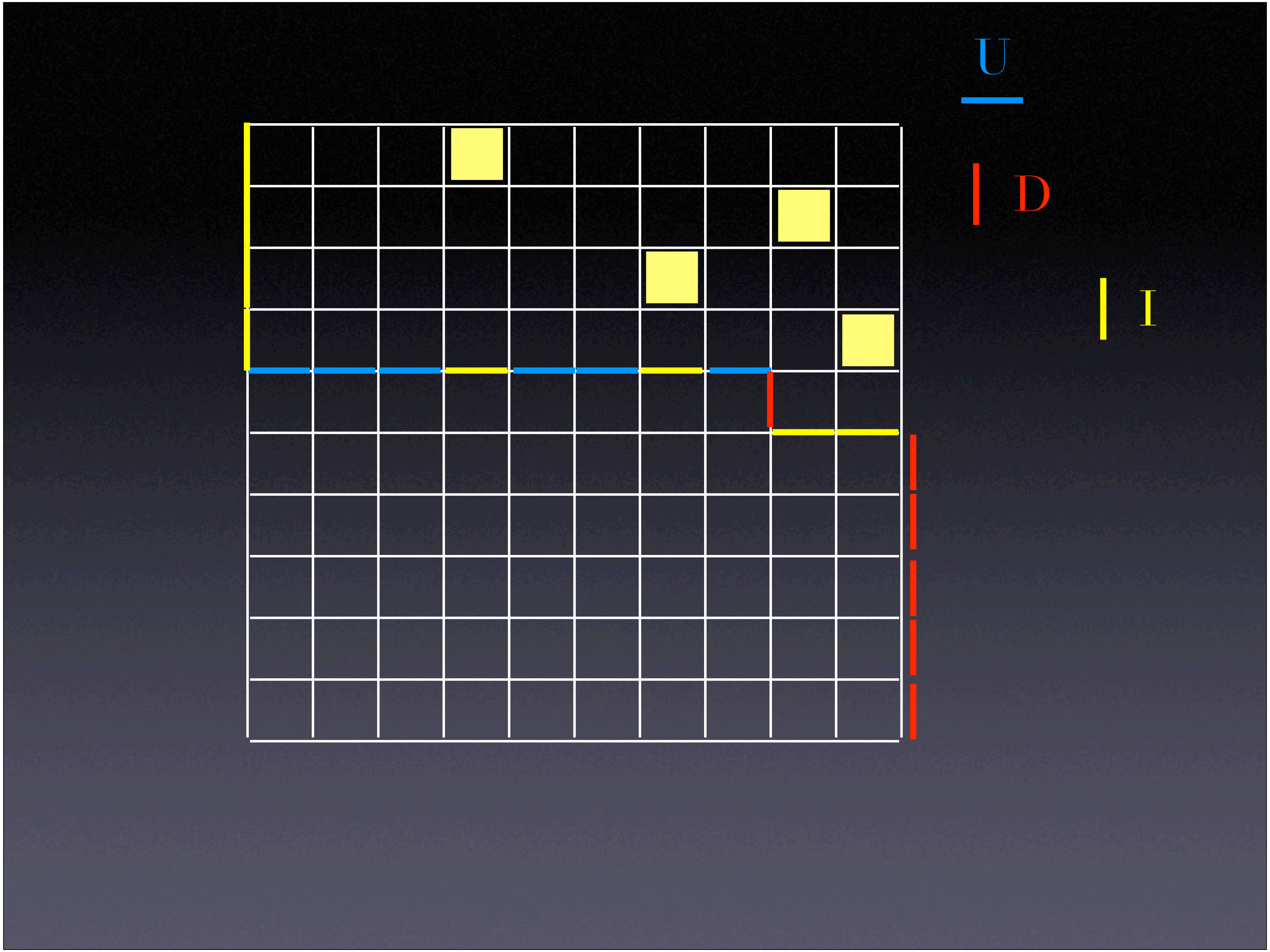
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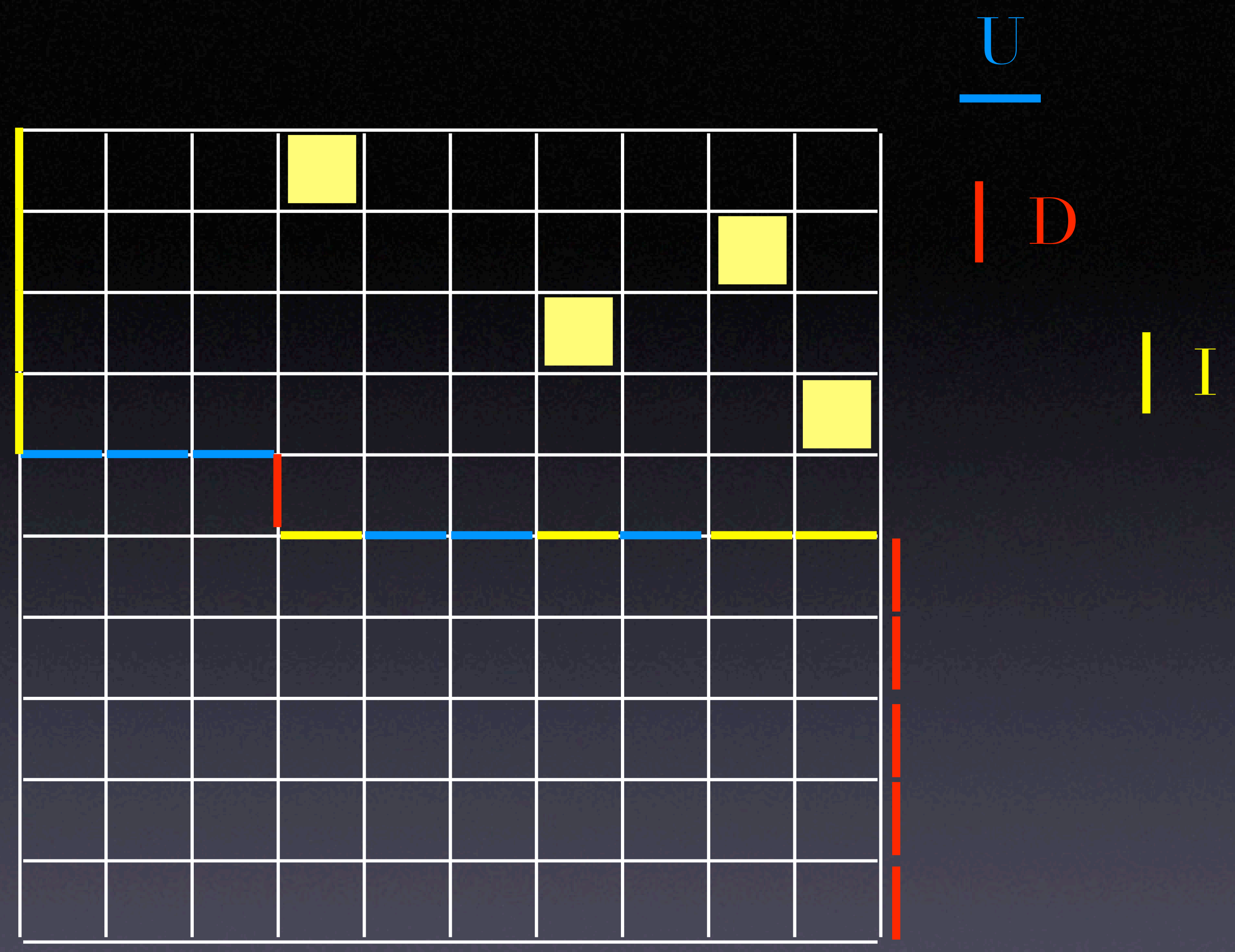
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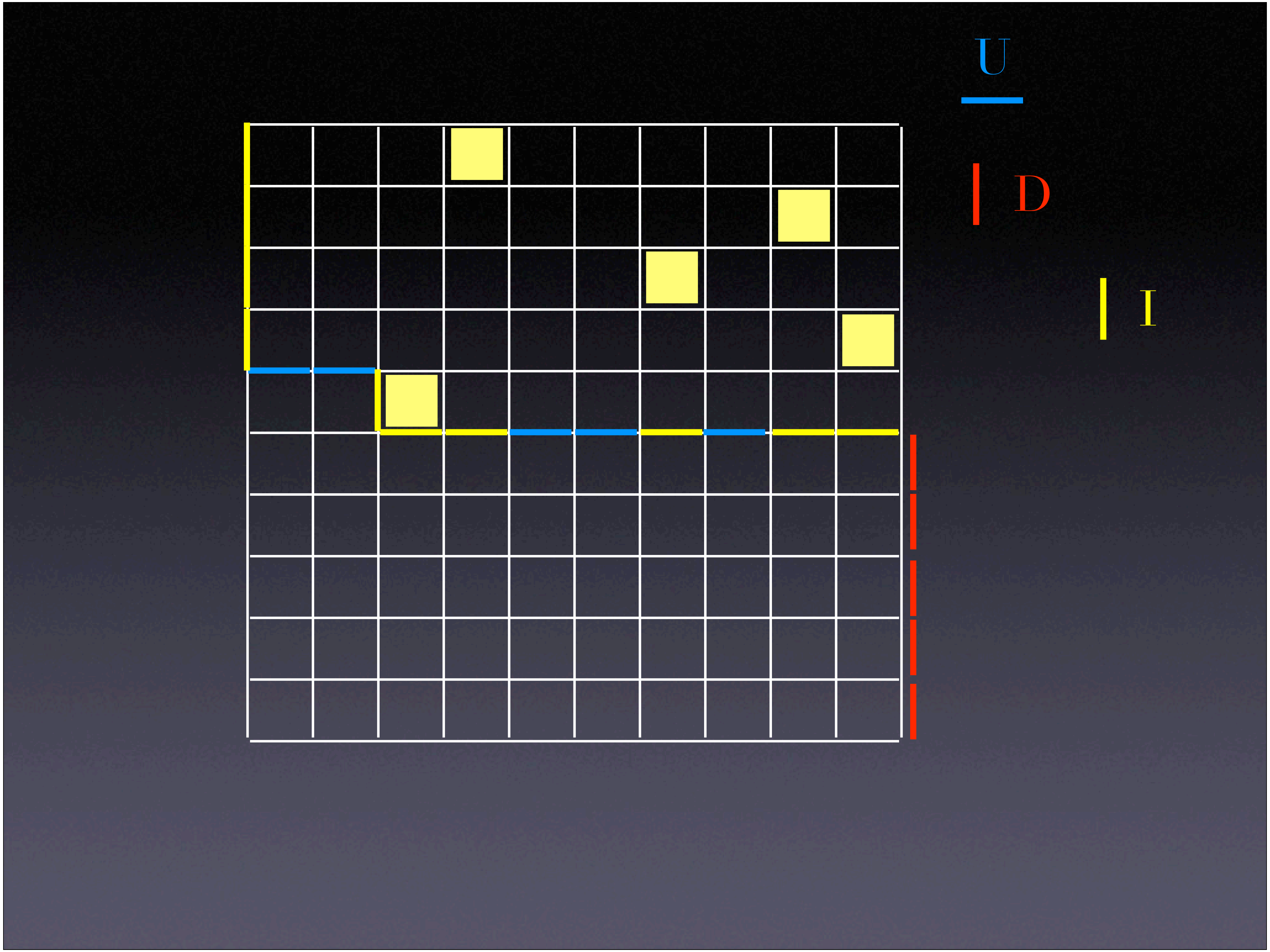
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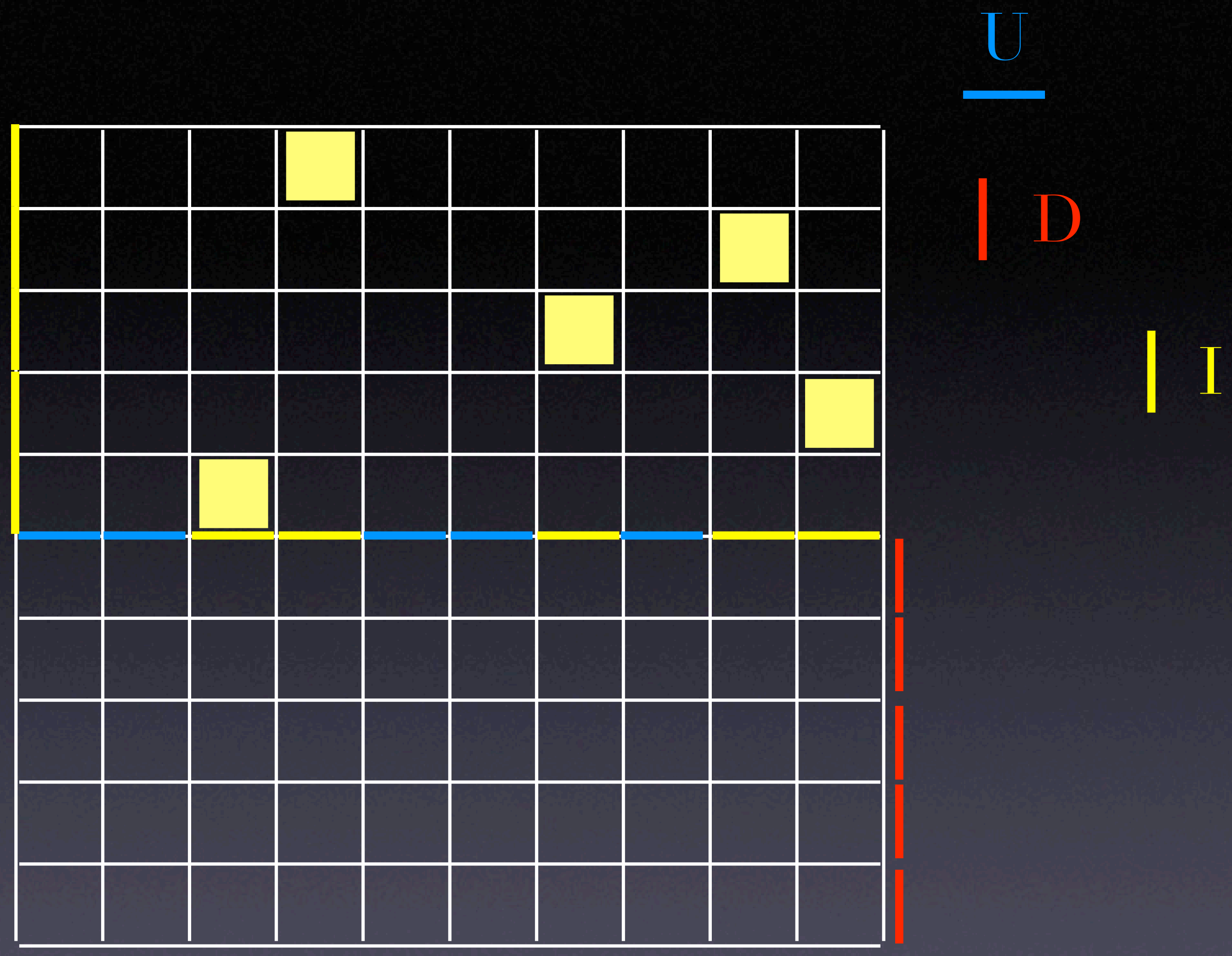
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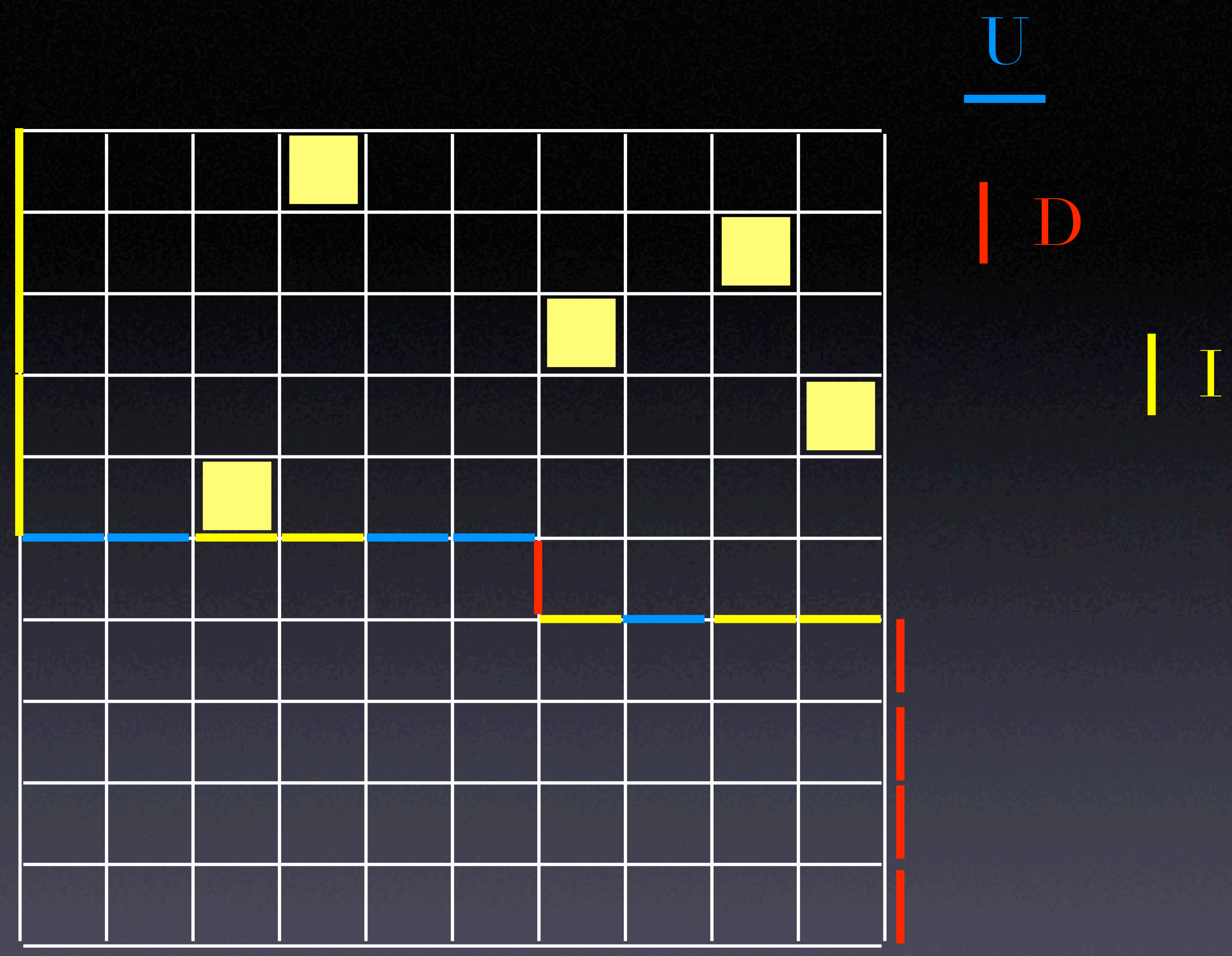
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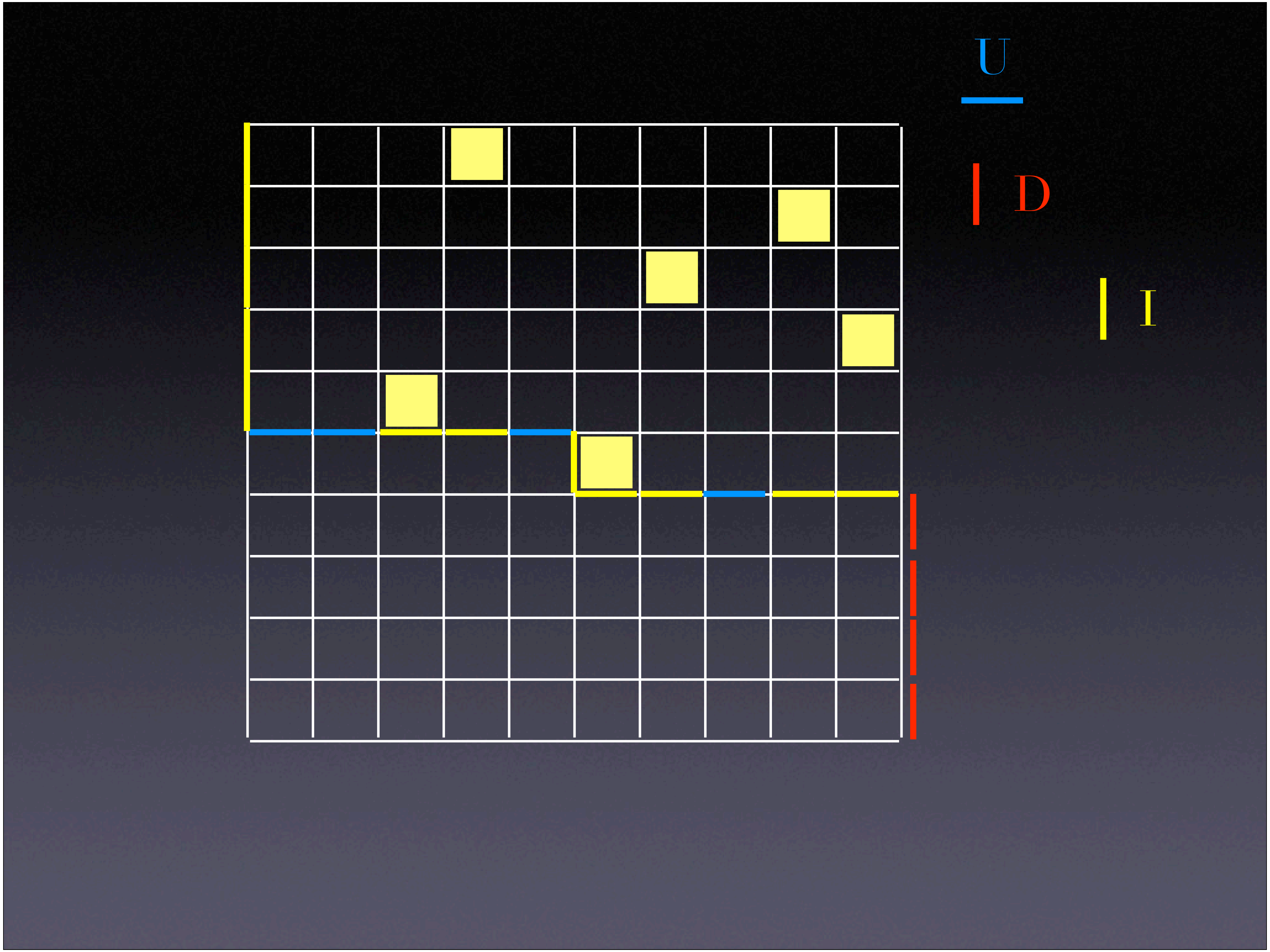

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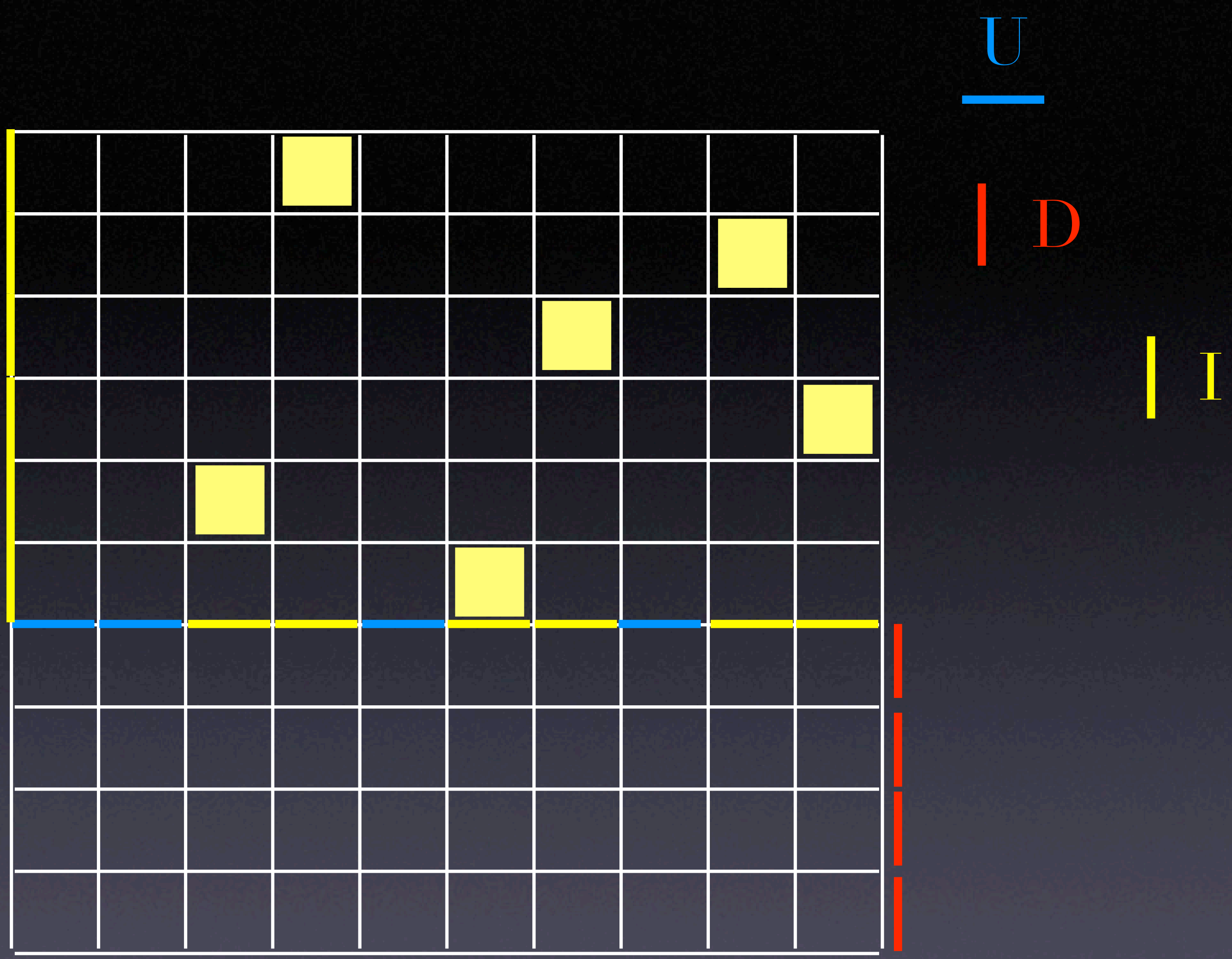
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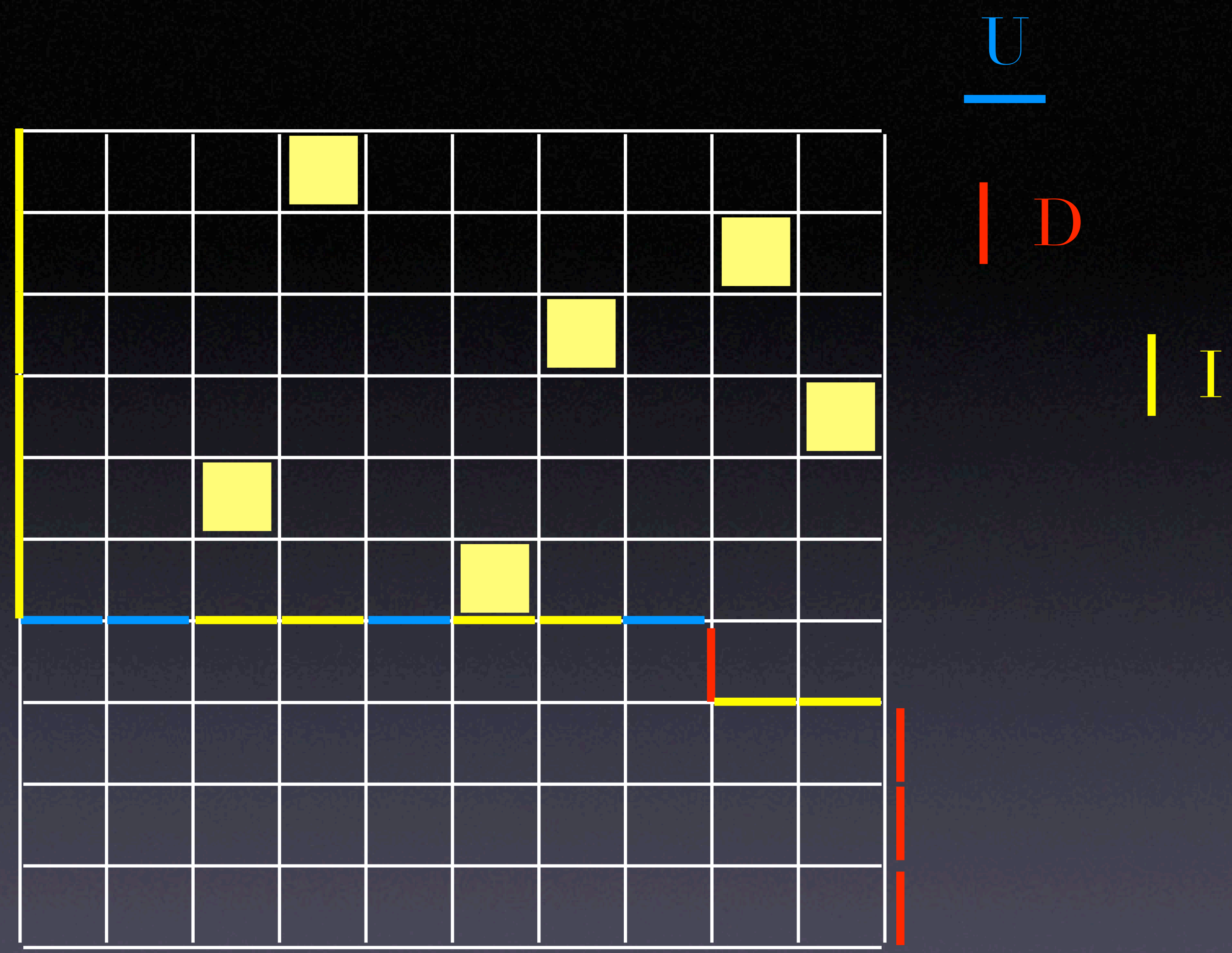

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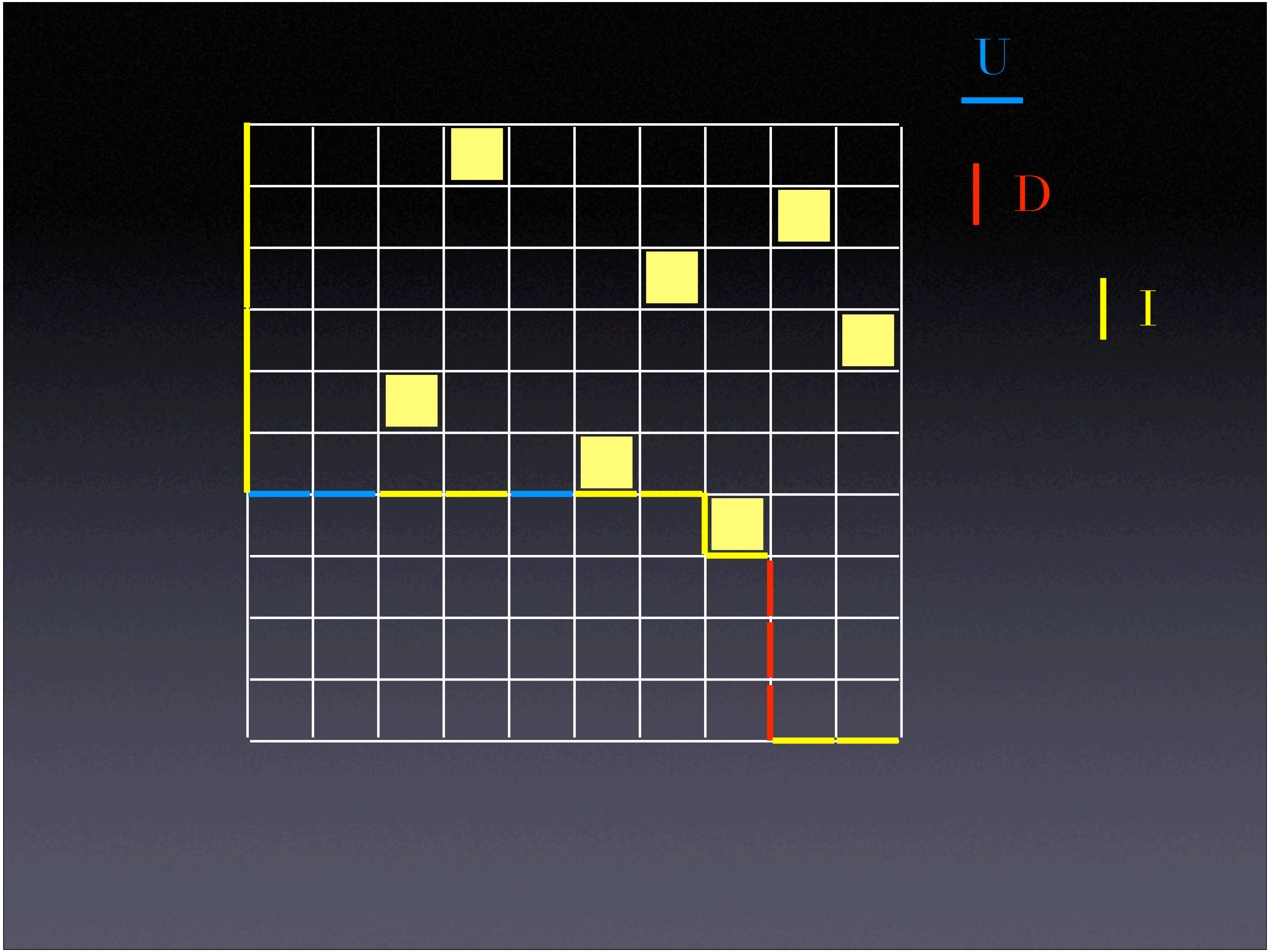
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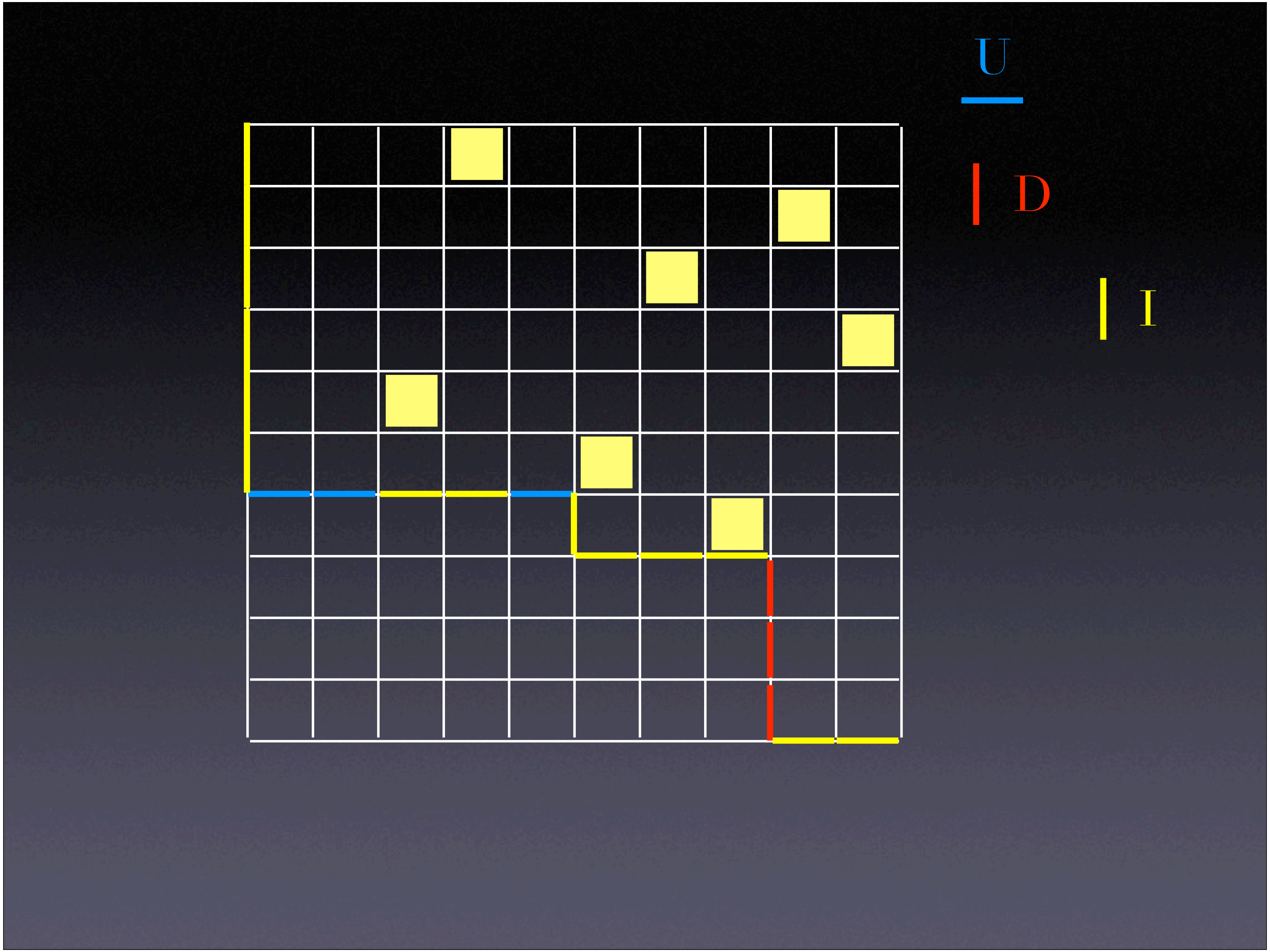
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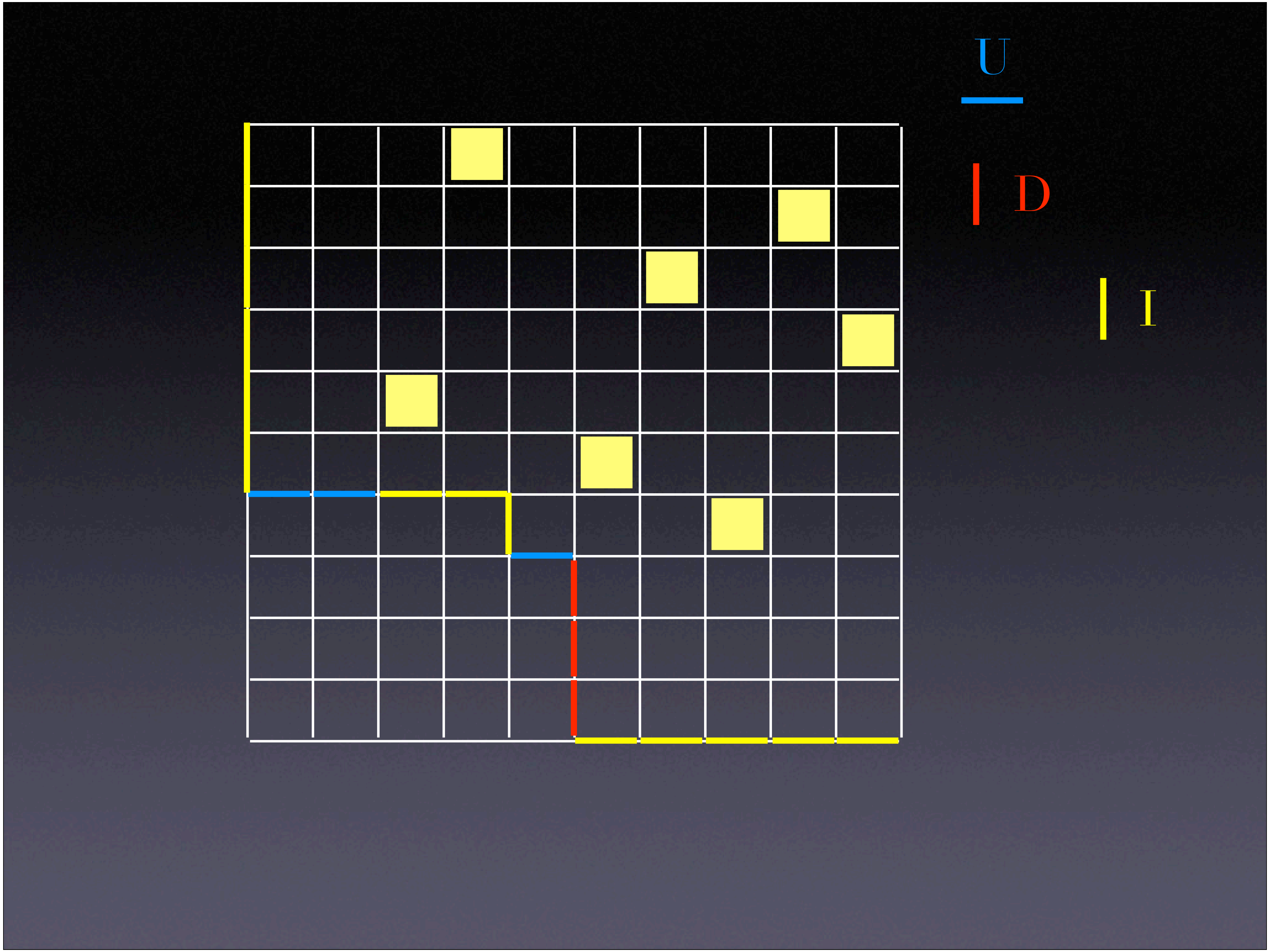
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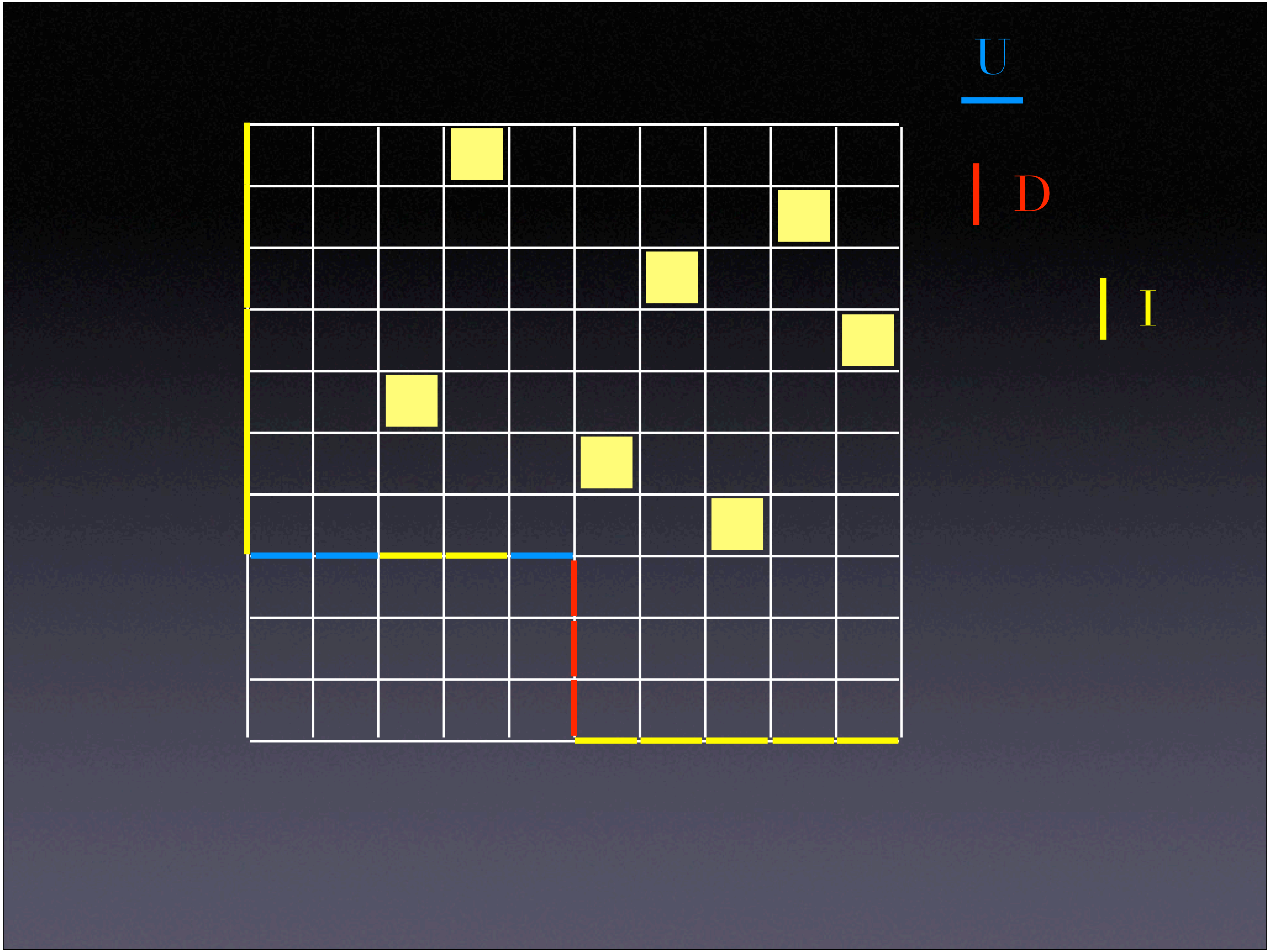
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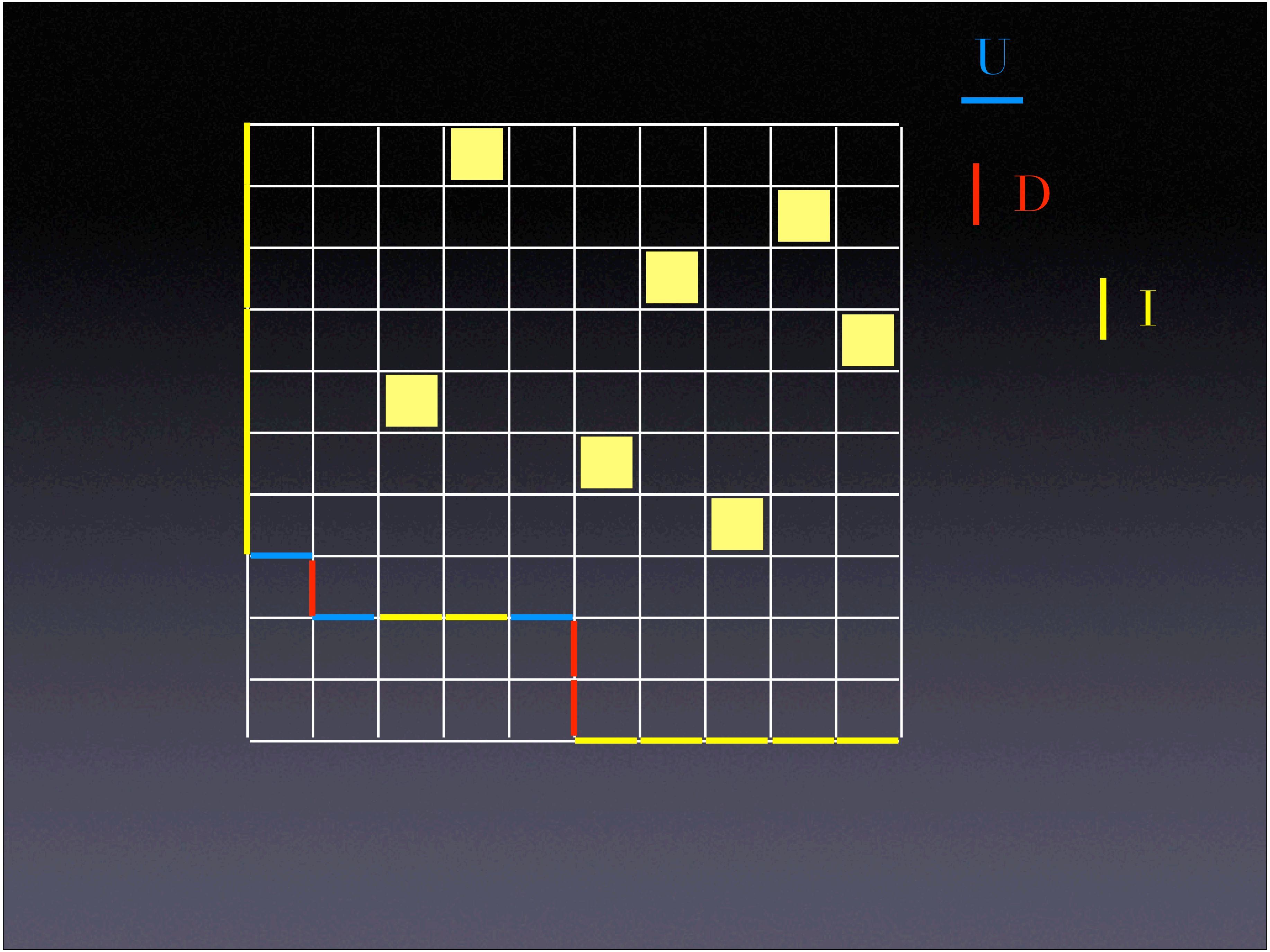
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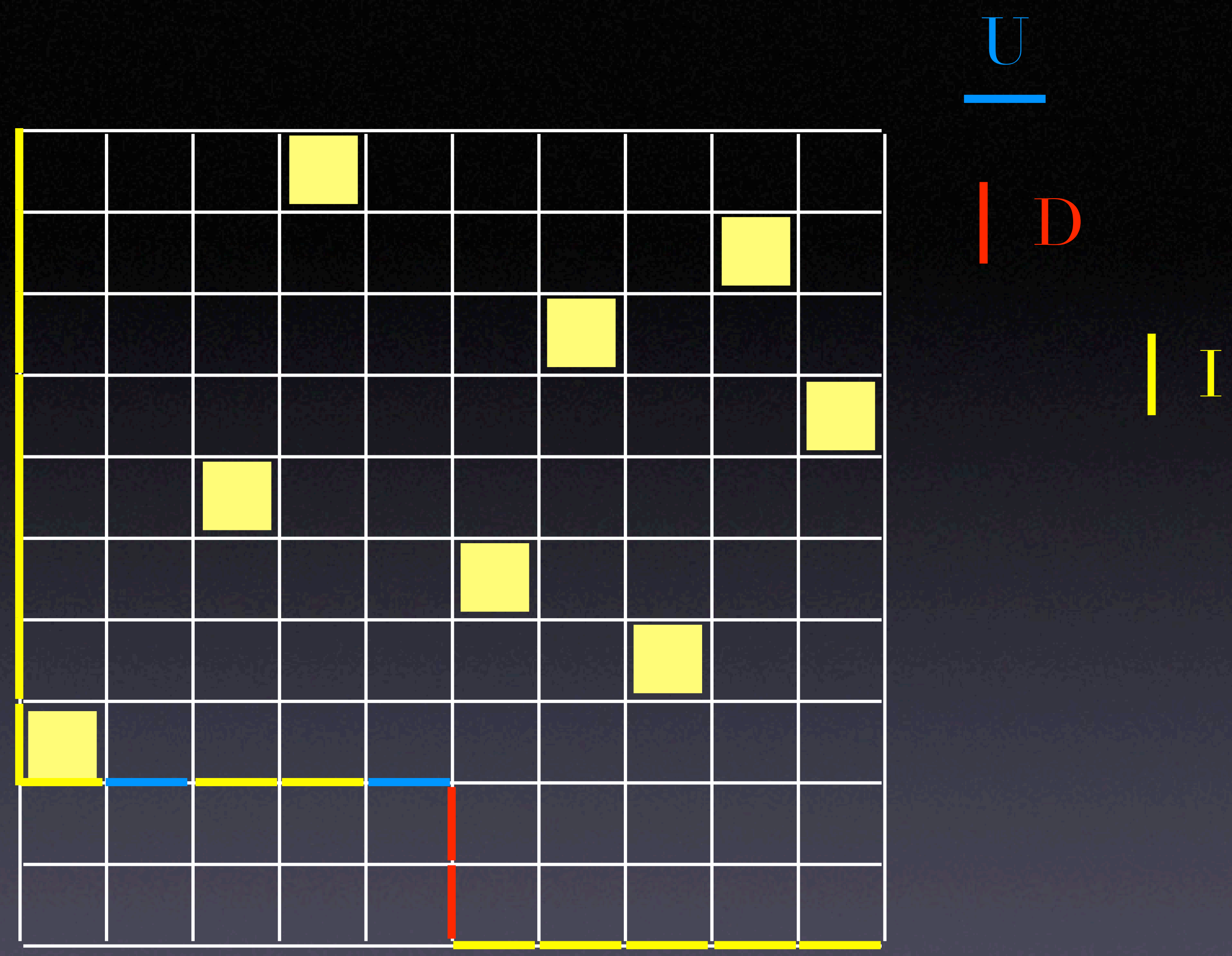

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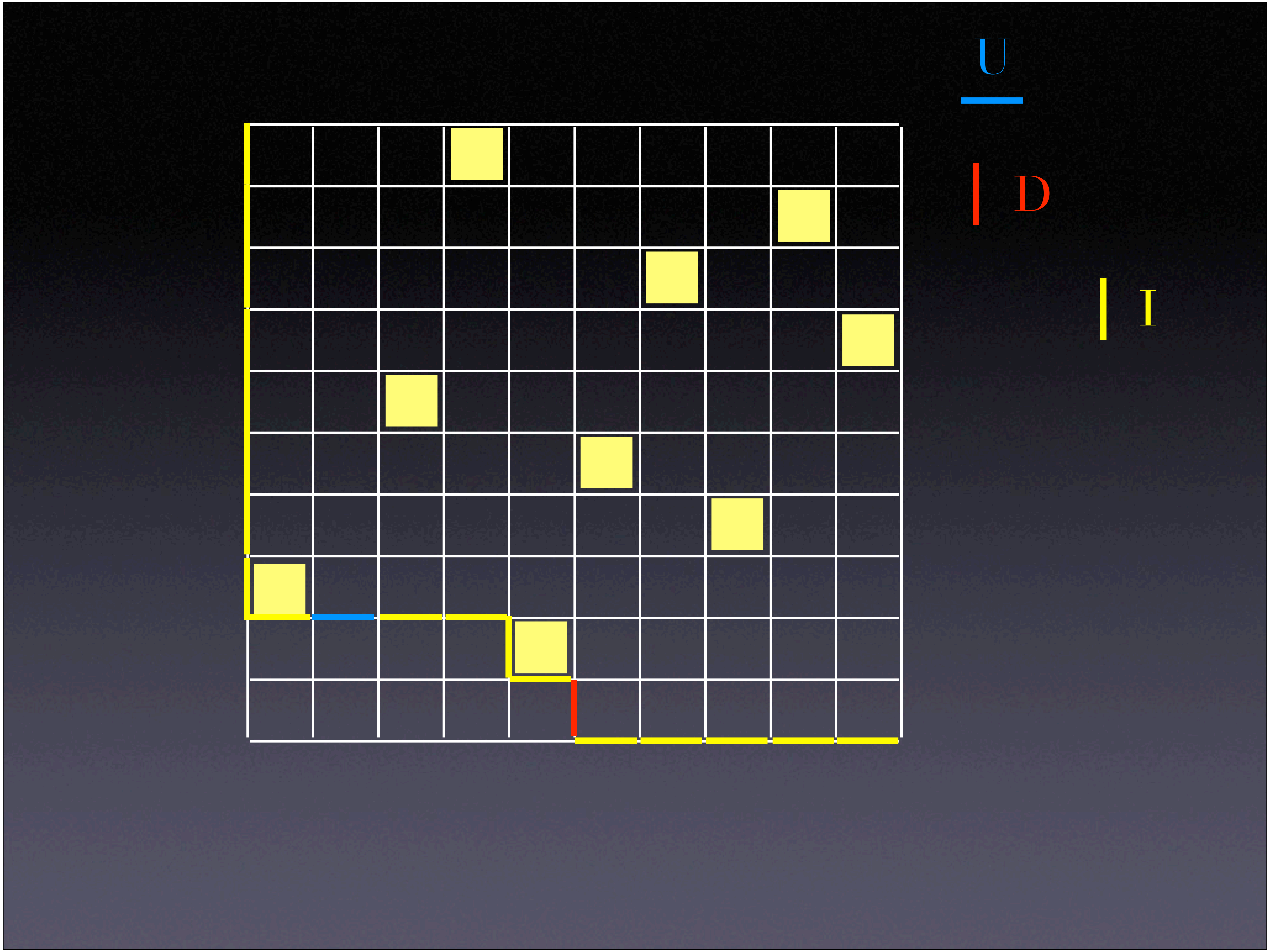

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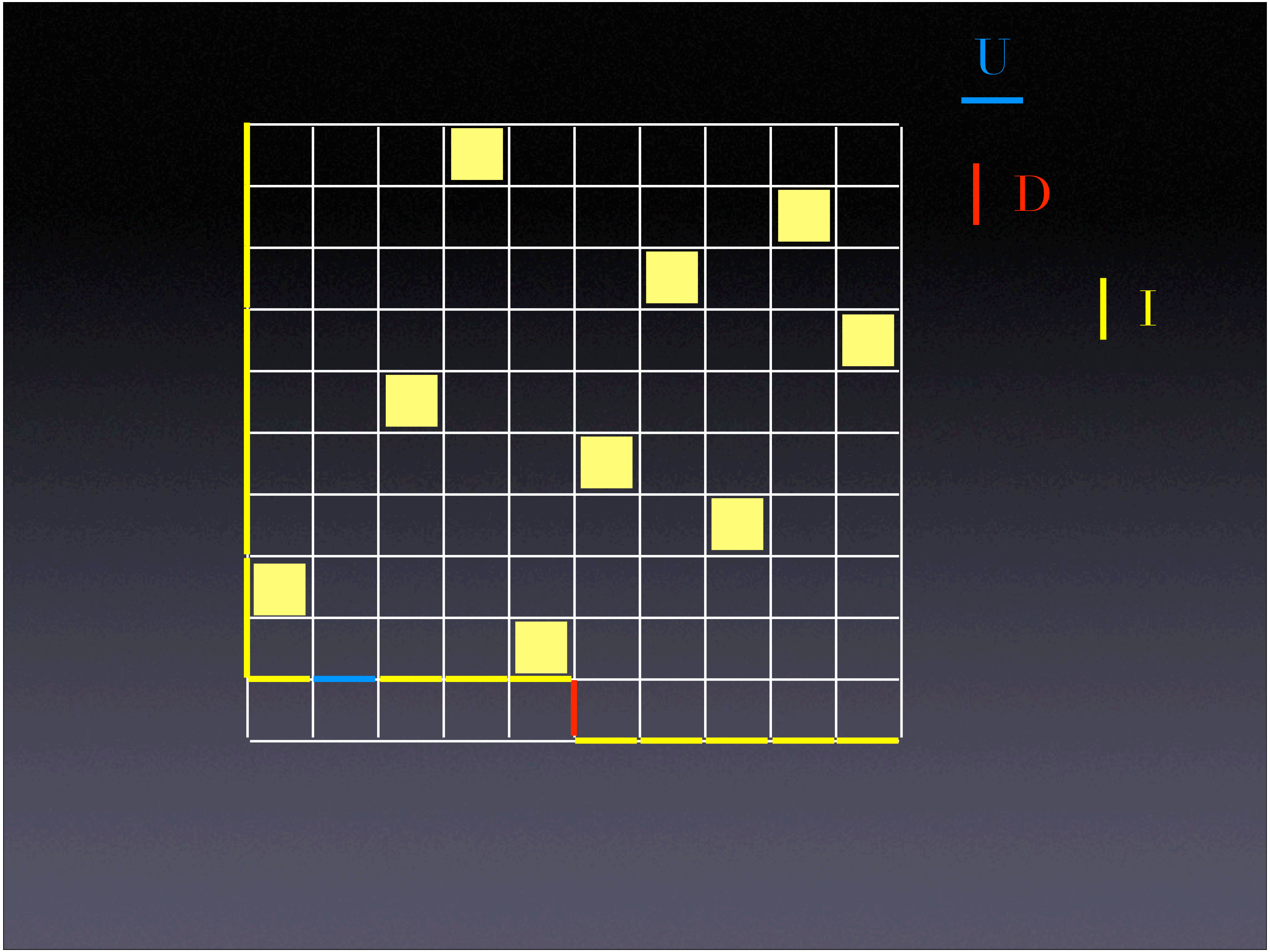
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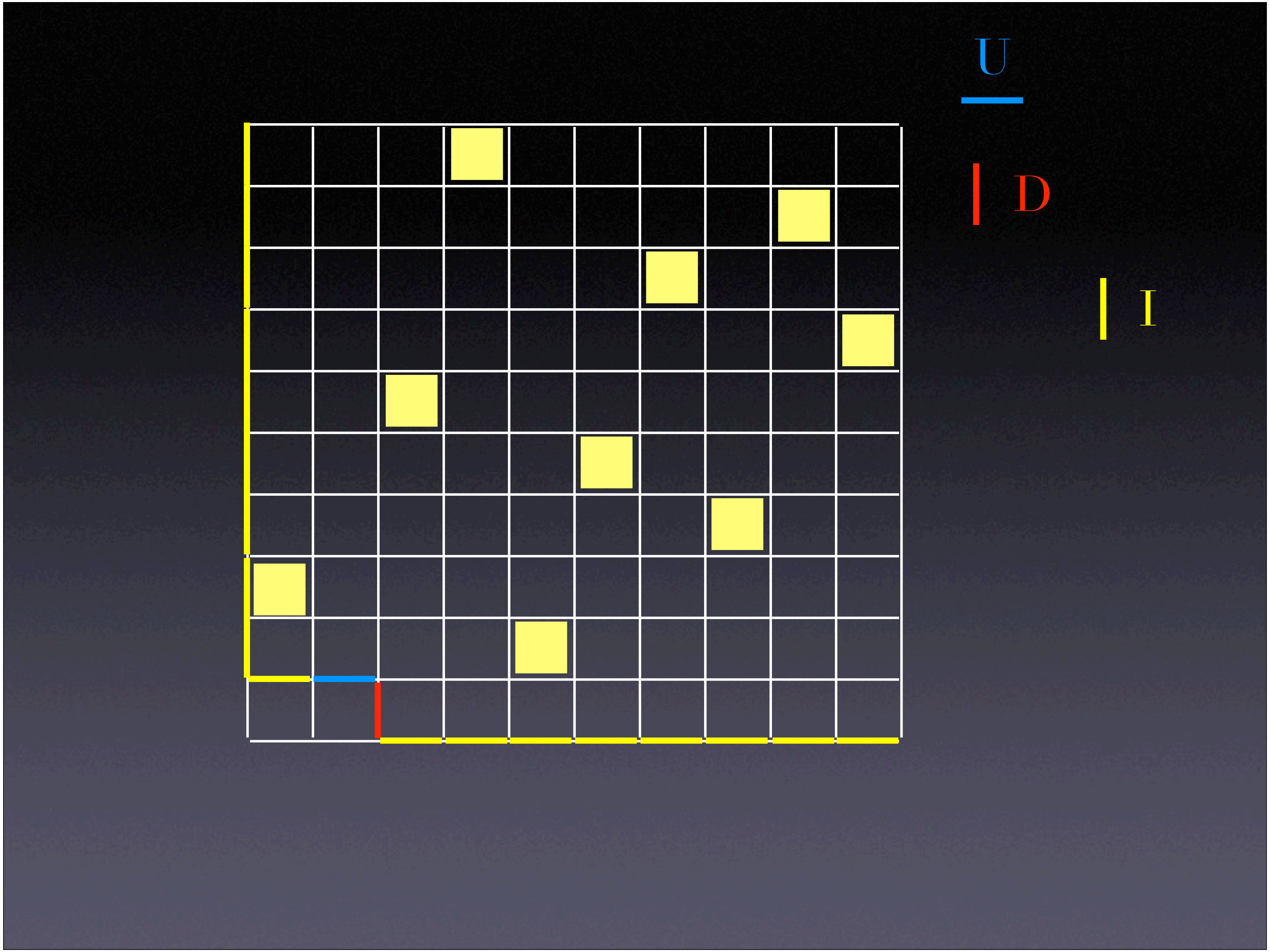
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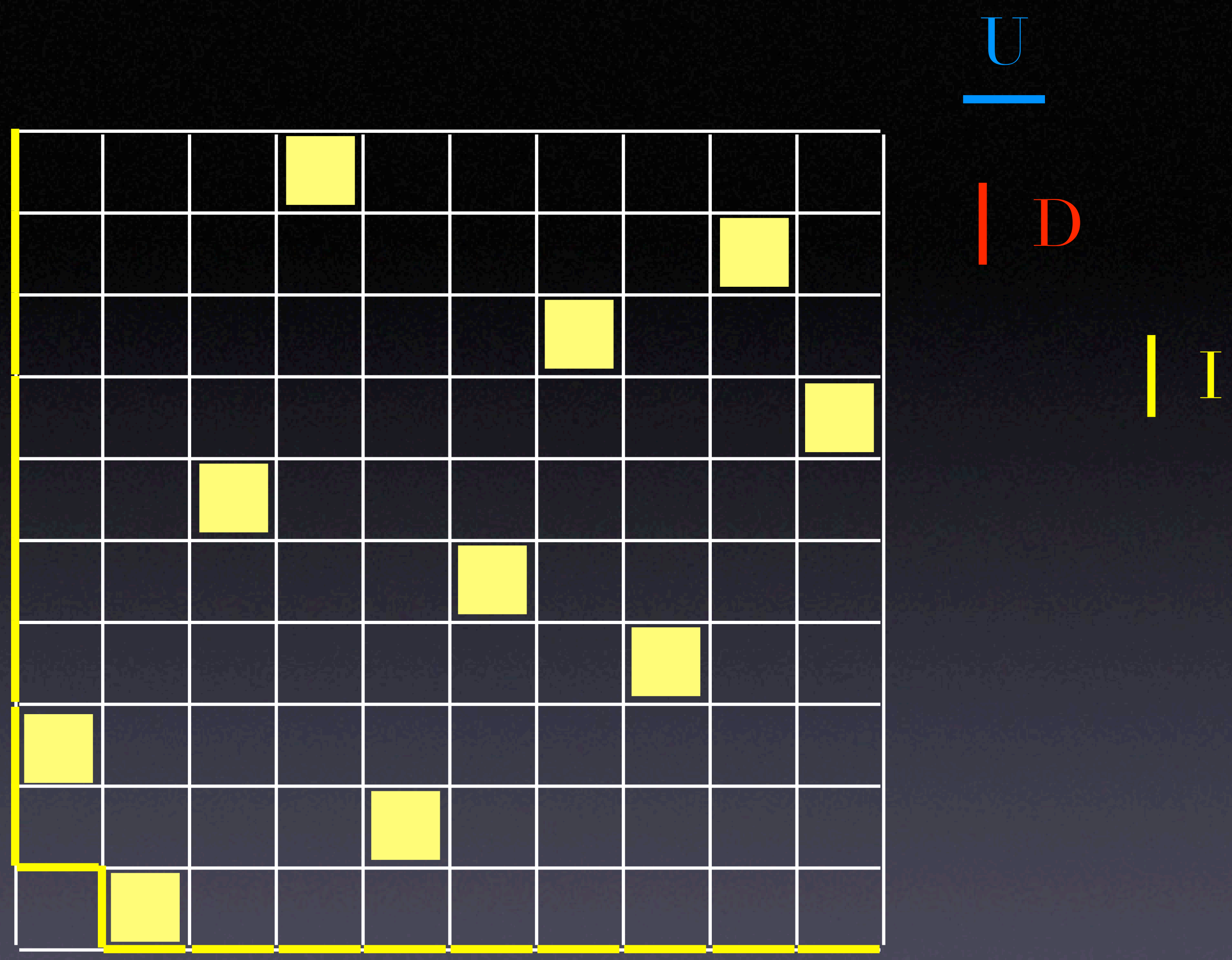
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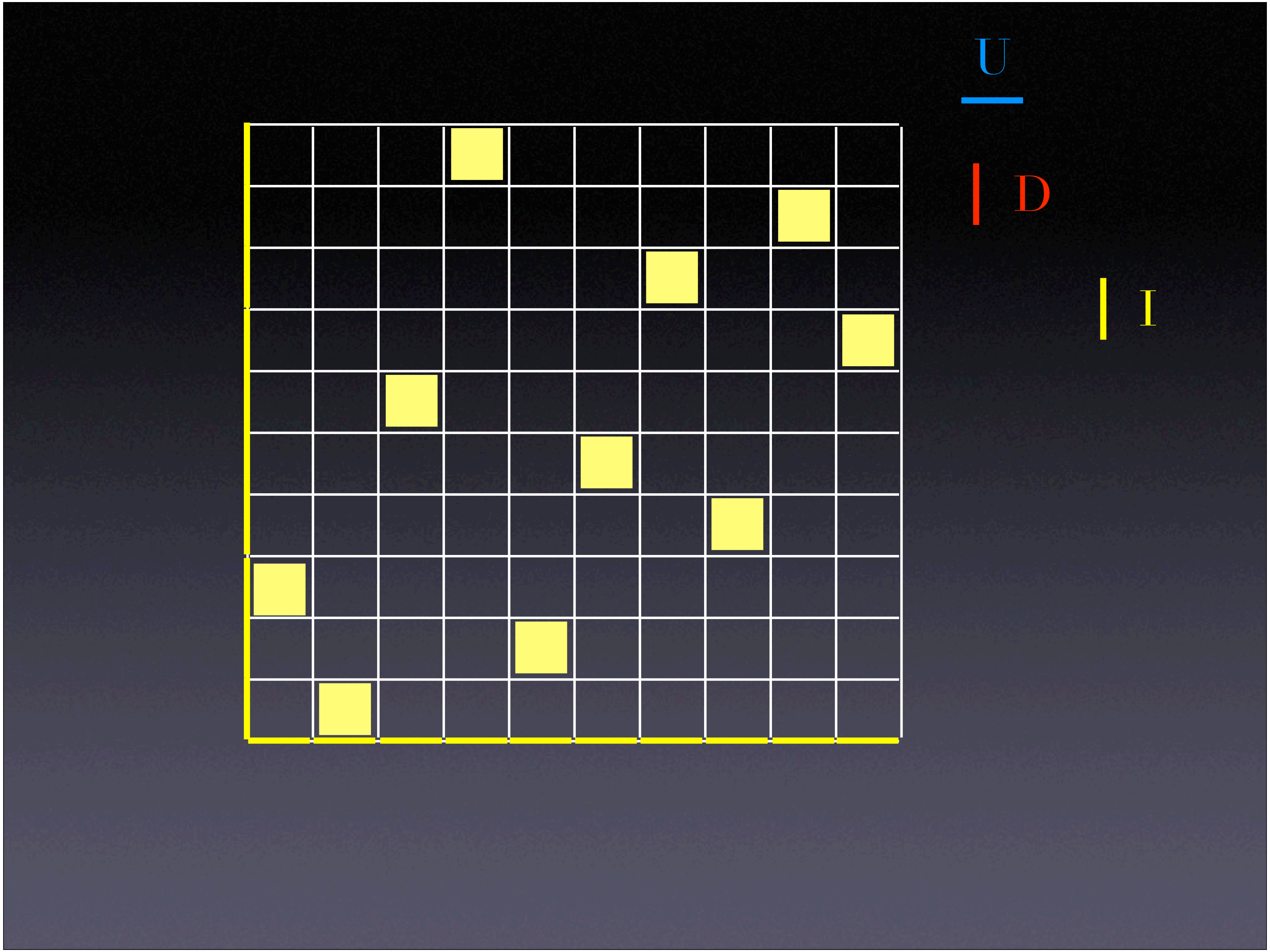
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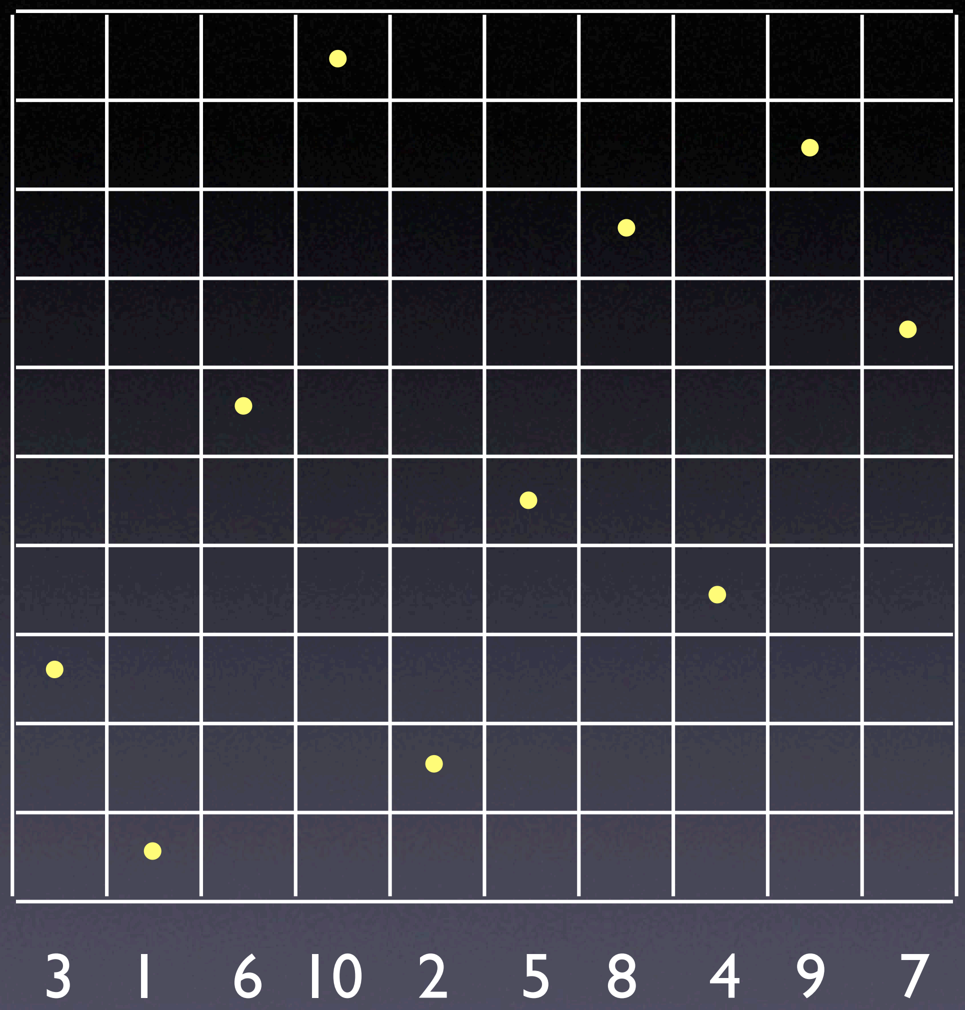
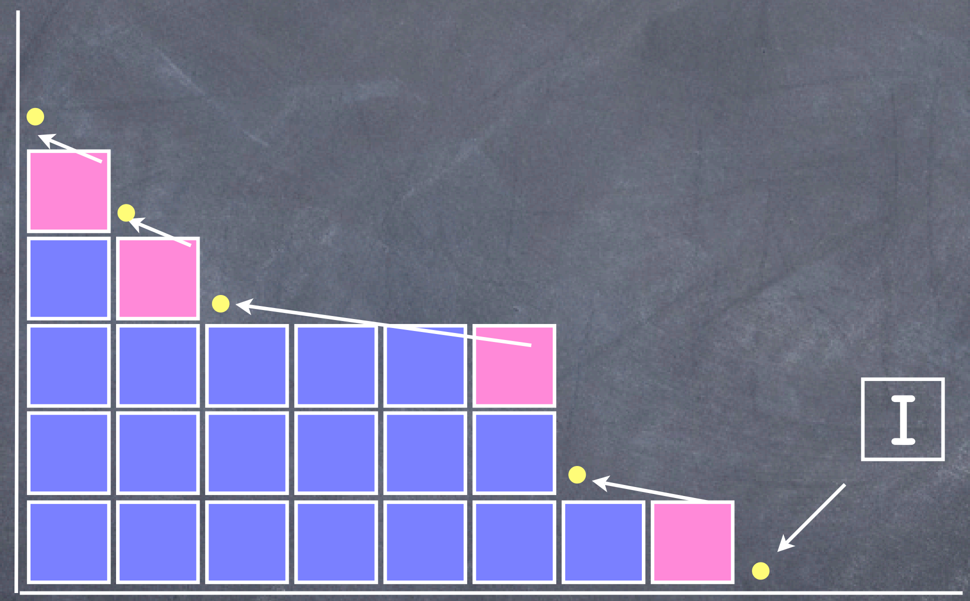
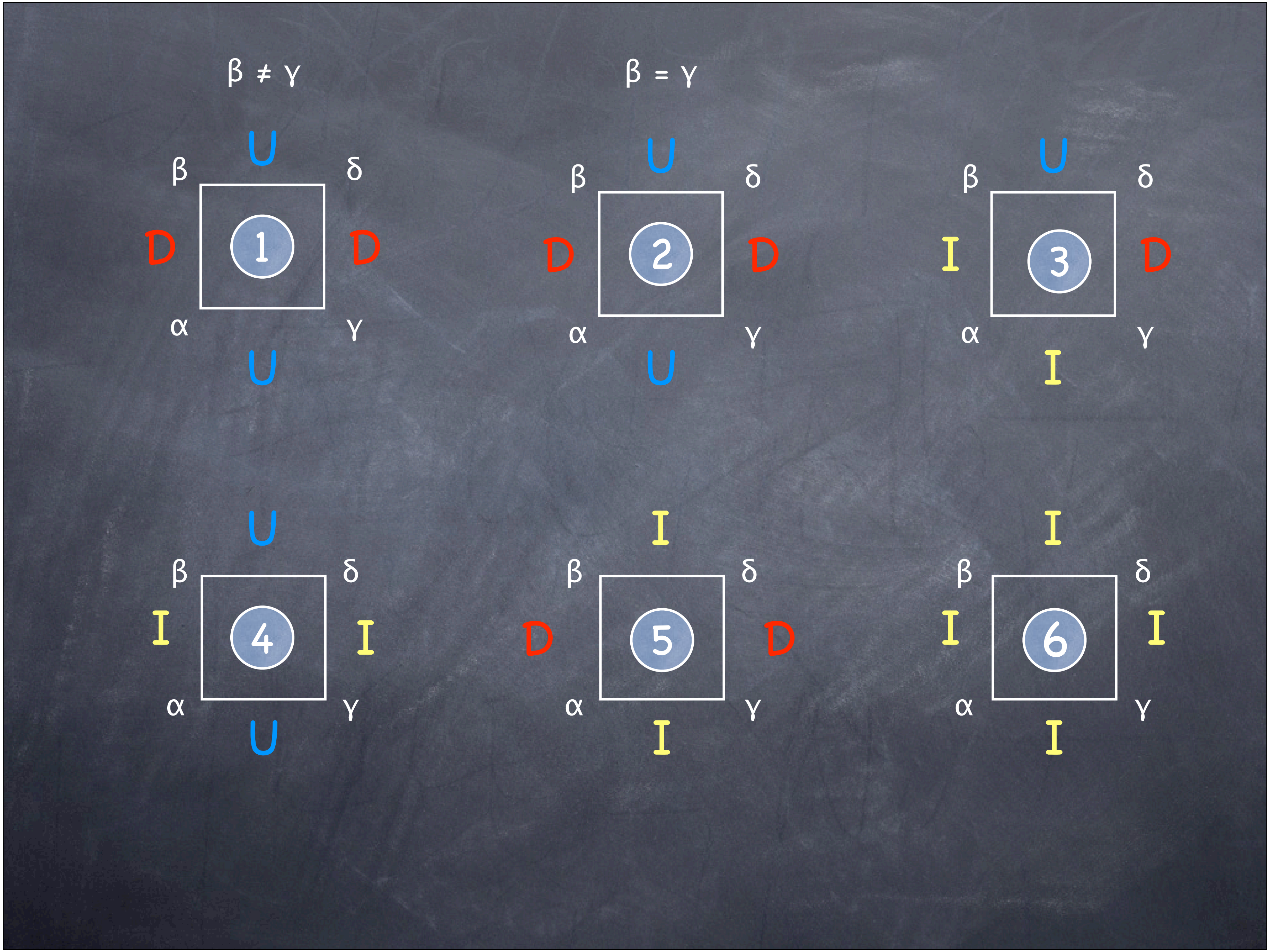

3 1 6 10 2 5 8 4 9 7

Un algrithme local sur une grille

$UD = DU + I$




β ≠ γ
β = γ
U
β
δ
D
1
D
α
γ
U
U
β
δ
D
2
D
α
γ
U
U
β
δ
I
3
D
α
γ
I
U
β
δ
I
4
I
α
γ
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I
β
δ
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α
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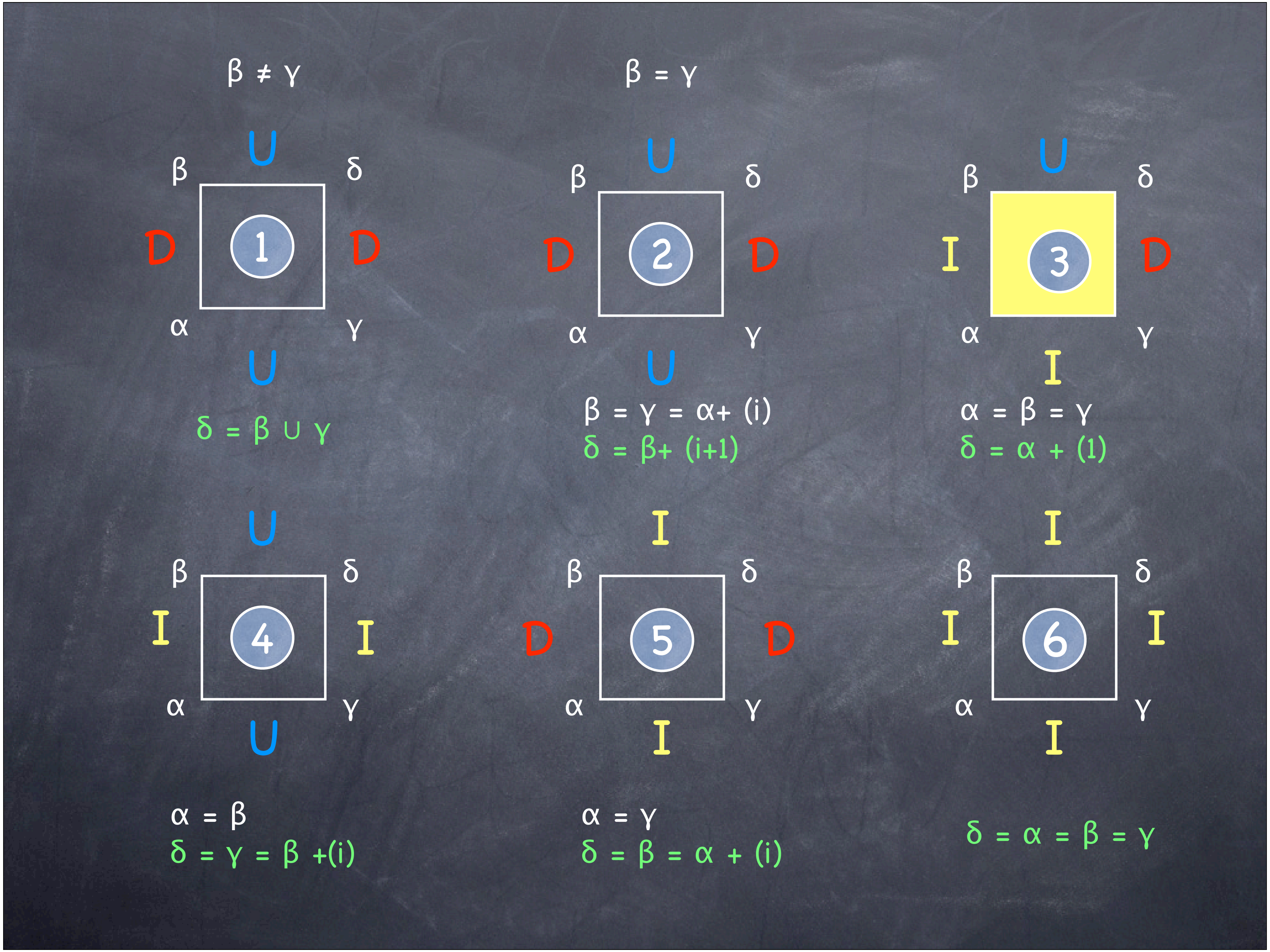

β ≠ γ
β = γ
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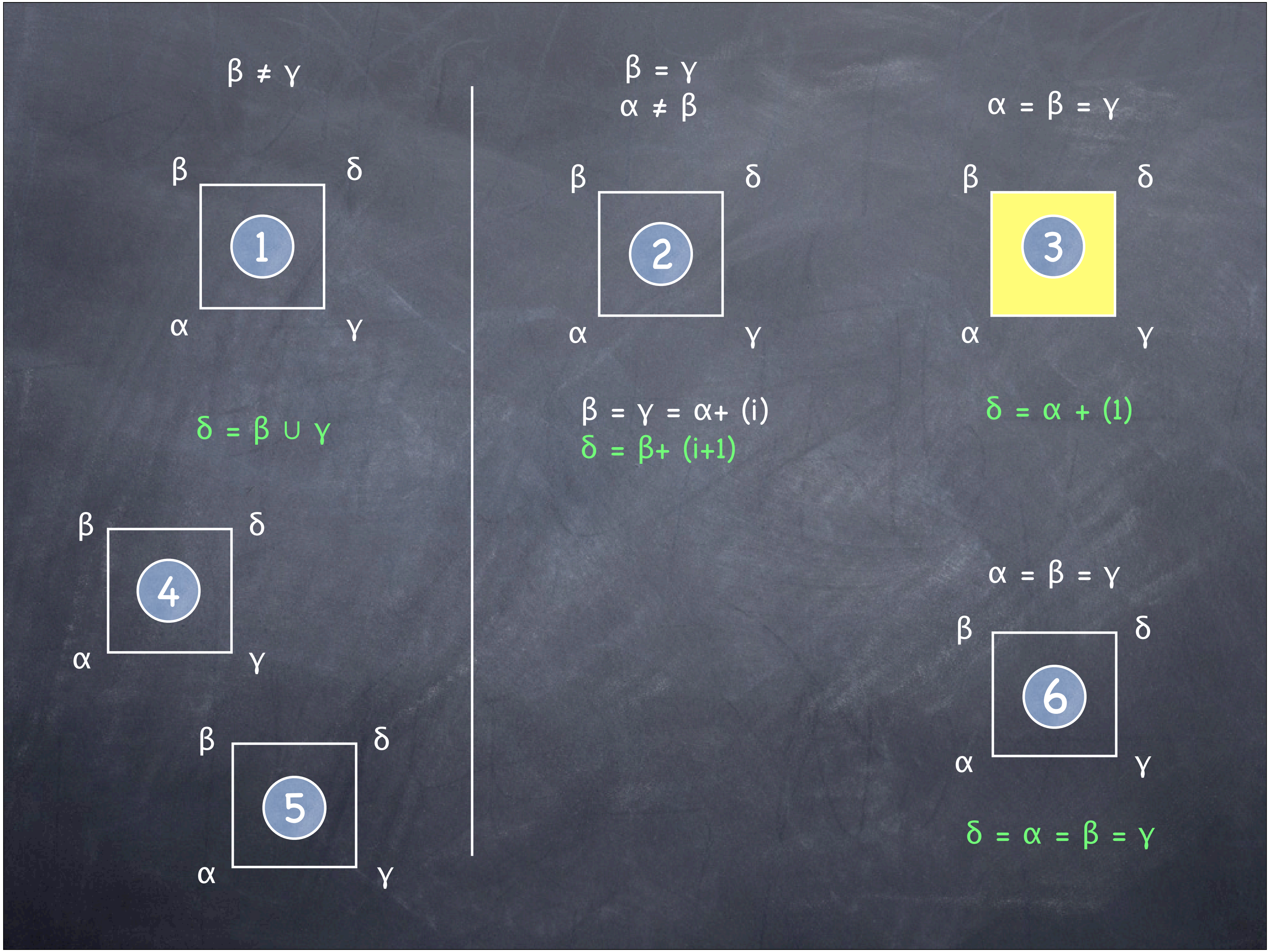
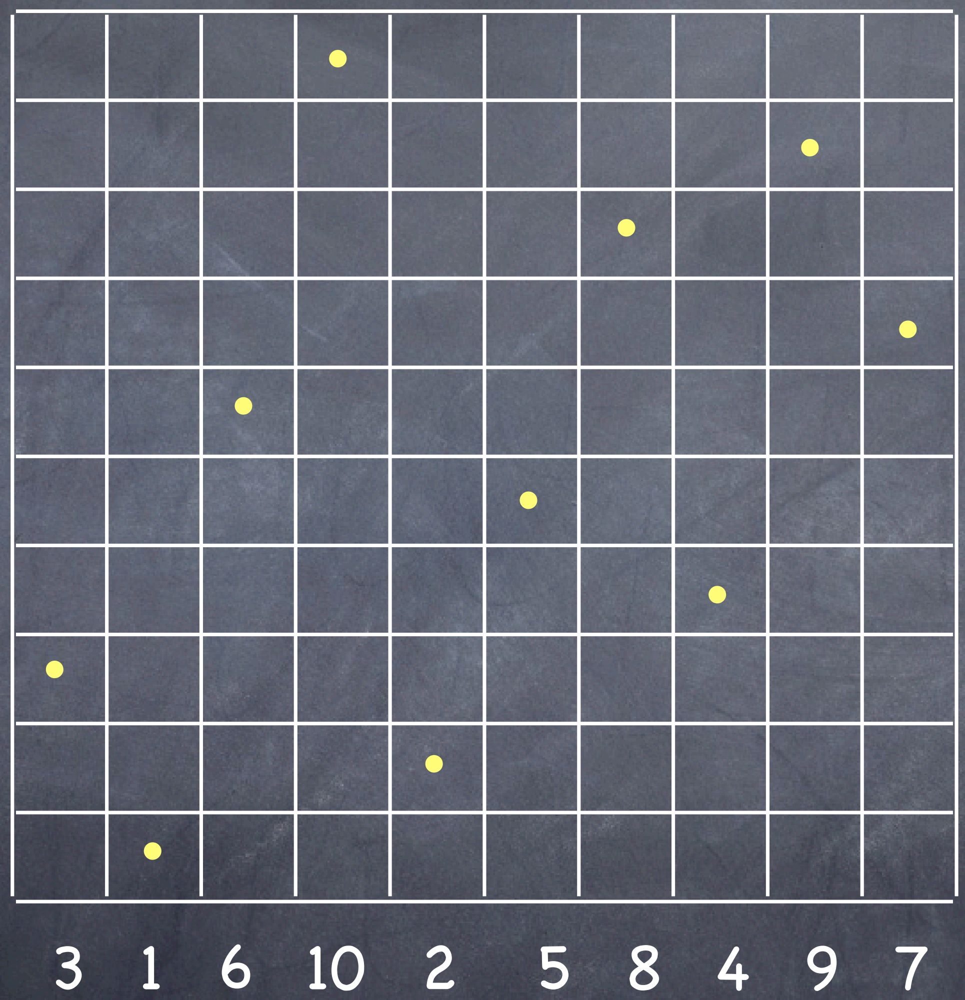
β δ
D 1 D
α γ
U
δ = β ∪ γ
U
β δ
D 2 D
α γ
U
β = γ = α+ (i)
δ = β+ (i+1)
U
β δ
I 3 D
α γ
I
α = β = γ
δ = α + (1)
U
β δ
I 4 I
α γ
U
α = β
δ = γ = β +(i)
I
β δ
D 5 D
α γ
I
α = γ
δ = β = α + (i)
I
β δ
I 6 I
α γ
I
δ = α = β = γ


β ≠ γ
β δ
1
α γ
δ = β ∪ γ
β δ
4
α γ
β δ
5
α γ
β = γ
α ≠ β
β δ
2
α γ
β = γ = α+ (i)
δ = β+ (i+1)
α = β = γ
β δ
3
α γ
δ = α + (1)
α = β = γ
β δ
6
α γ
δ = α = β = γ


3 1 6 10 2 5 8 4 9 7

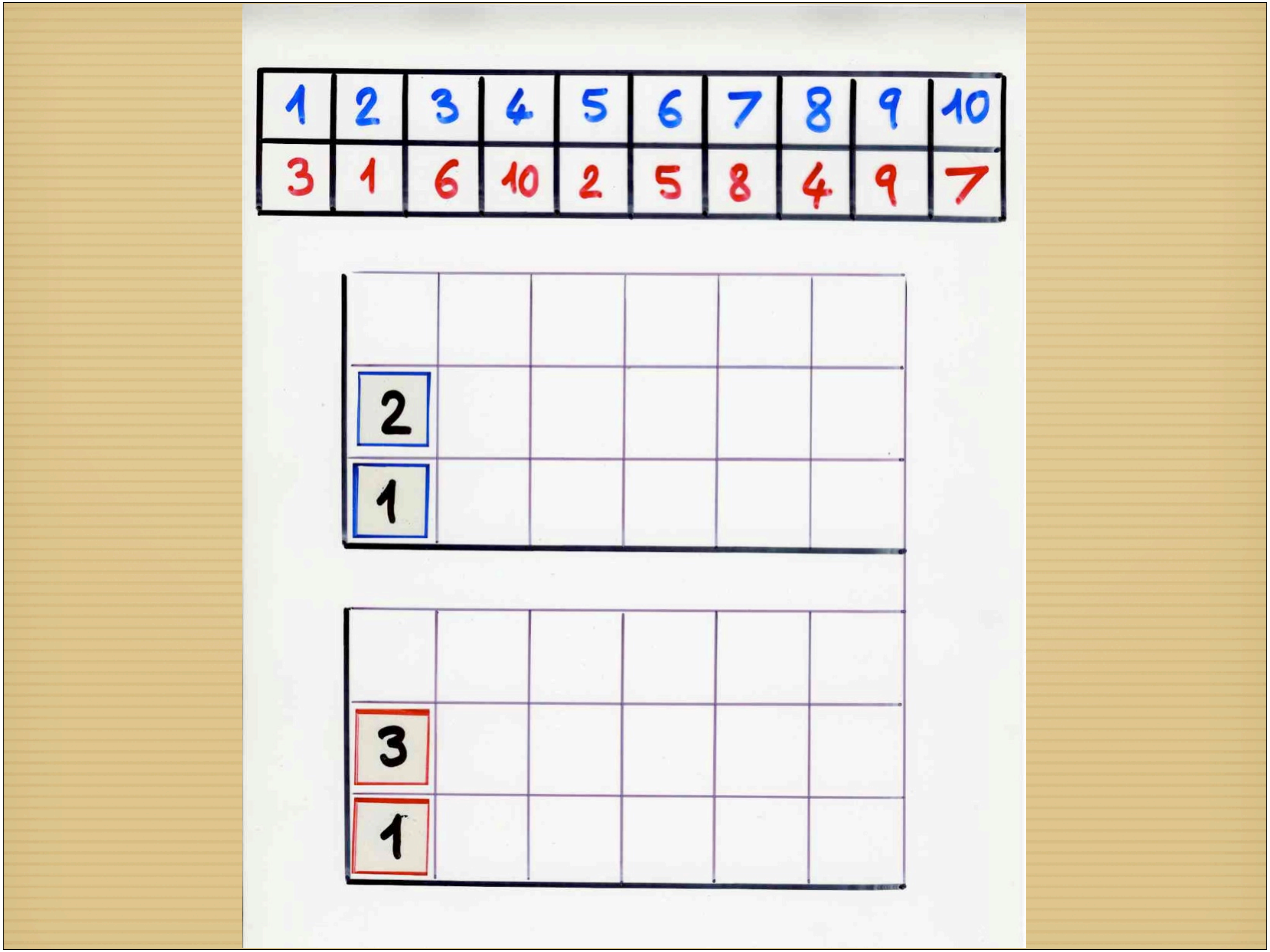
RSK avec les insertions de Schensted

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

1					

3					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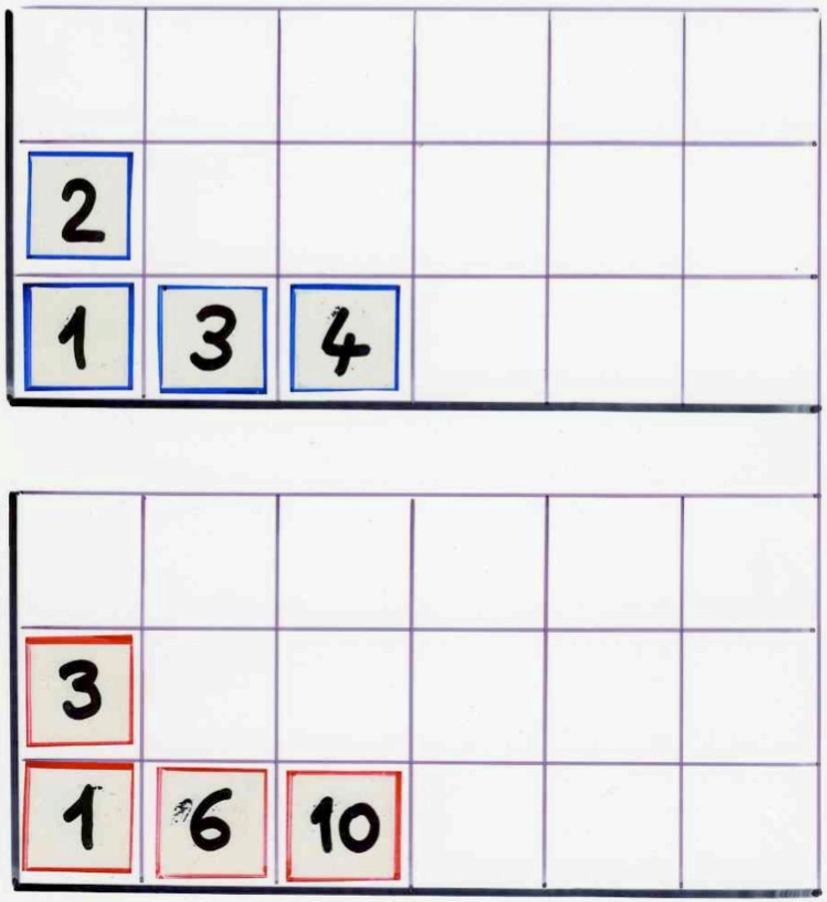
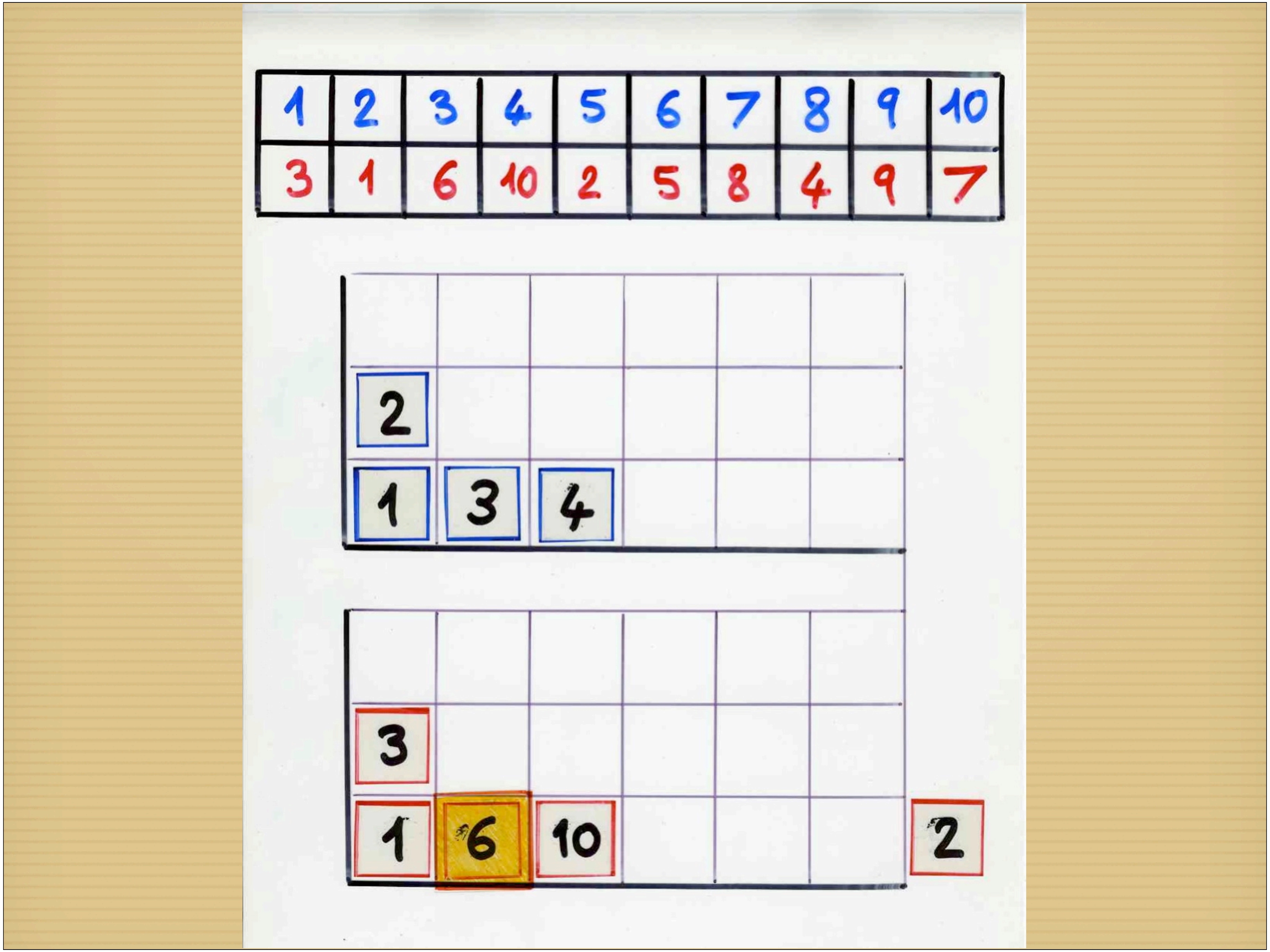
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

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3 1 6 10 2 5 8 4 9 7
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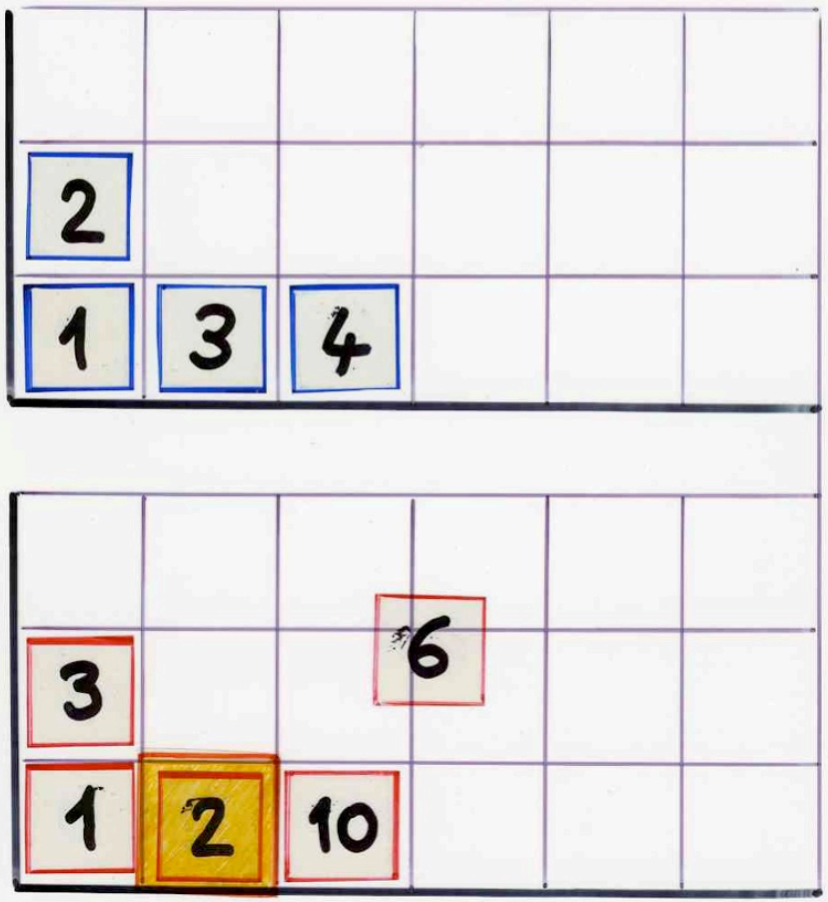
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



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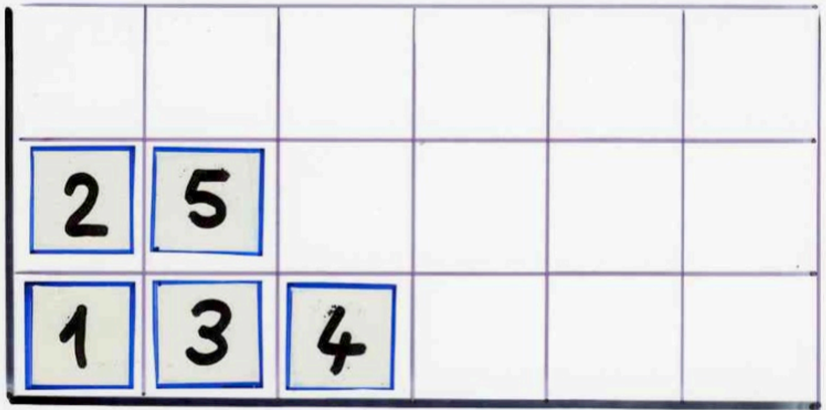
3	1	6	10	2	5	8	4	9	7



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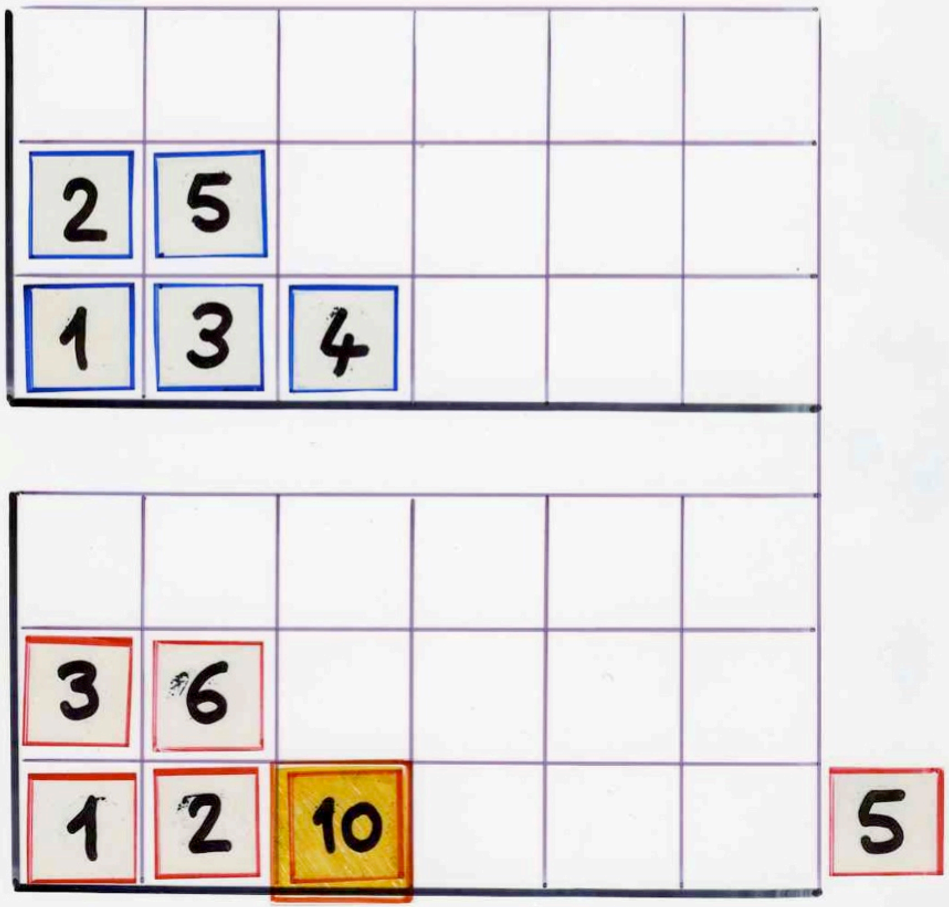

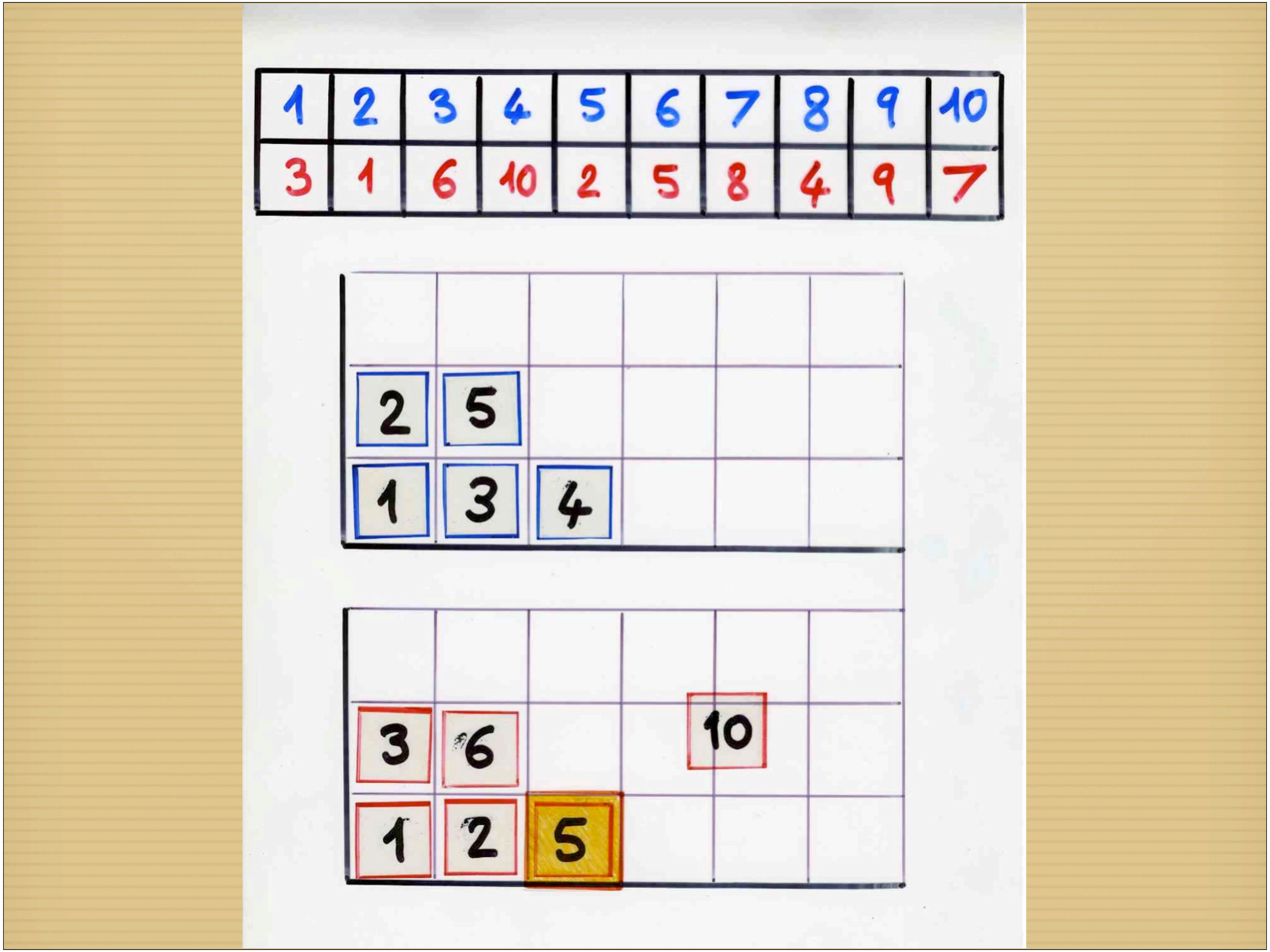
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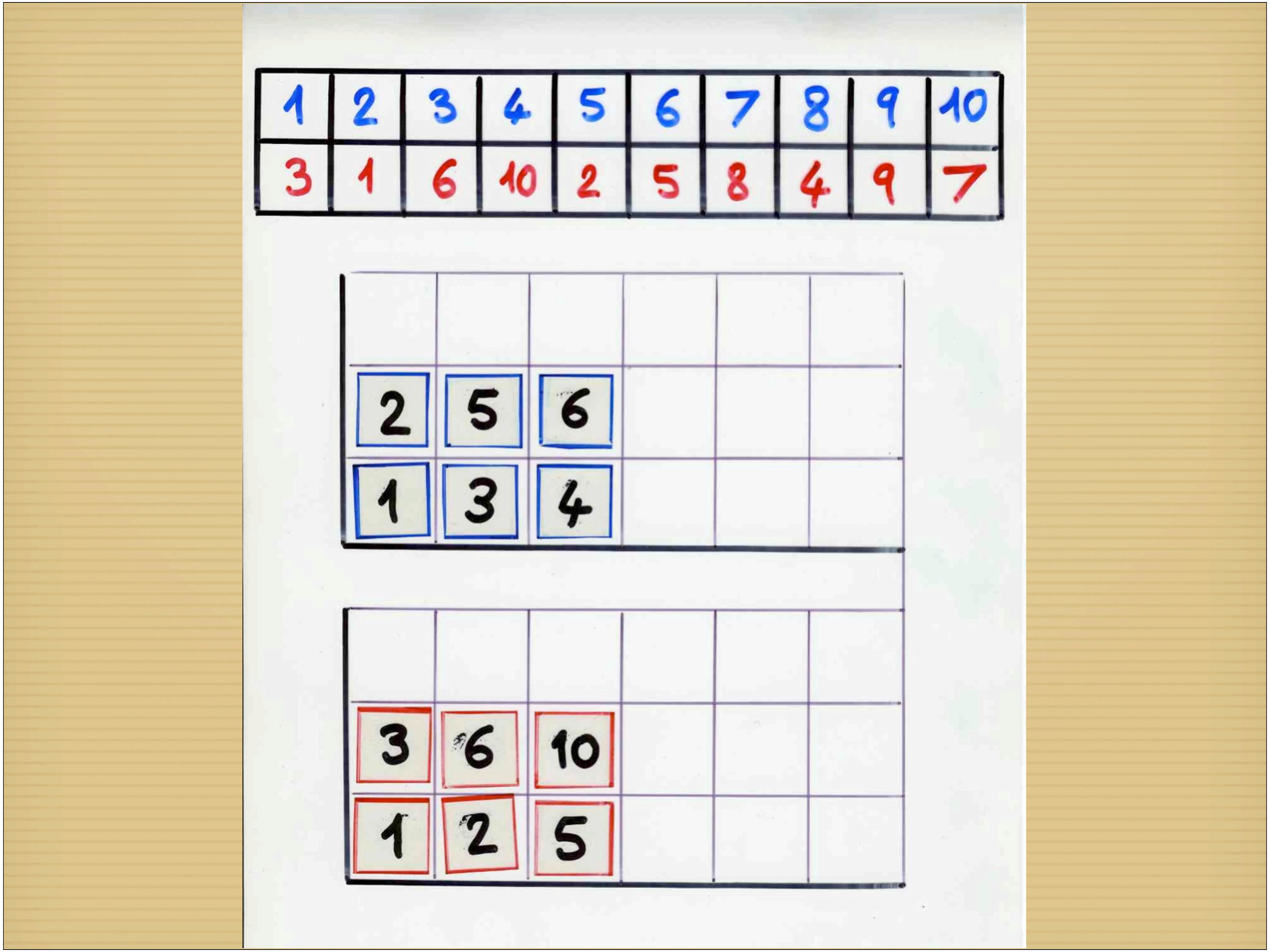
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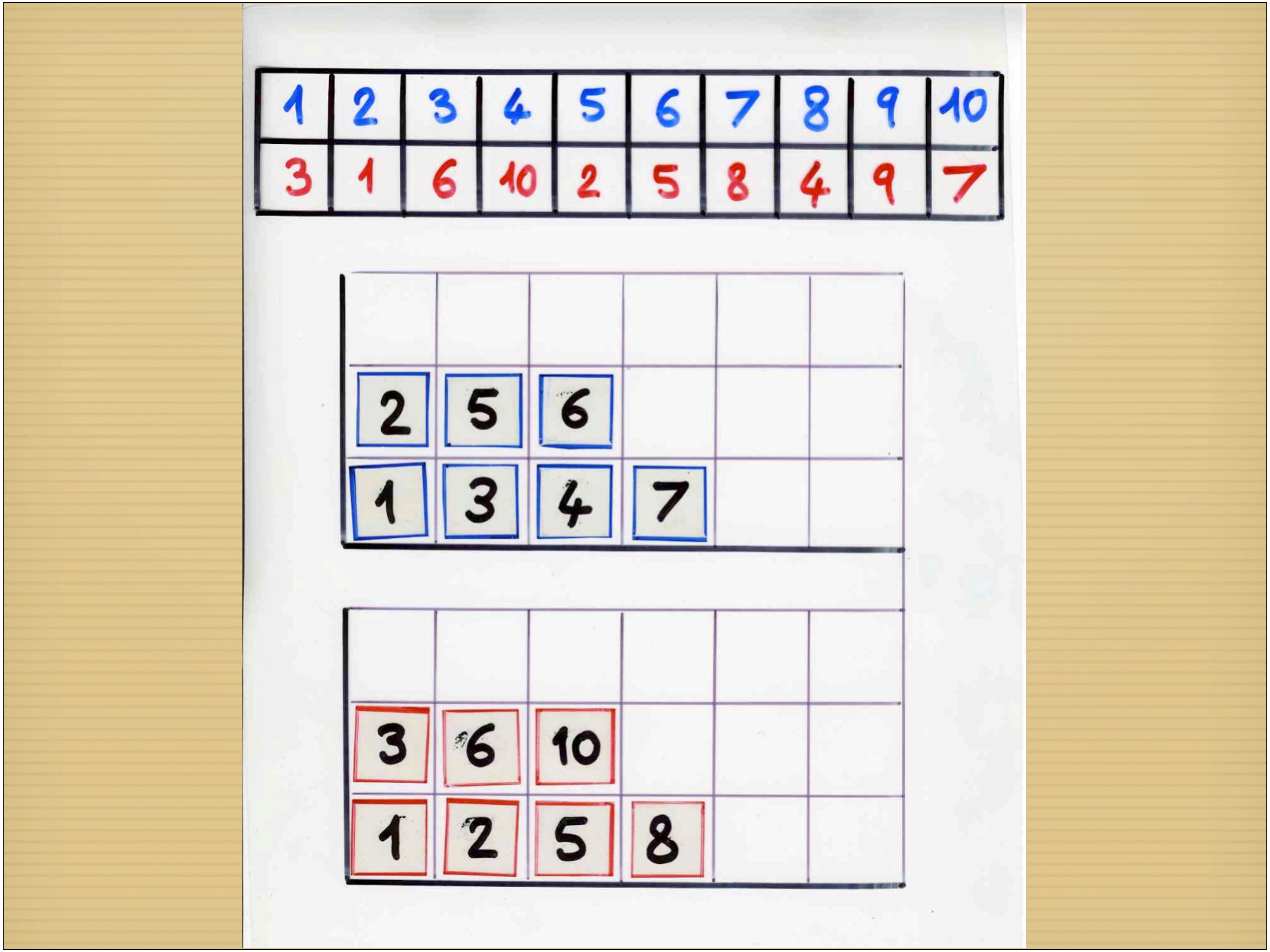
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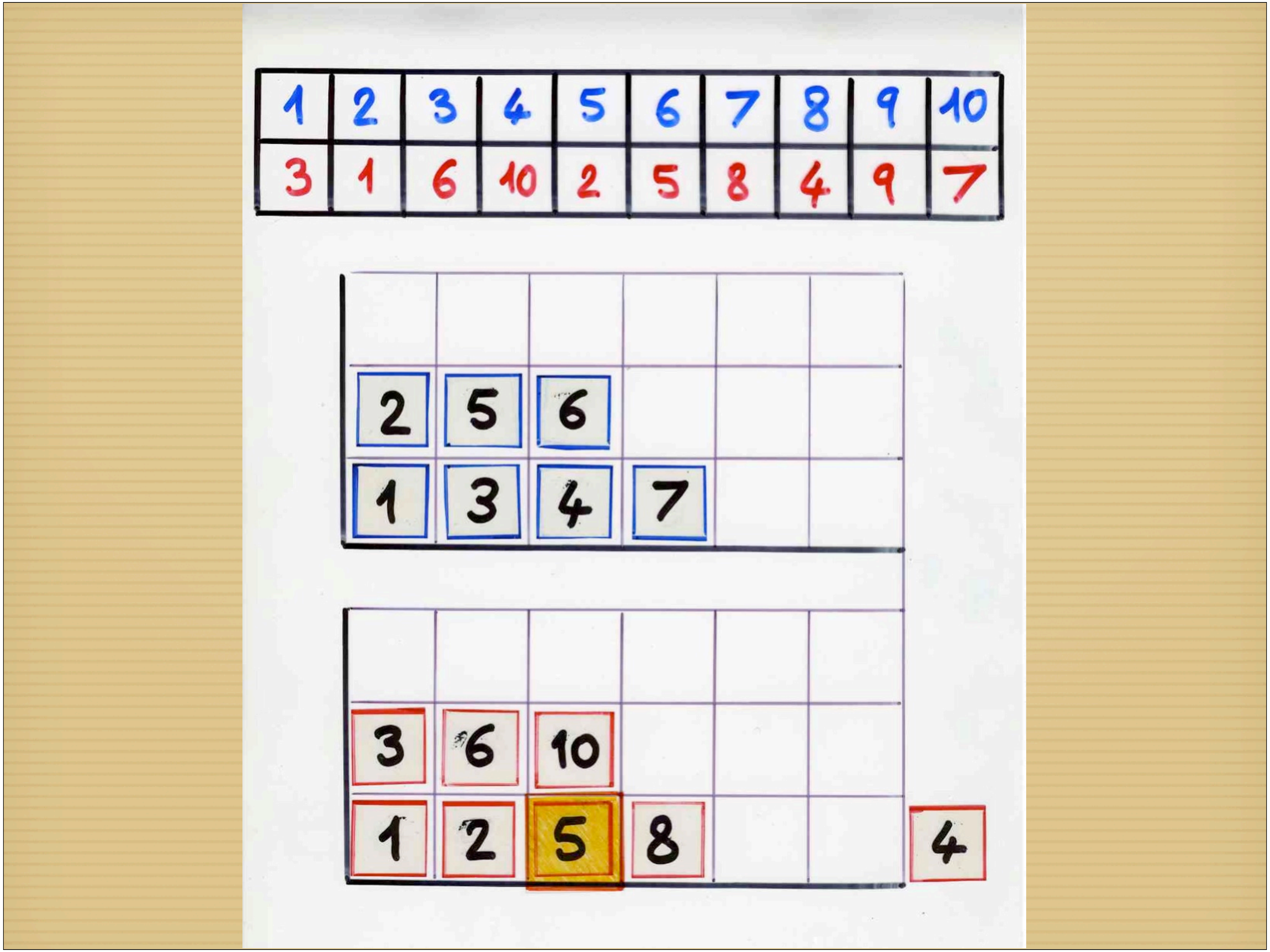
1 2 3 4 5 6 7 8 9 10
3 1 6 10 2 5 8 4 9 7
2 5 6
1 3 4 7
3 6 10
1 2 5 8
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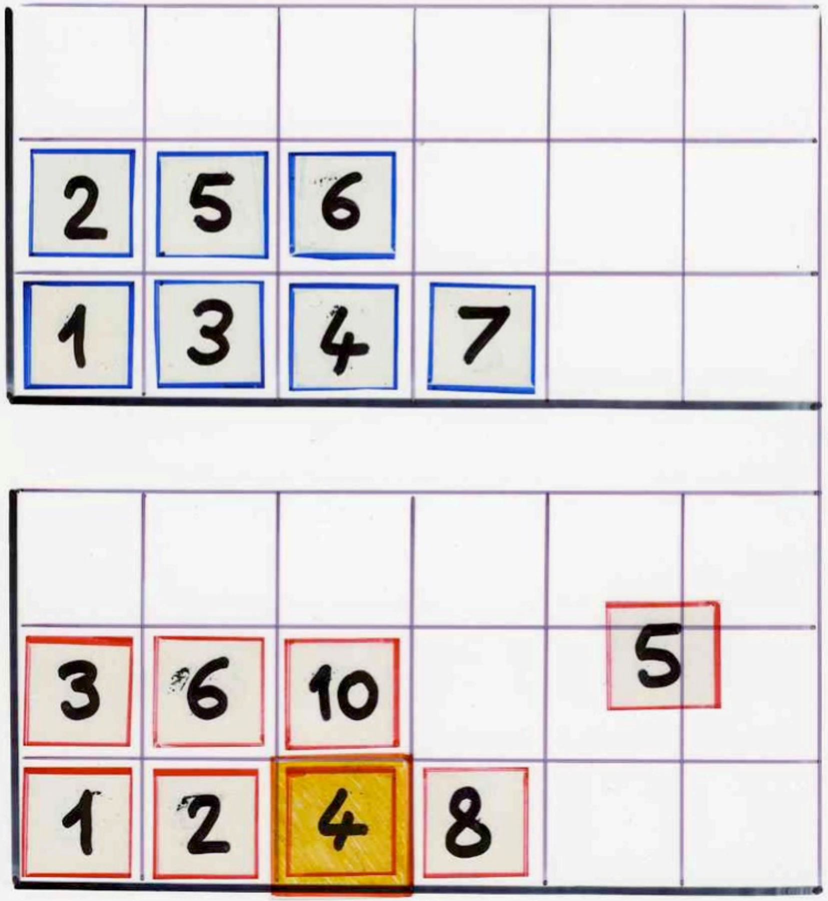
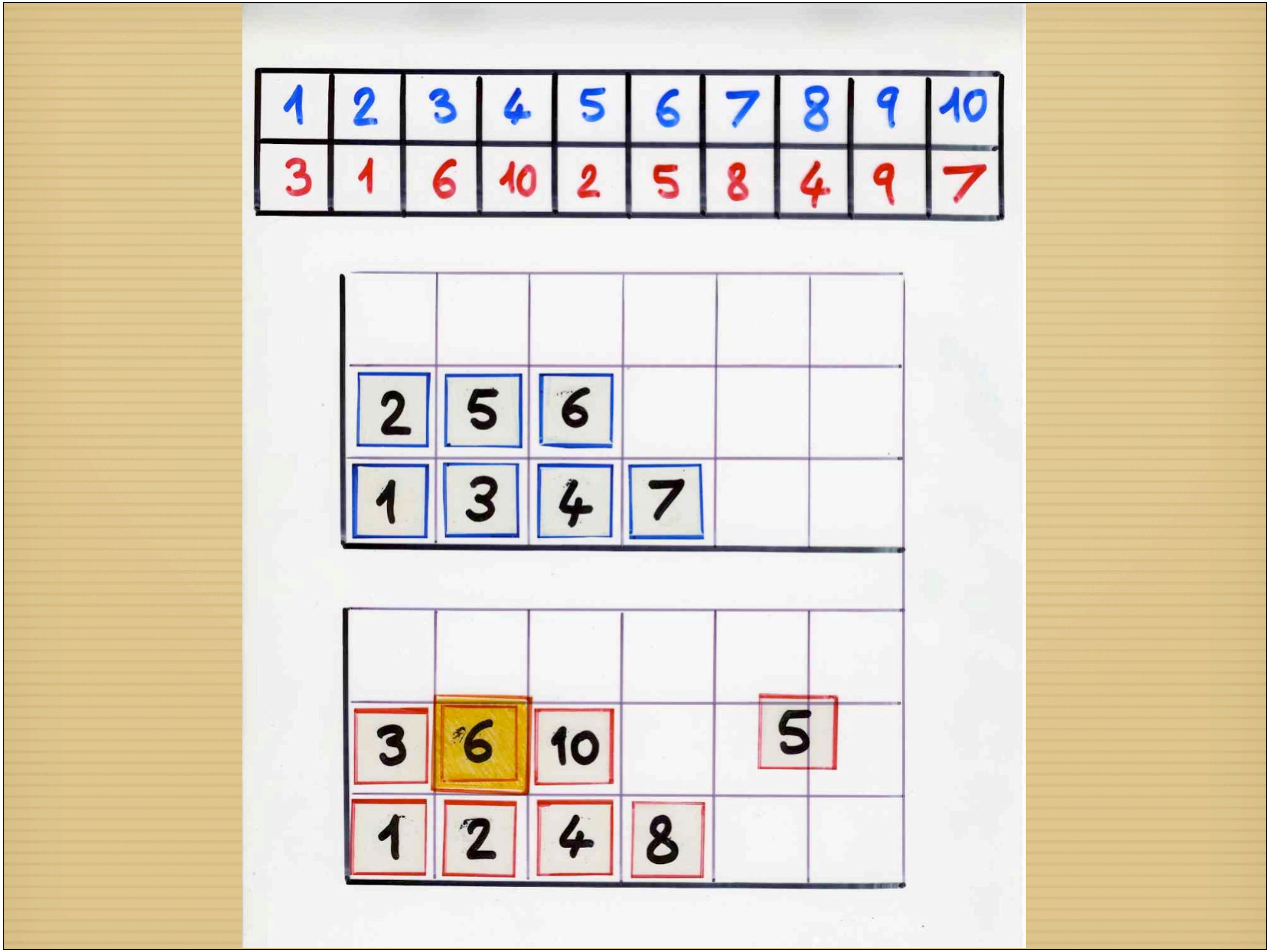
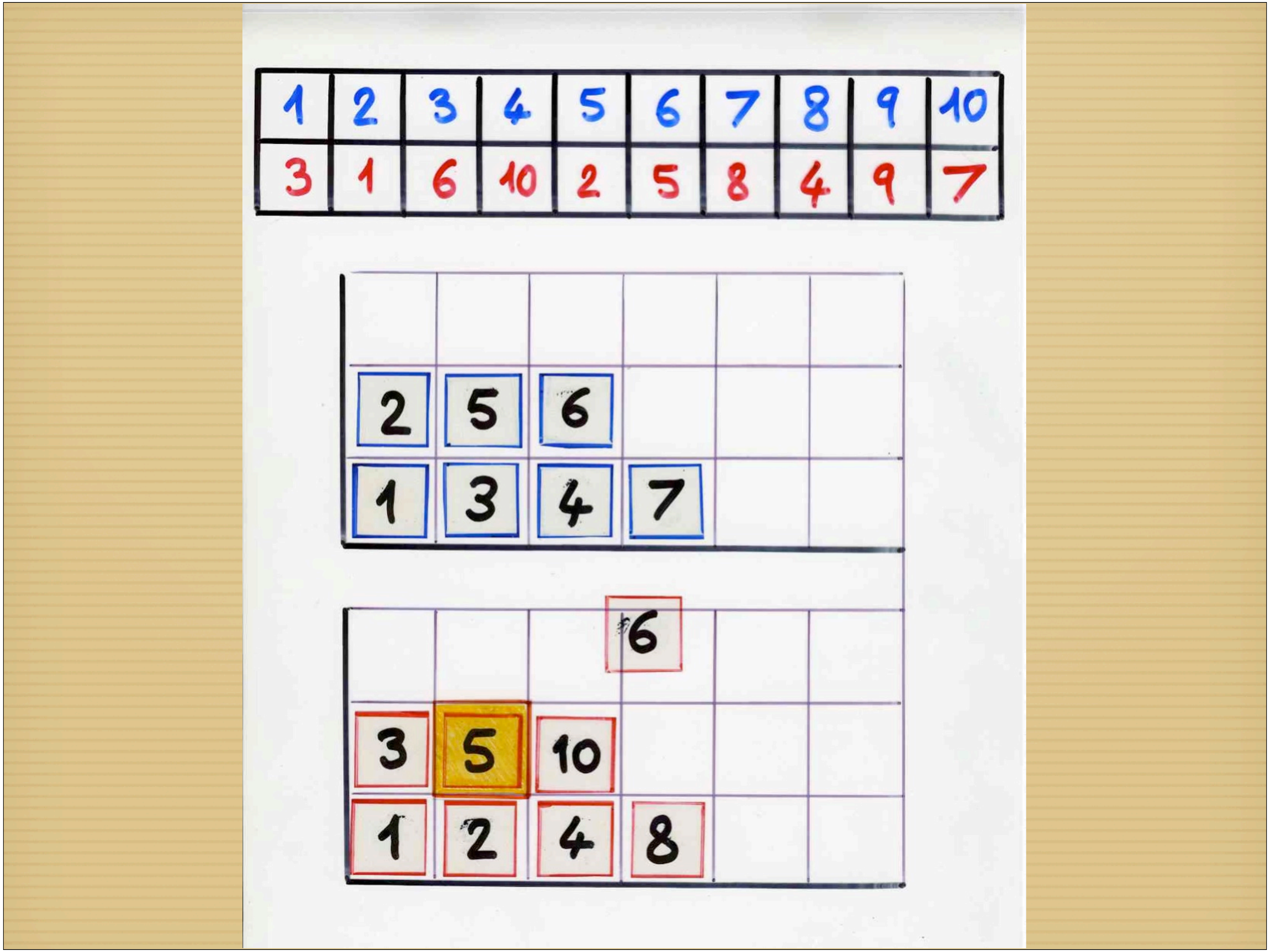
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7




1 2 3 4 5 6 7 8 9 10
3 1 6 10 2 5 8 4 9 7
2 5 6
1 3 4 7
3 6 10 5
1 2 4 8


1 2 3 4 5 6 7 8 9 10
3 1 6 10 2 5 8 4 9 7
2 5 6
1 3 4 7
6
3 5 10
1 2 4 8

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7		

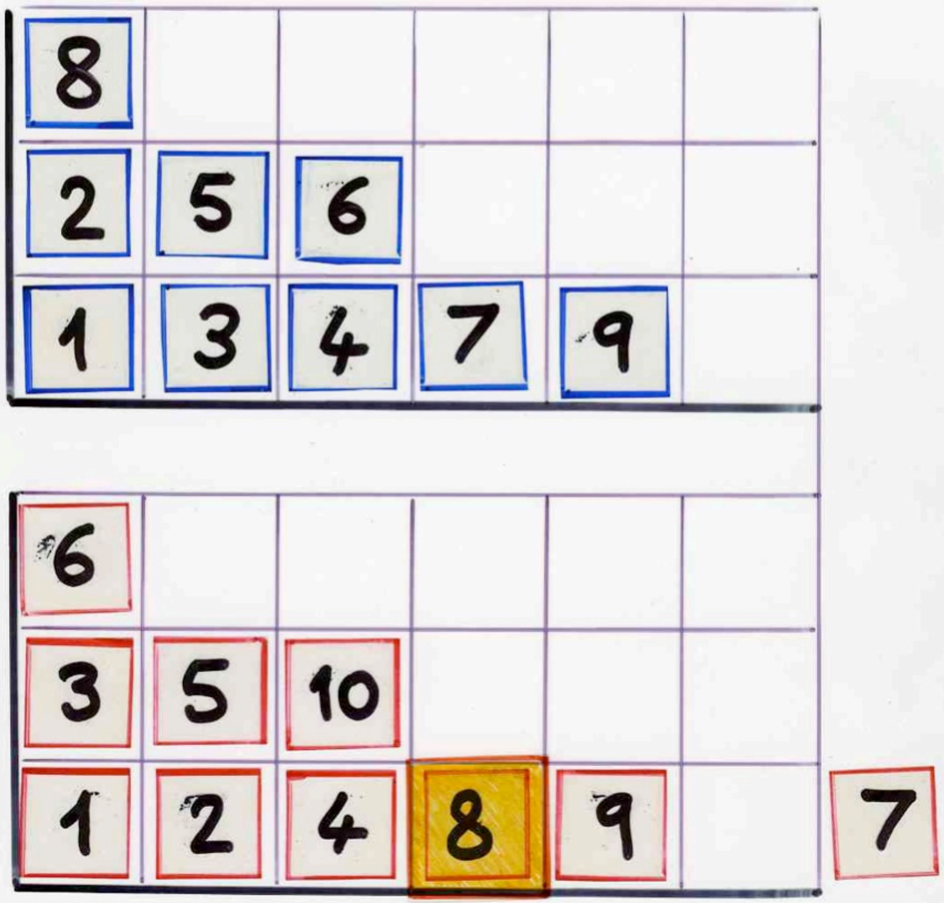
6					
3	5	10			
1	2	4	8		

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8					
2	5	6			
1	3	4	7	9	

6					
3	5	10			
1	2	4	8	9	

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

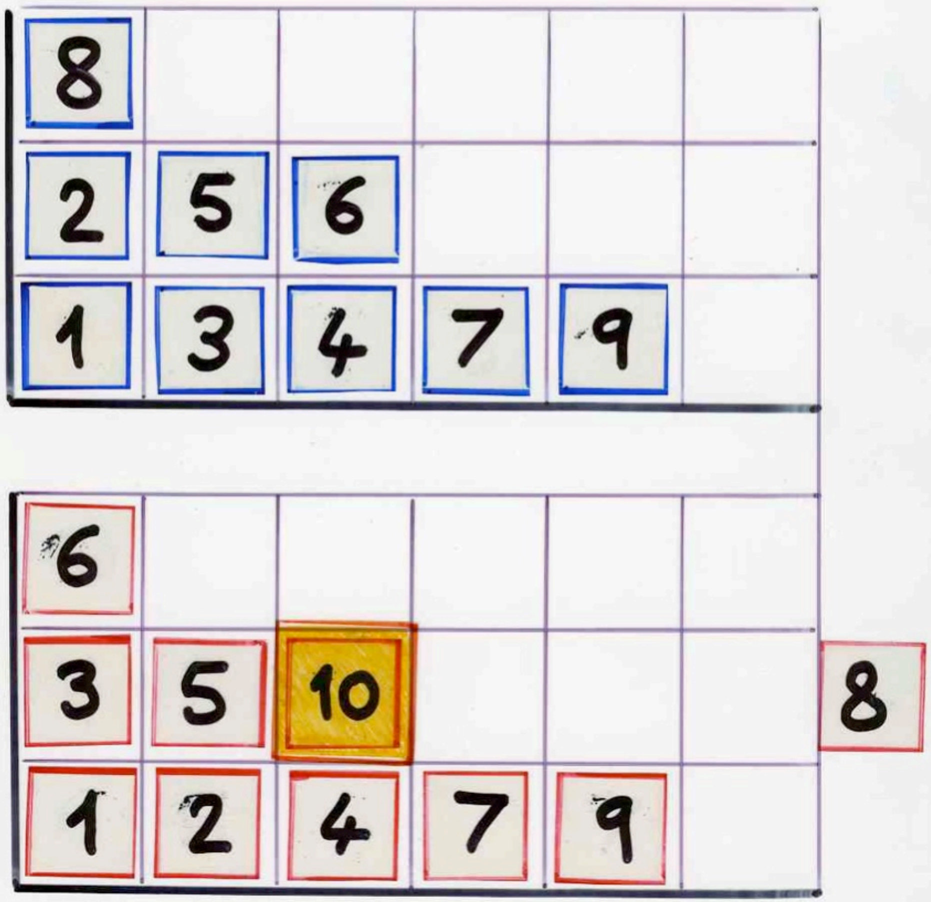


1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

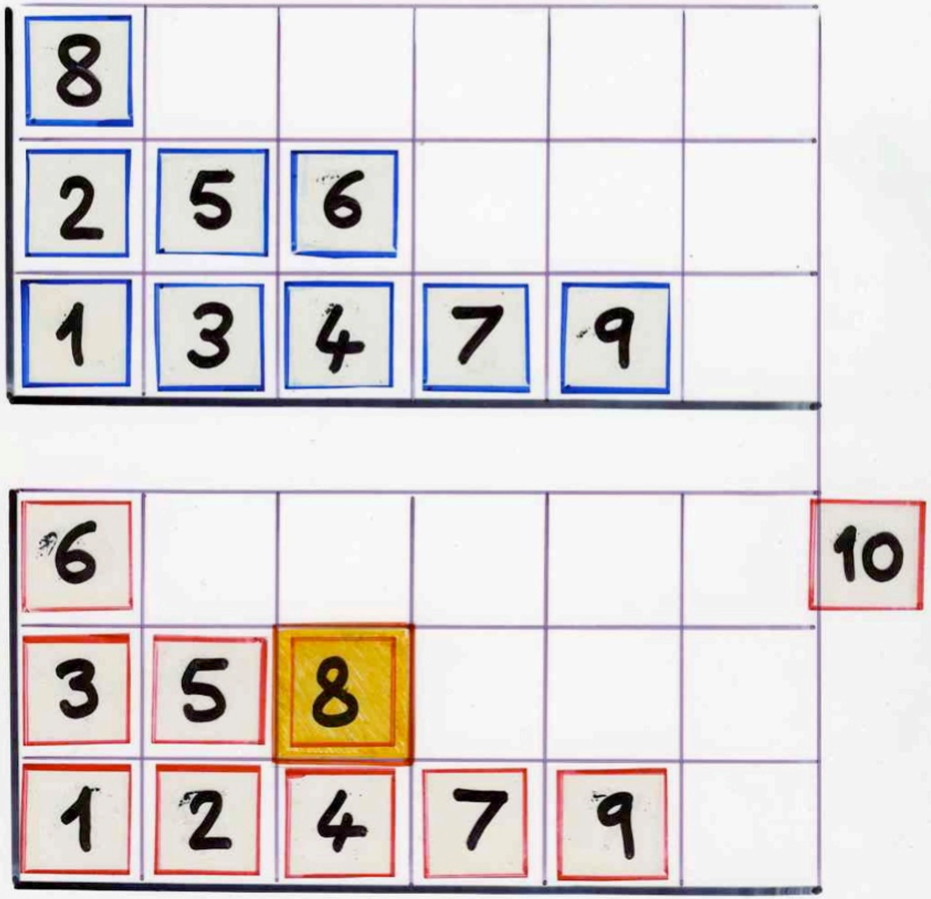
8					
2	5	6			
1	3	4	7	9	

6						
3	5	10				8
1	2	4	7	9		

1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



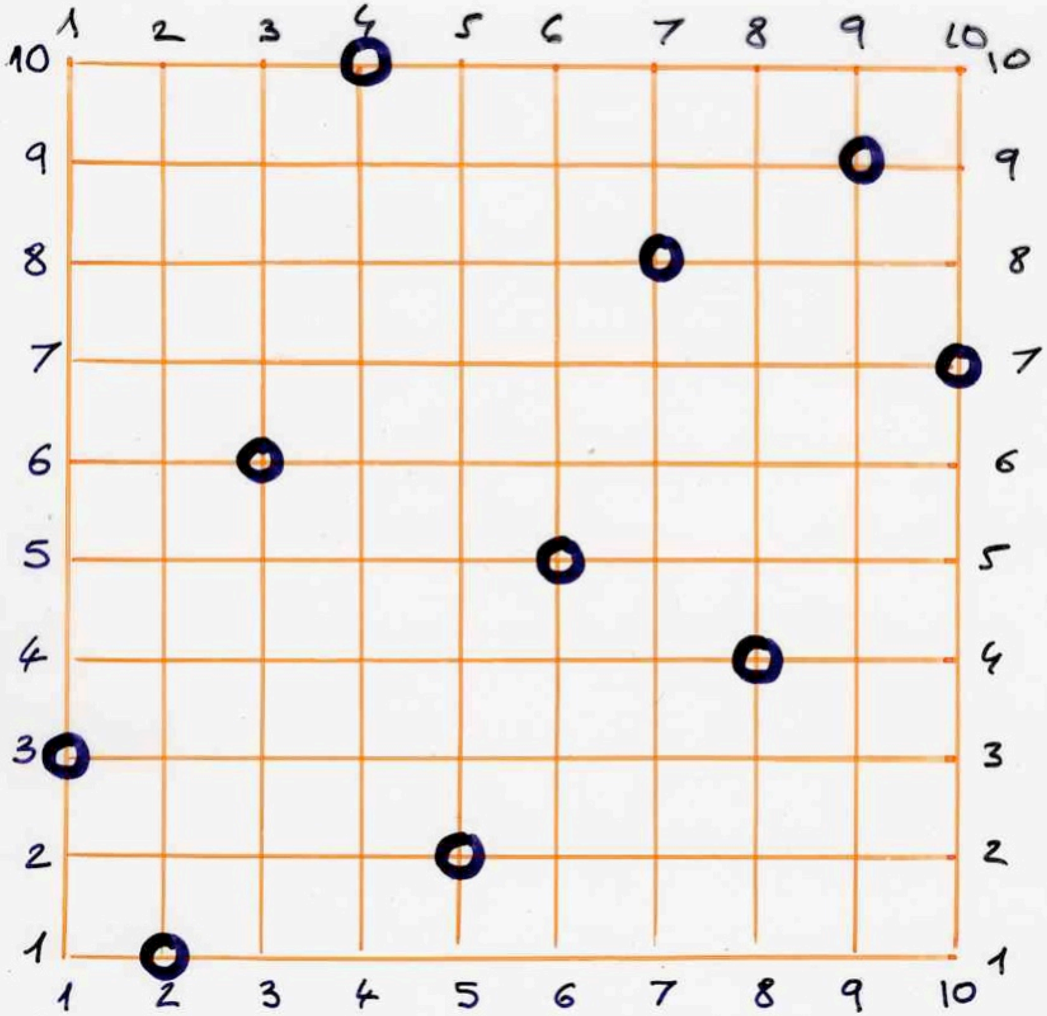
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

8	10				
2	5	6			
1	3	4	7	9	

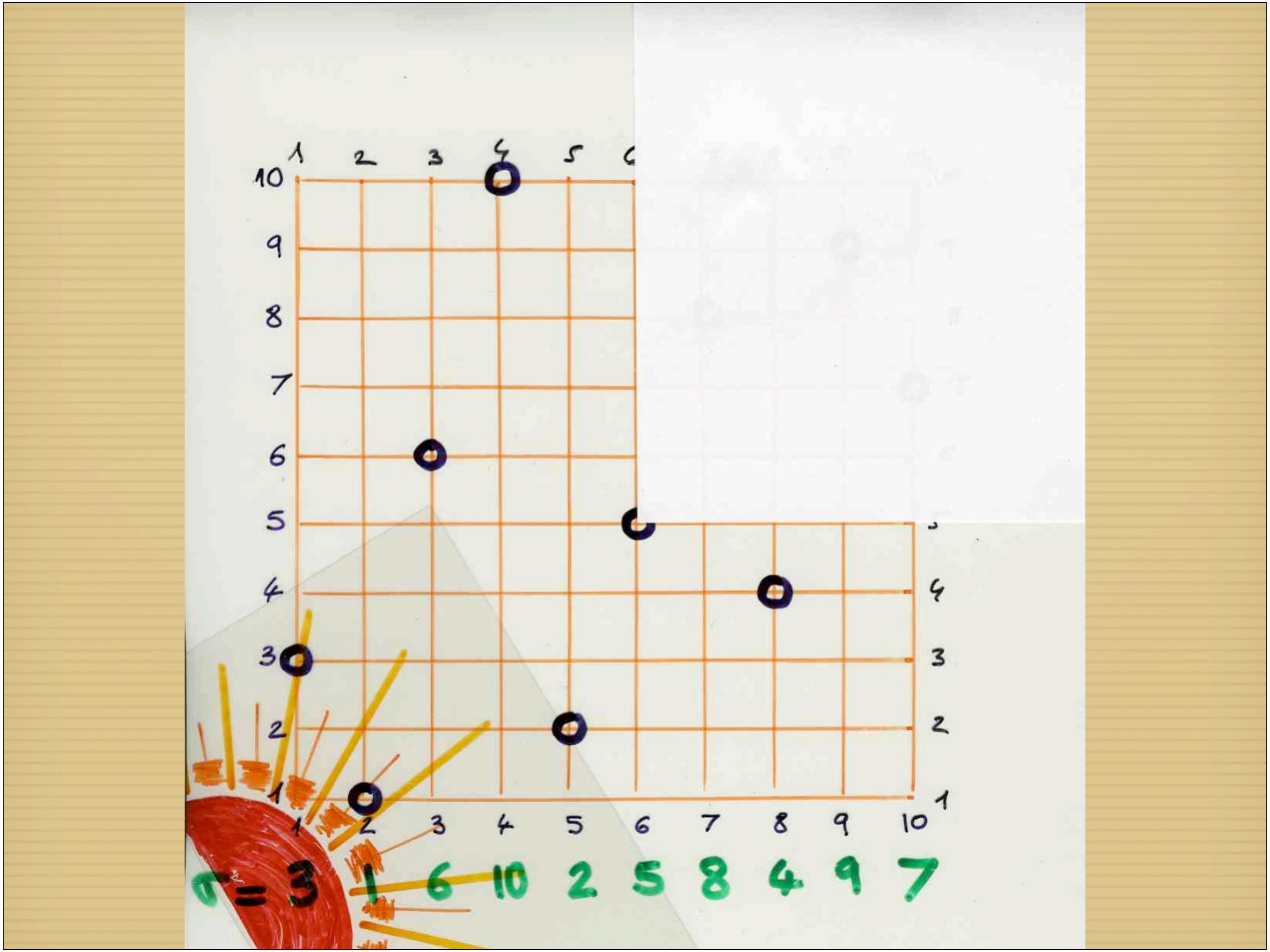
6	10				
3	5	8			
1	2	4	7	9	

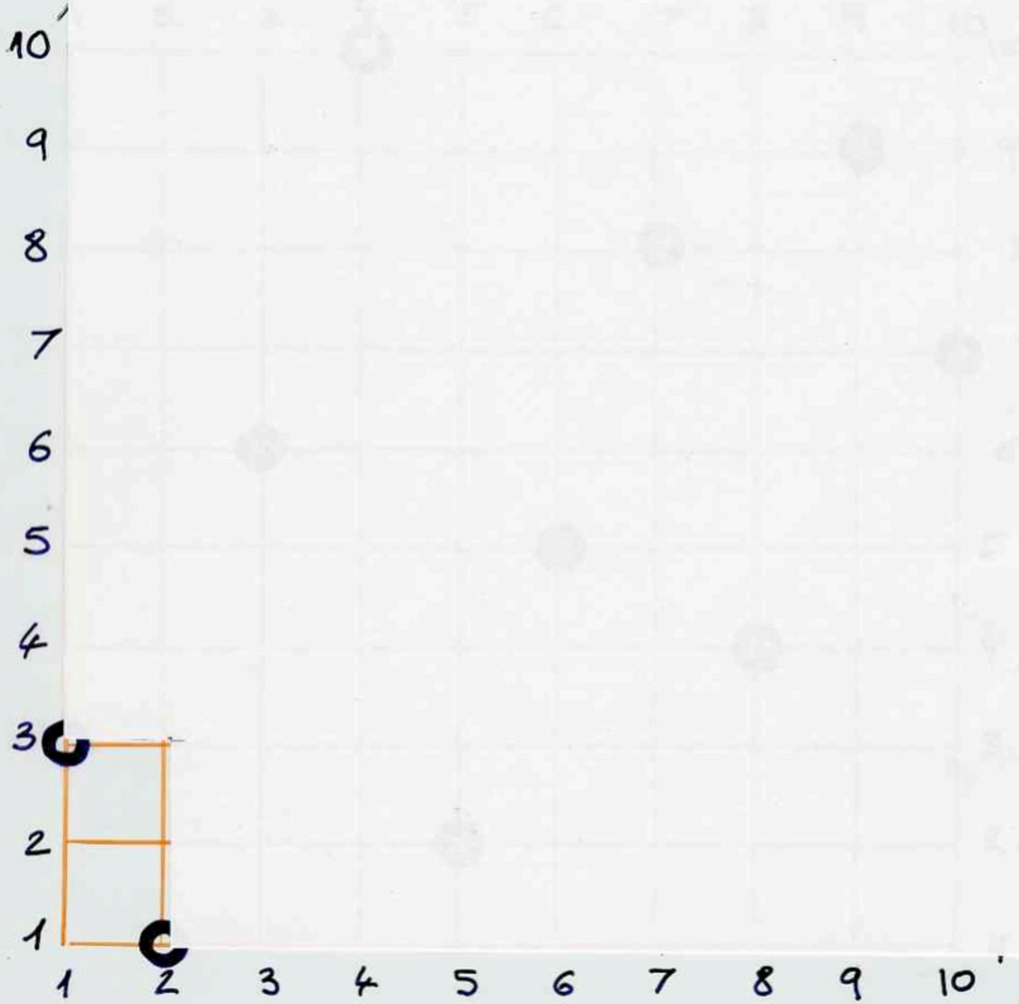
Une forme géométrique de RSK

(x.g. viennot, 1976)

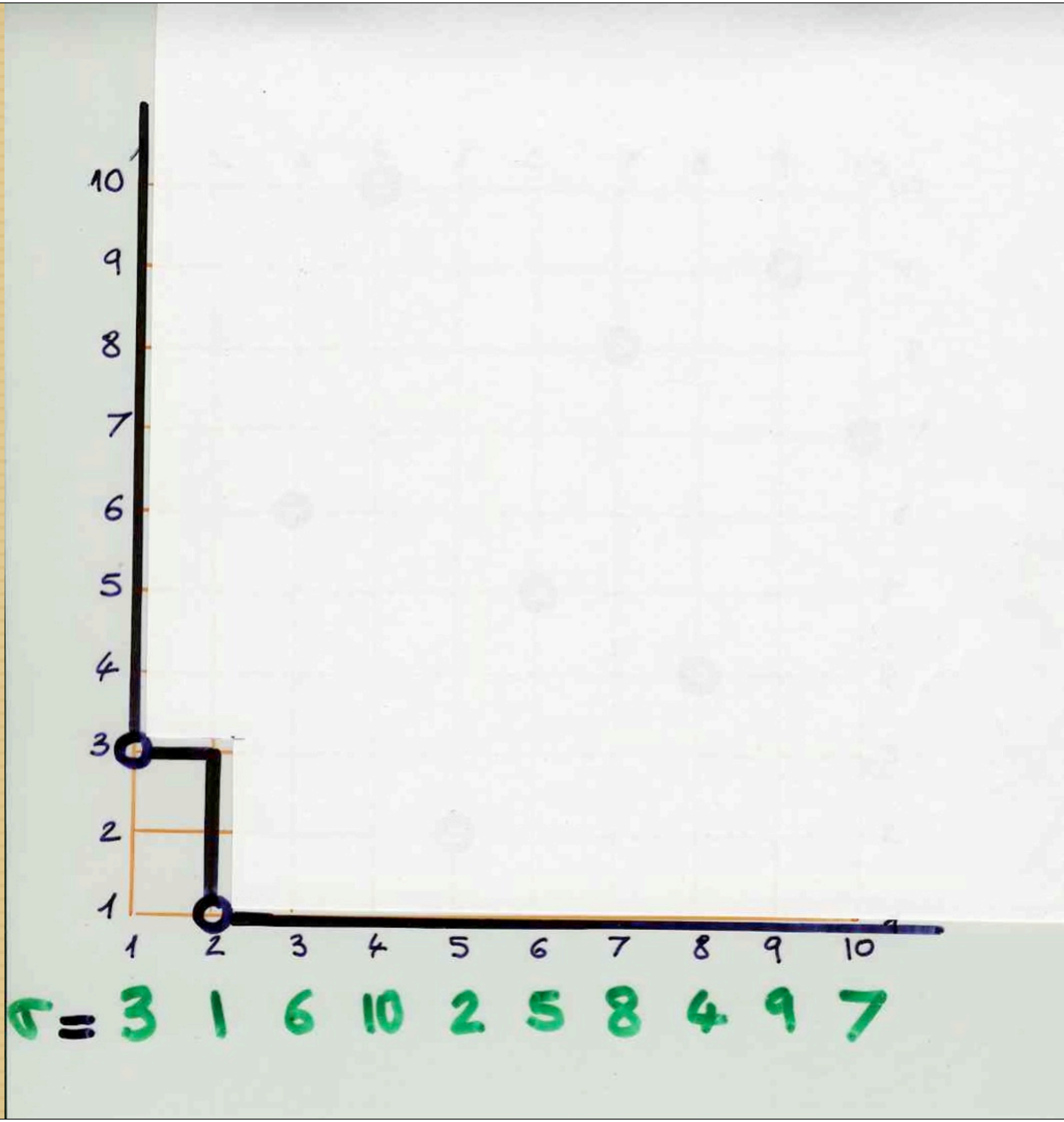


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

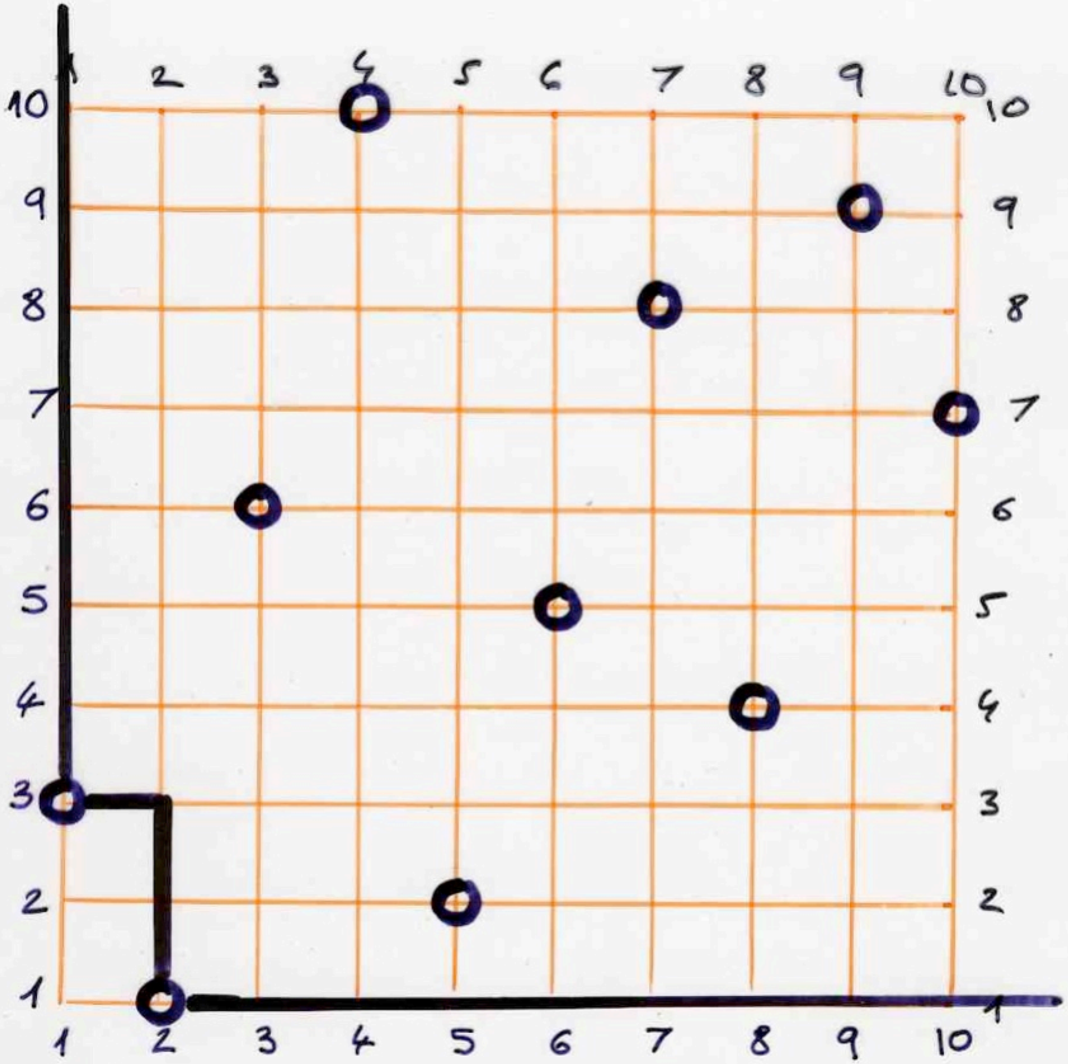

σ = 3 1 6 10 2 5 8 4 9 7



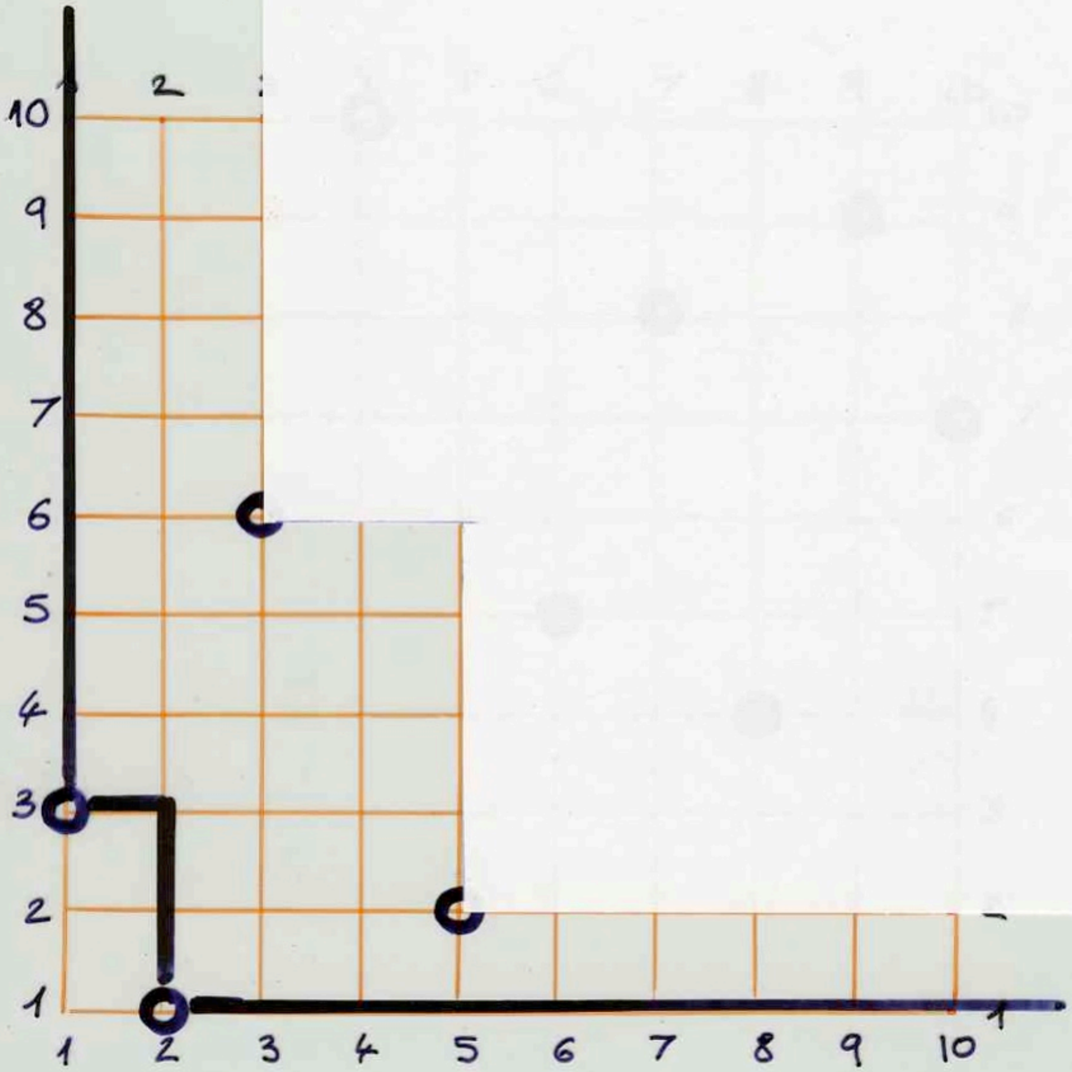
$\sigma = 3 \quad 1 \quad 6 \quad 10 \quad 2 \quad 5 \quad 8 \quad 4 \quad 9 \quad 7$



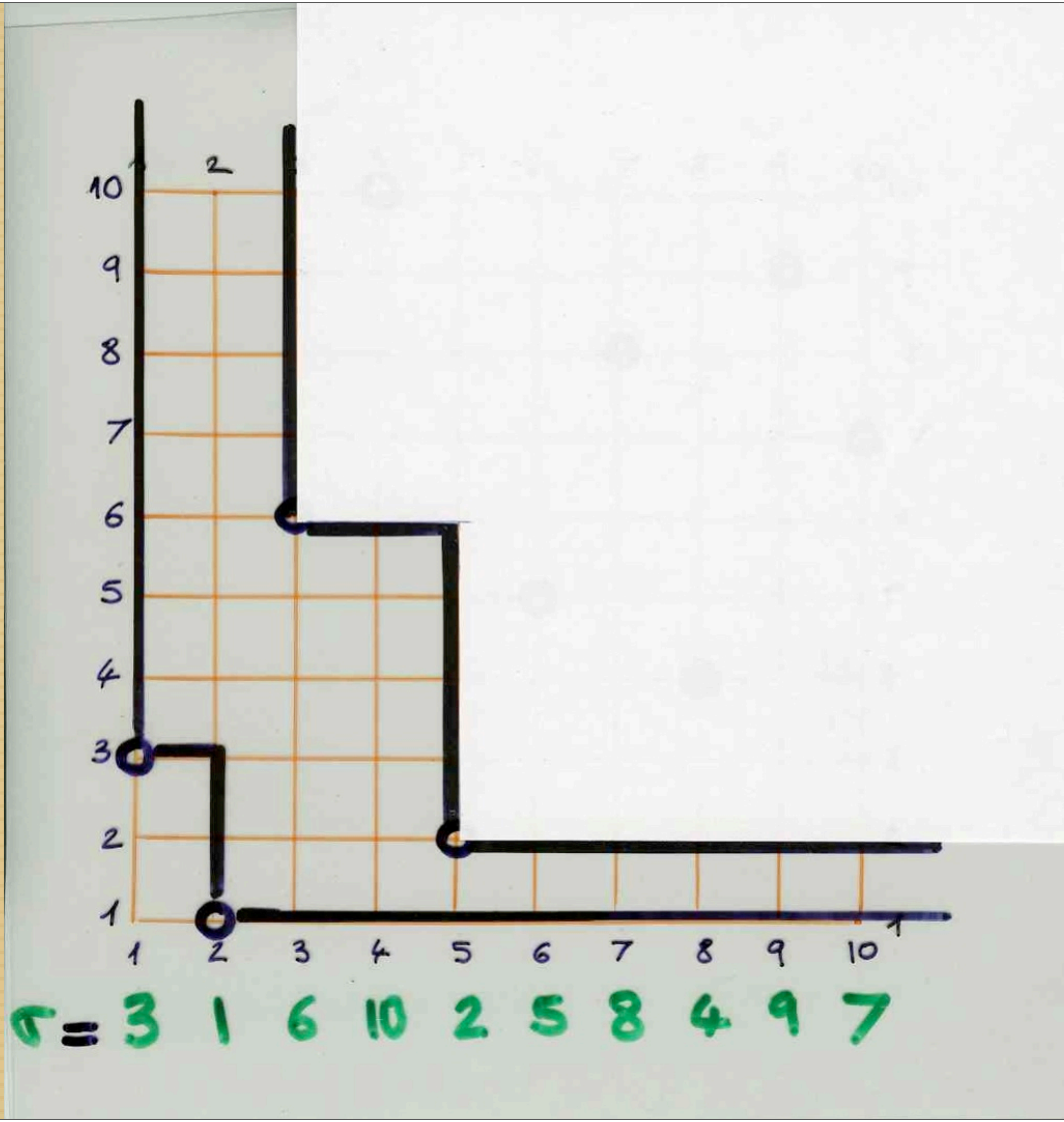
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10
σ = 3 1 6 10 2 5 8 4 9 7



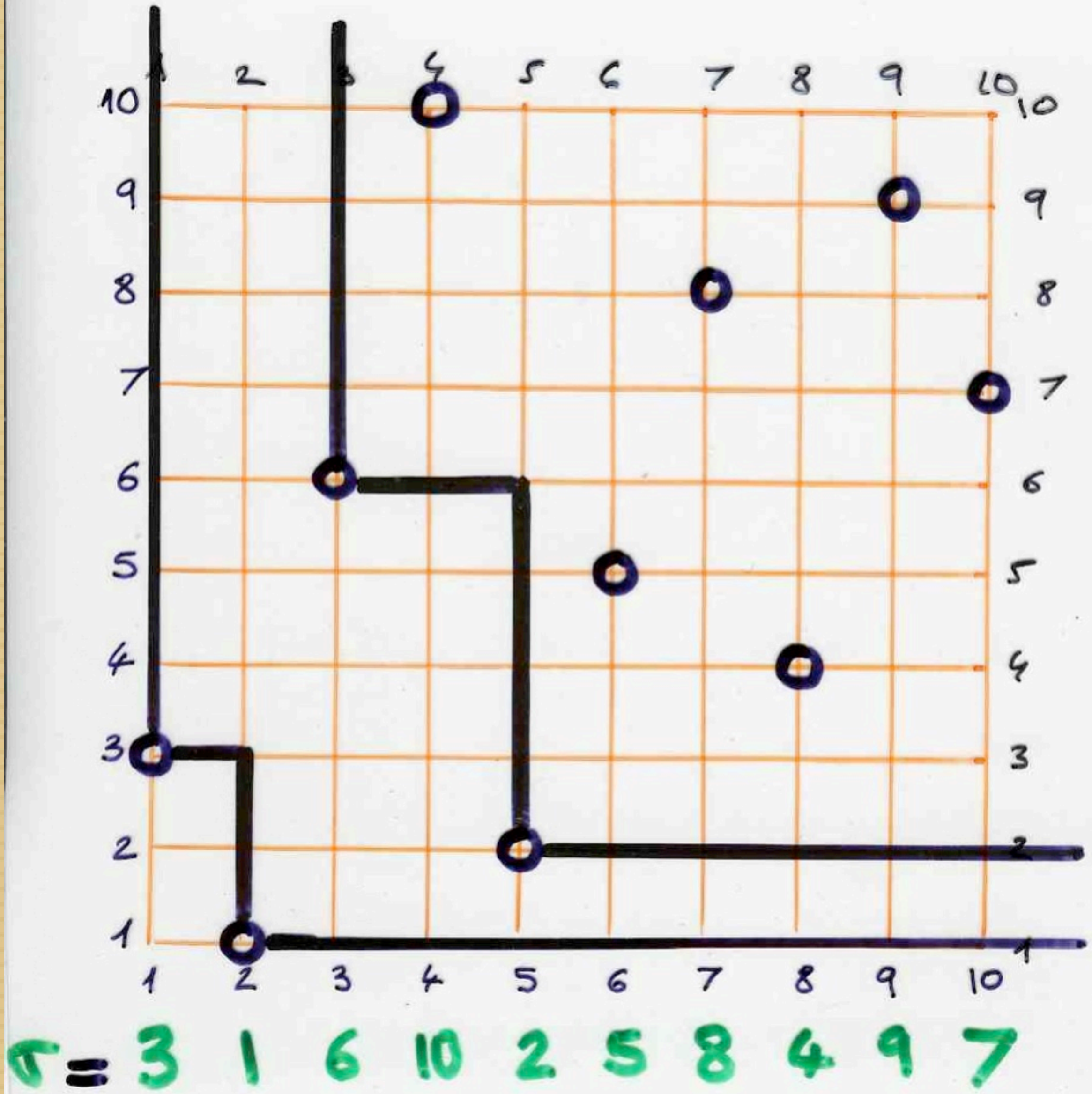
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



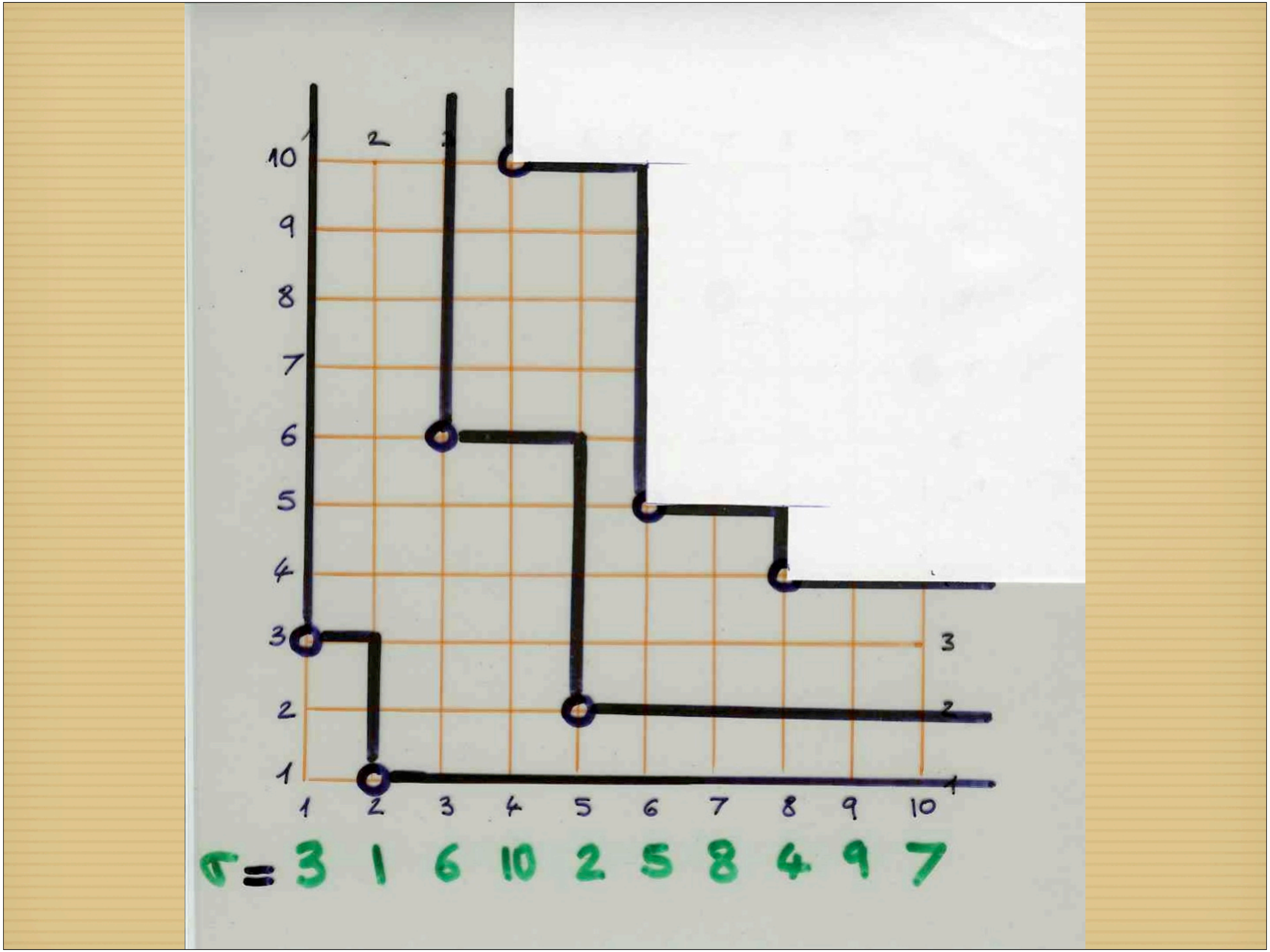
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



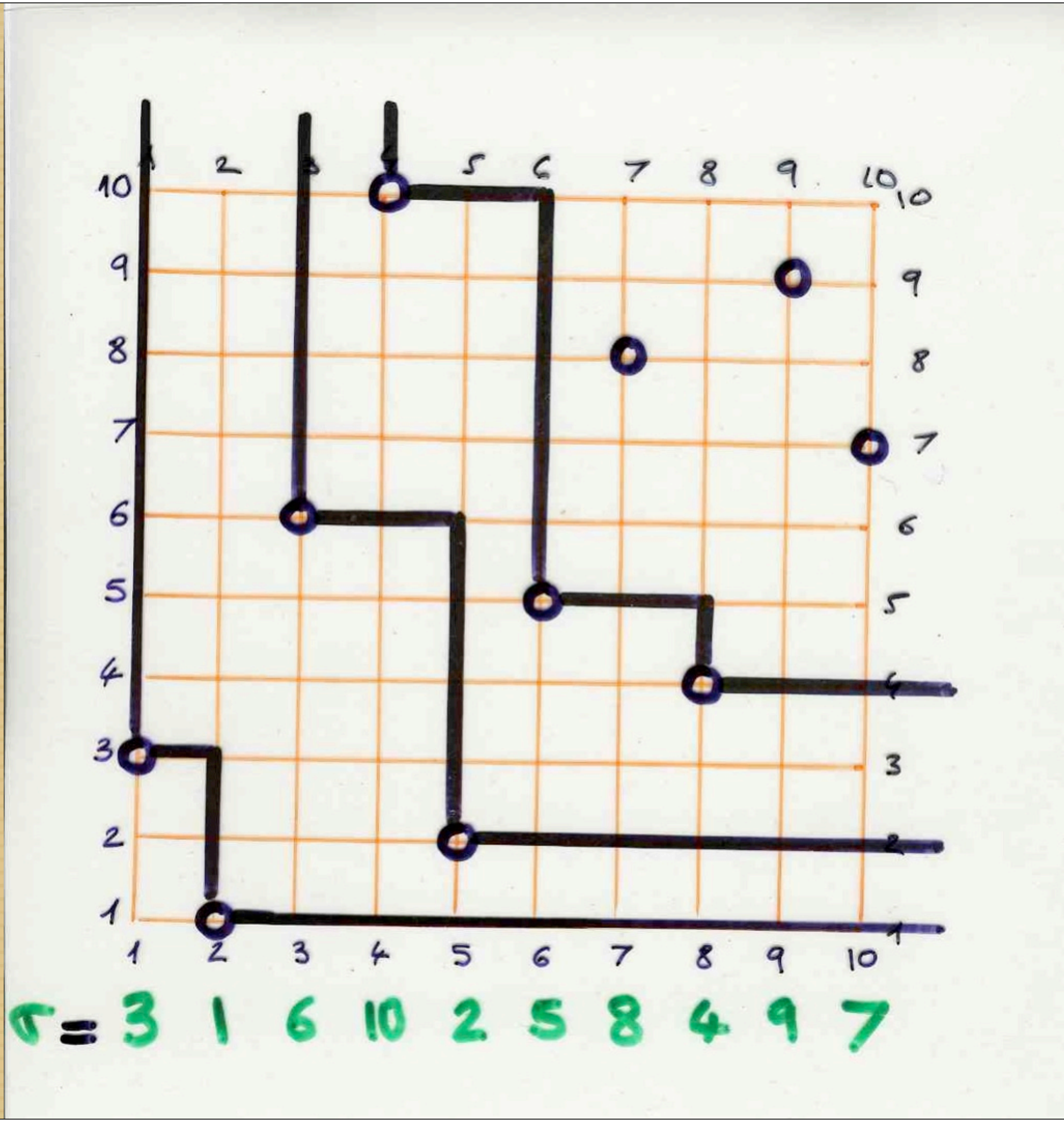
1 2
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10
σ = 3 1 6 10 2 5 8 4 9 7



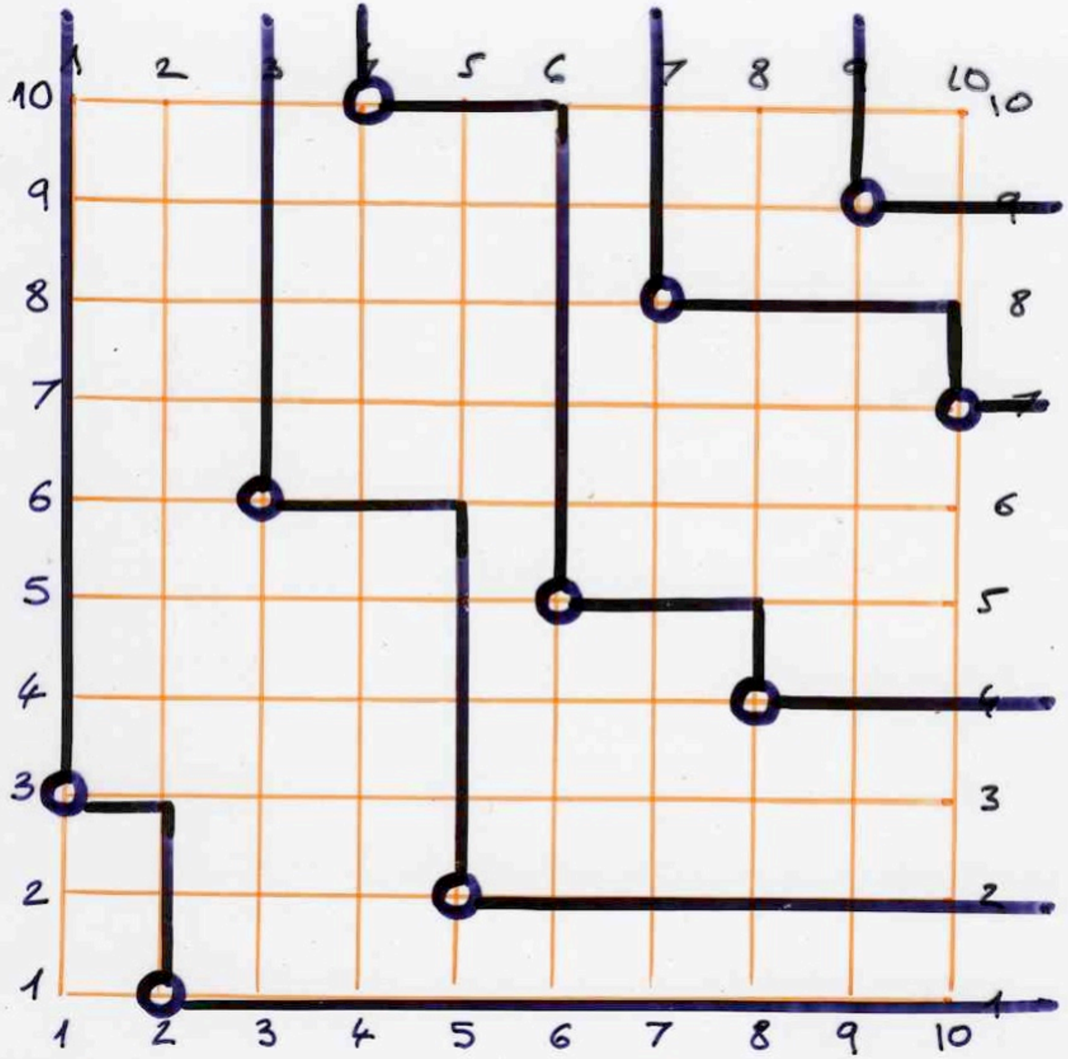
1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1
σ = 3 1 6 10 2 5 8 4 9 7



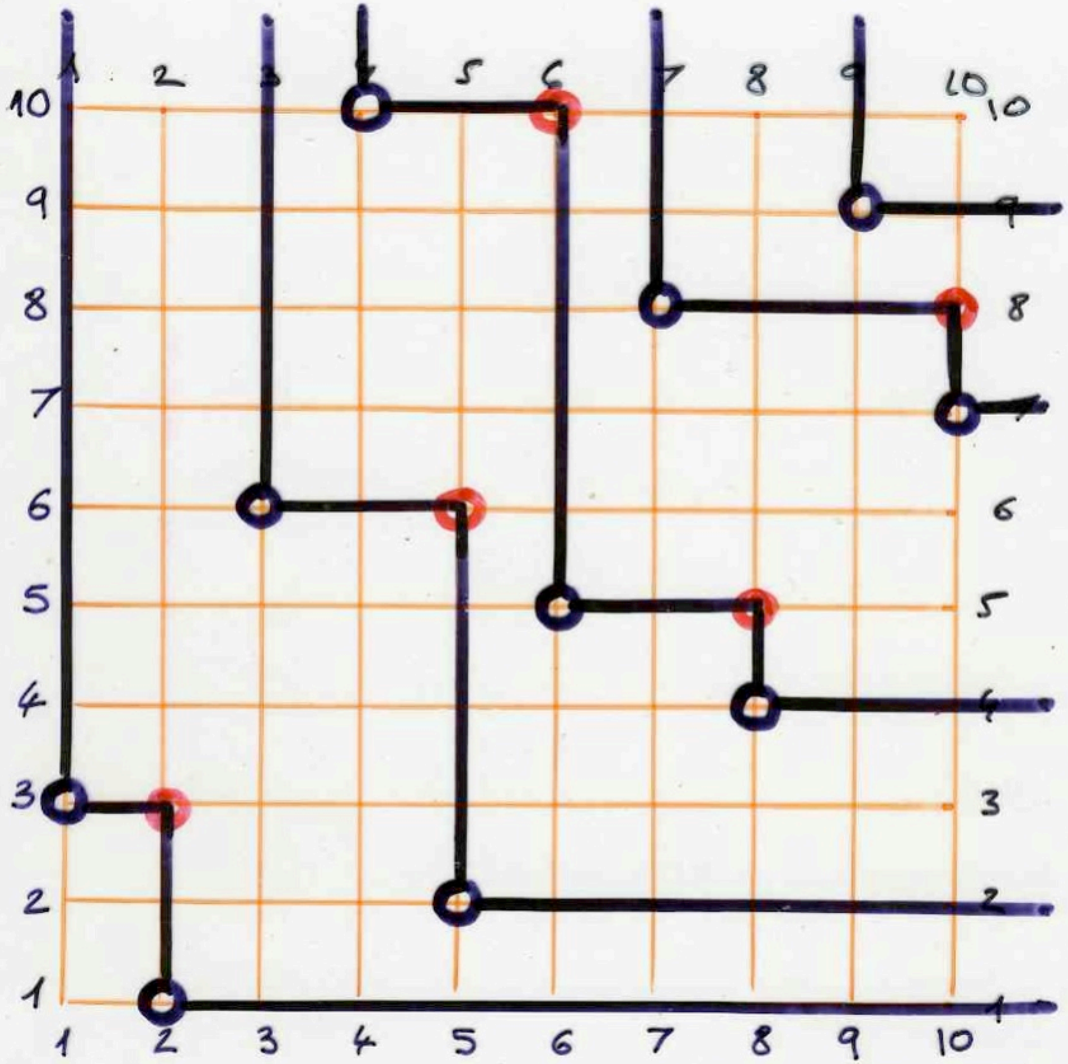
σ = 3 1 6 10 2 5 8 4 9 7



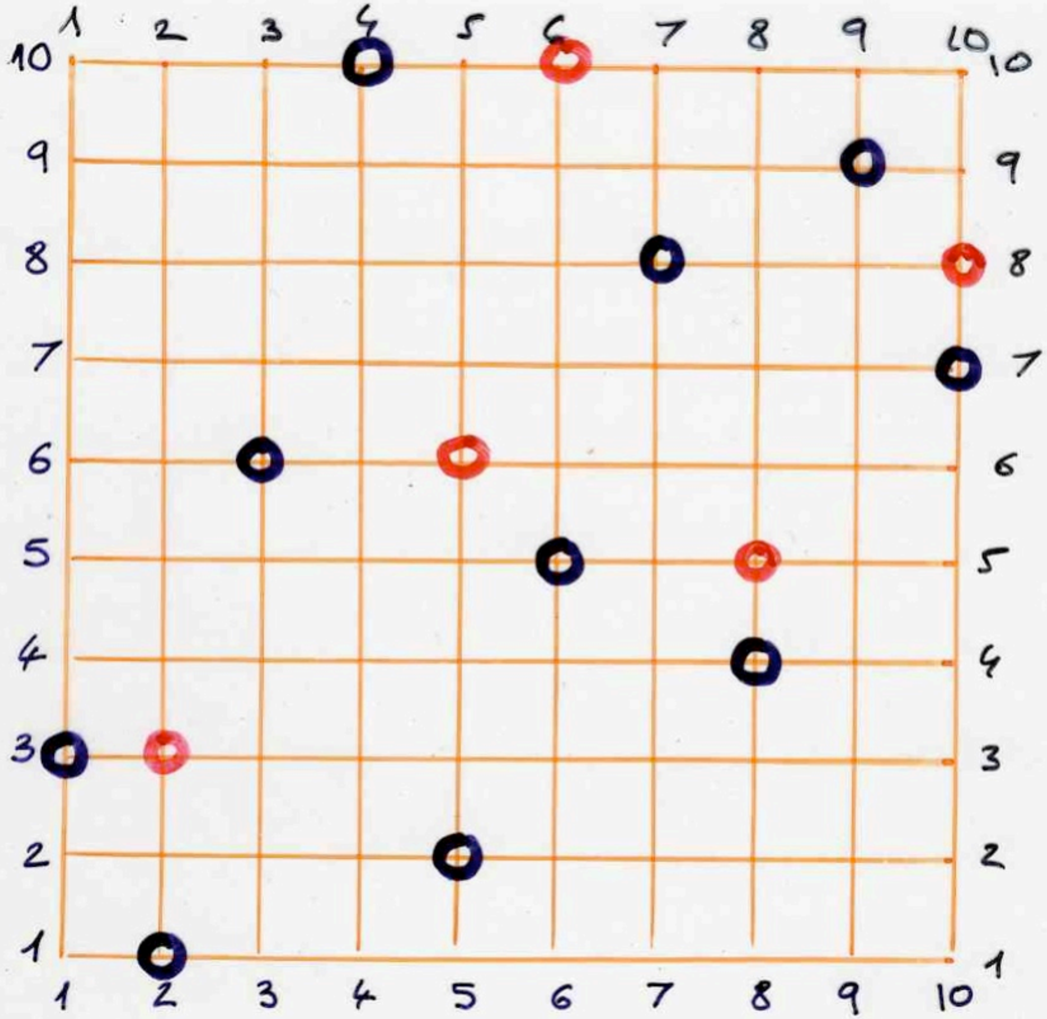
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10
σ = 3 1 6 10 2 5 8 4 9 7



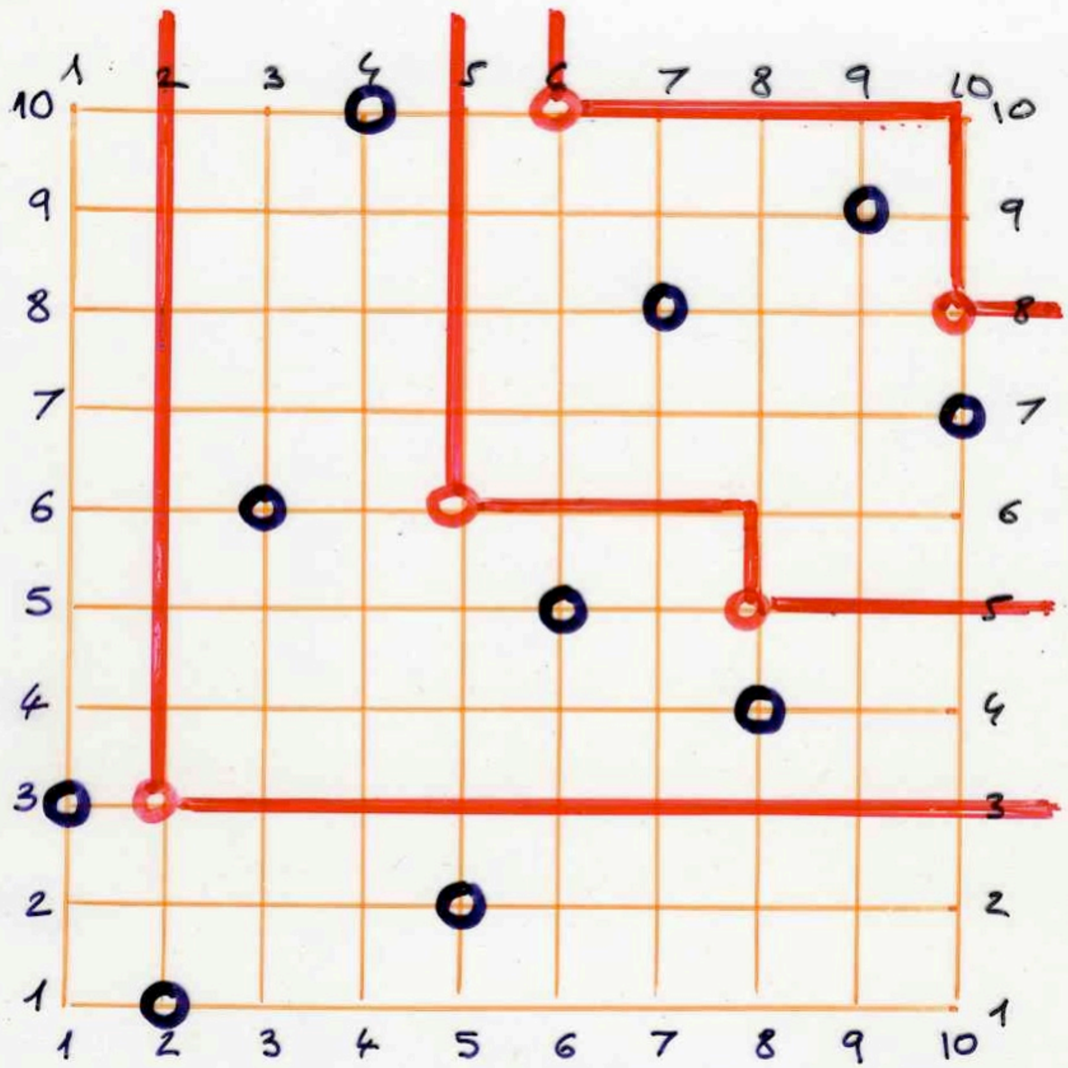
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



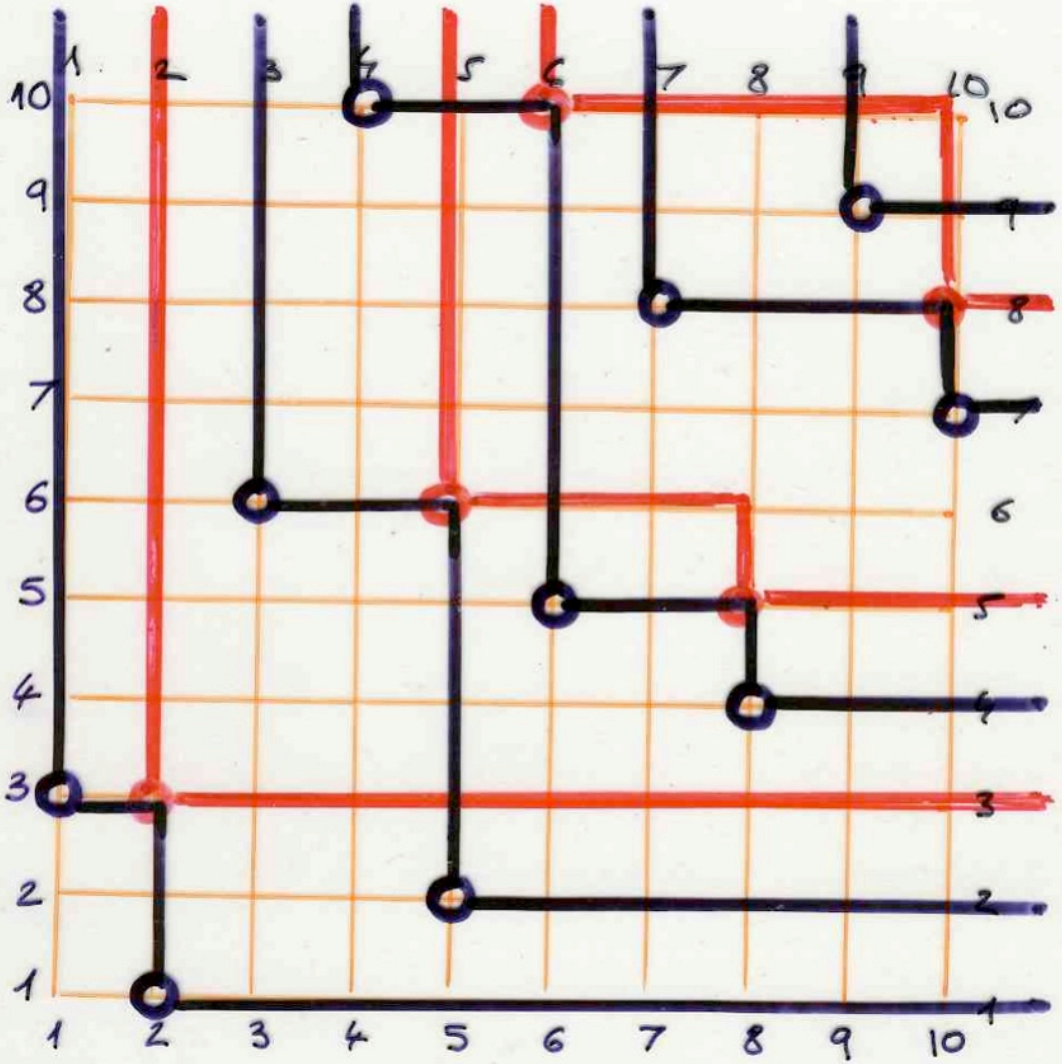
$\sigma = 3 \quad 1 \quad 6 \quad 10 \quad 2 \quad 5 \quad 8 \quad 4 \quad 9 \quad 7$



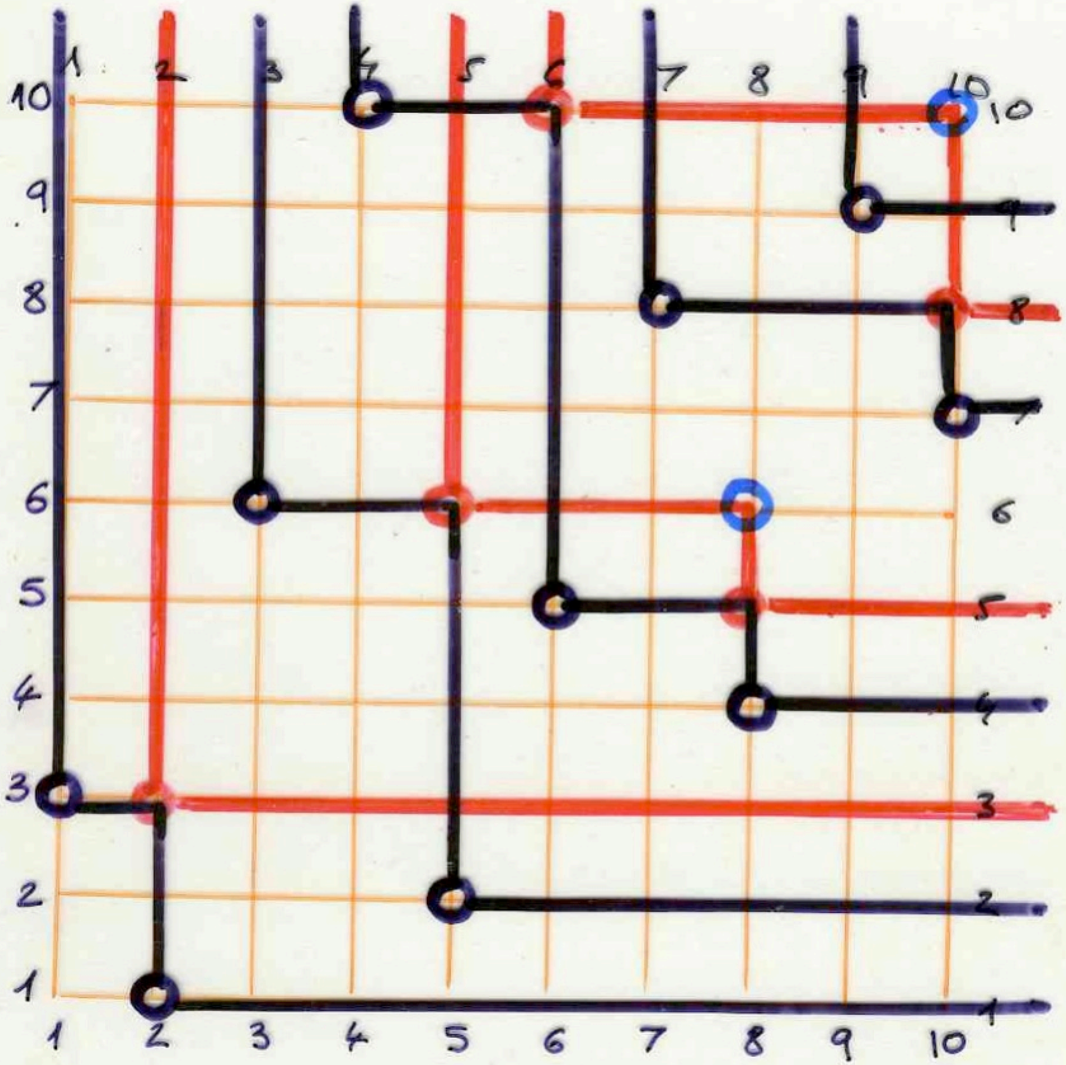
$\sigma = 3 \quad 1 \quad 6 \quad 10 \quad 2 \quad 5 \quad 8 \quad 4 \quad 9 \quad 7$



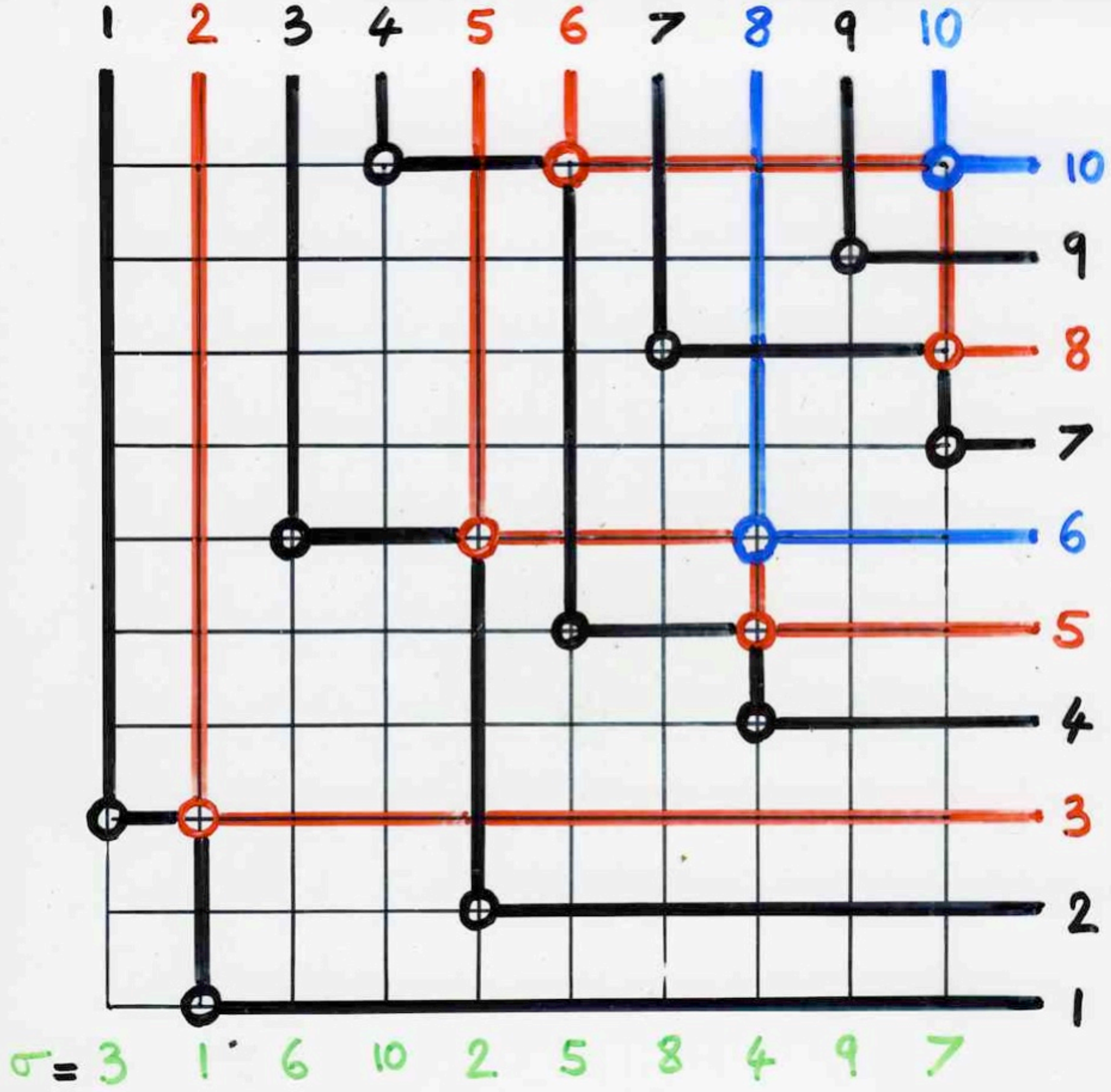
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$



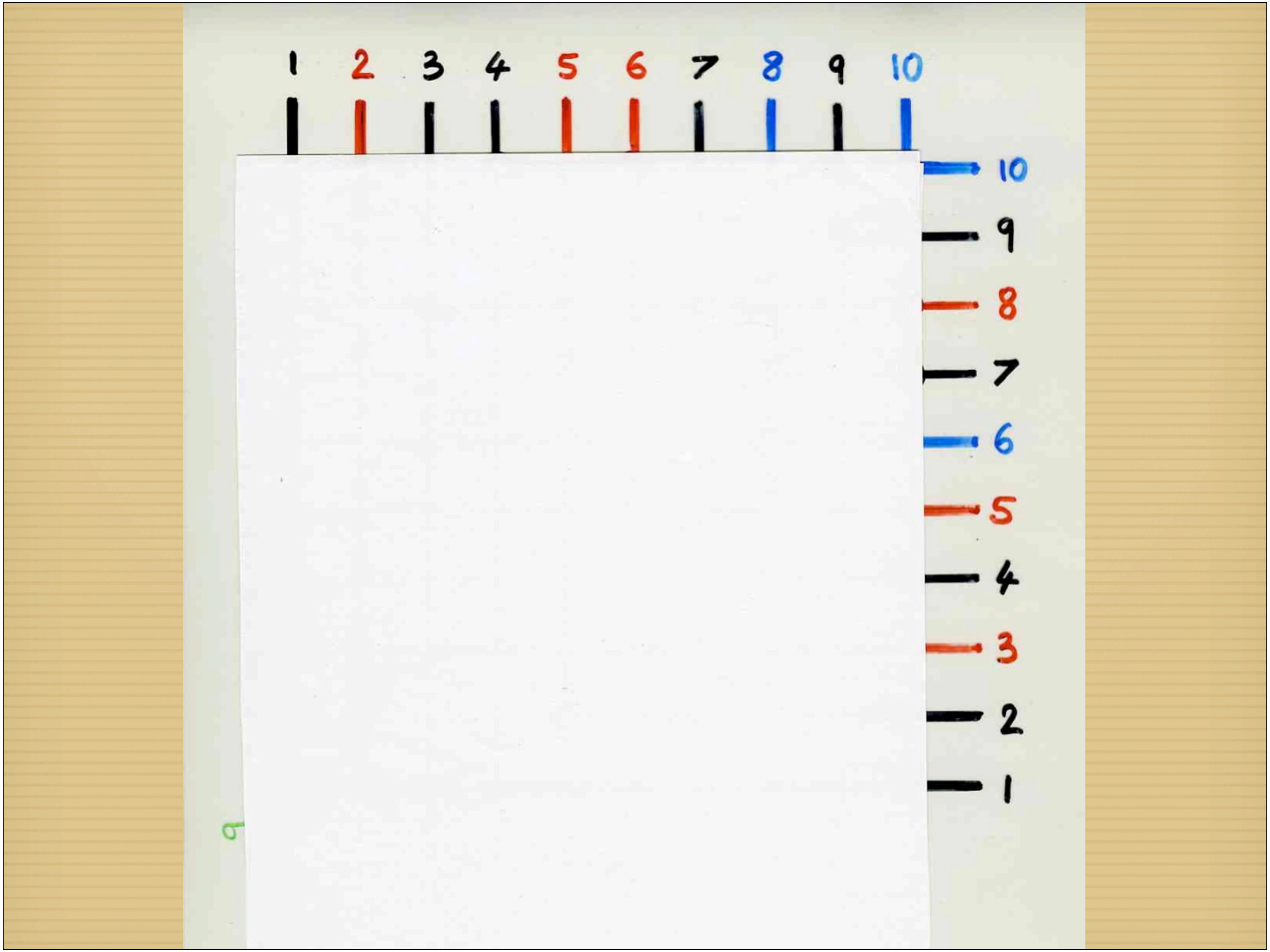
$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

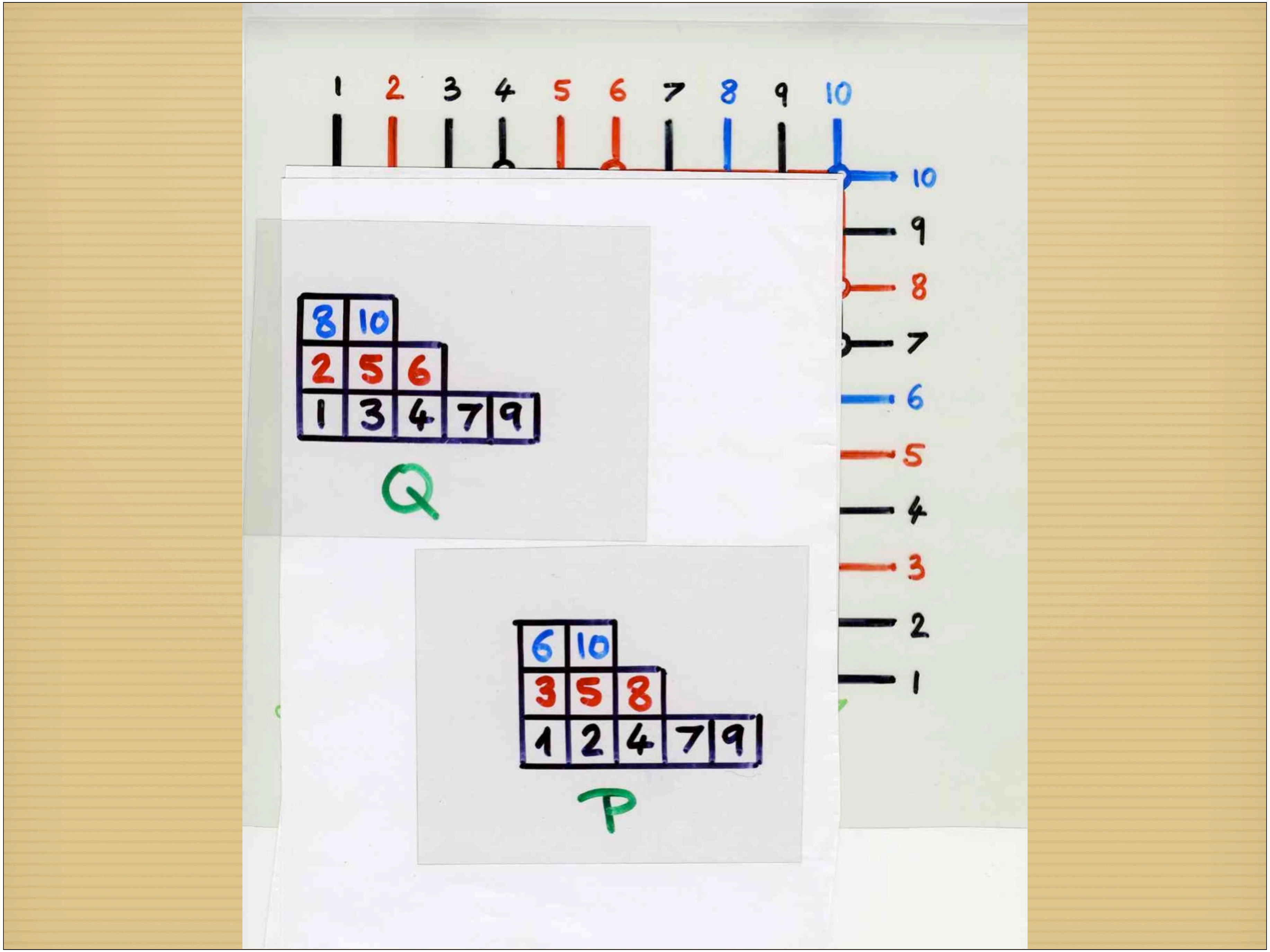


$\sigma = 3\ 1\ 6\ 10\ 2\ 5\ 8\ 4\ 9\ 7$

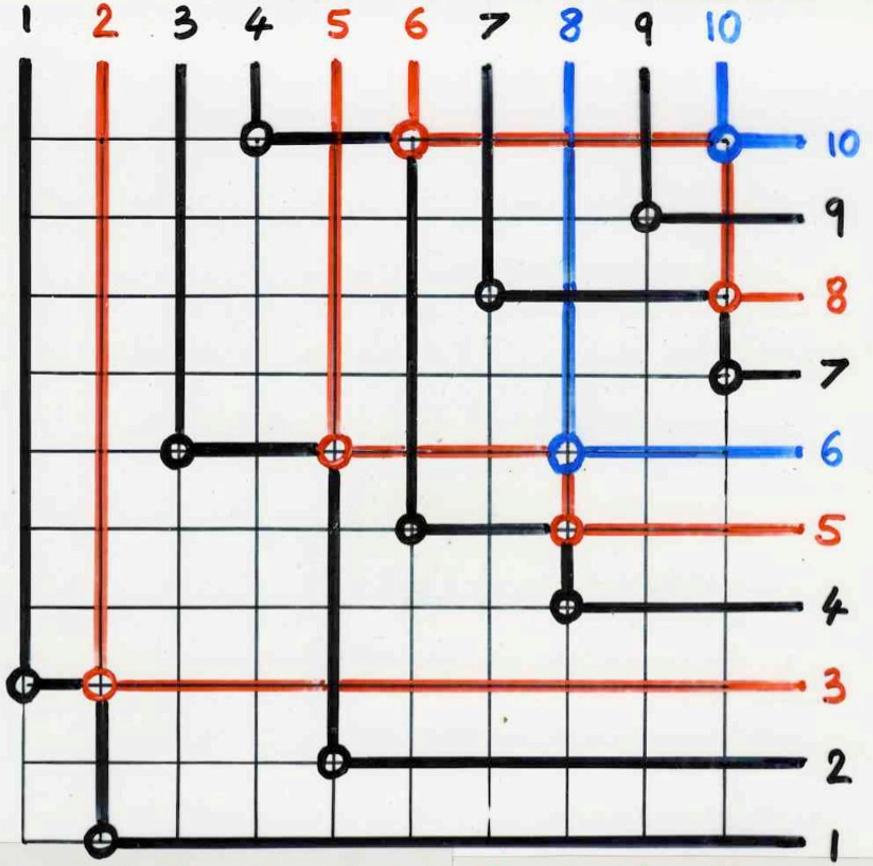


1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
σ = 3 1 6 10 2 5 8 4 9 7


1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1



1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1
8 10
2 5 6
1 3 4 7 9
Q
6 10
3 5 8
1 2 4 7 9
P



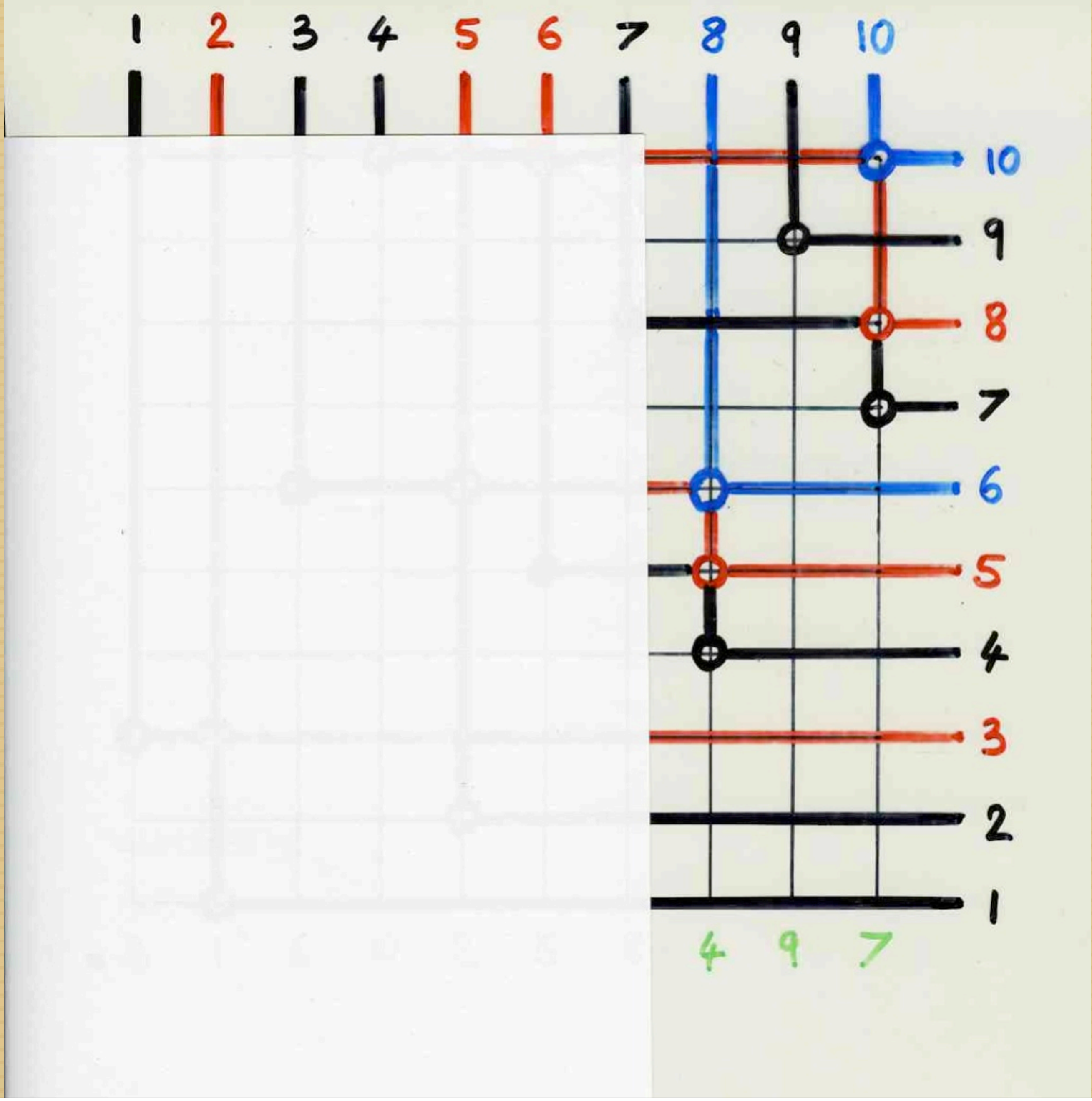
$\sigma = 3 \; 1 \; 6 \; 10 \; 2 \; 5 \; 8 \; 4 \; 9 \; 7$

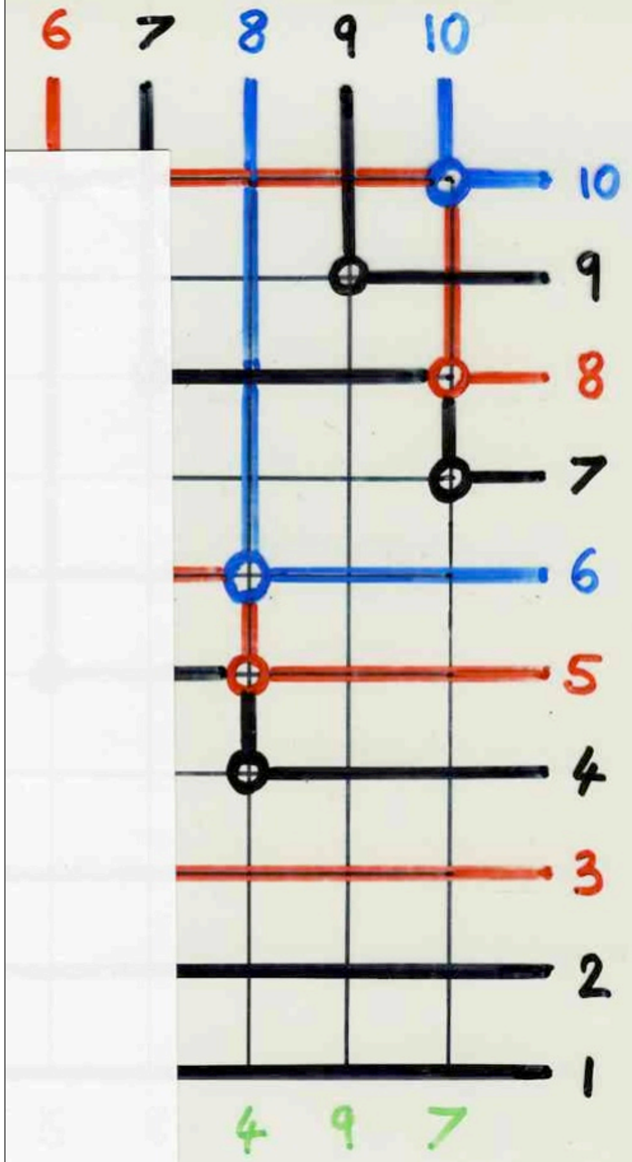
6	10			
3	5	8		
1	2	4	7	9

P

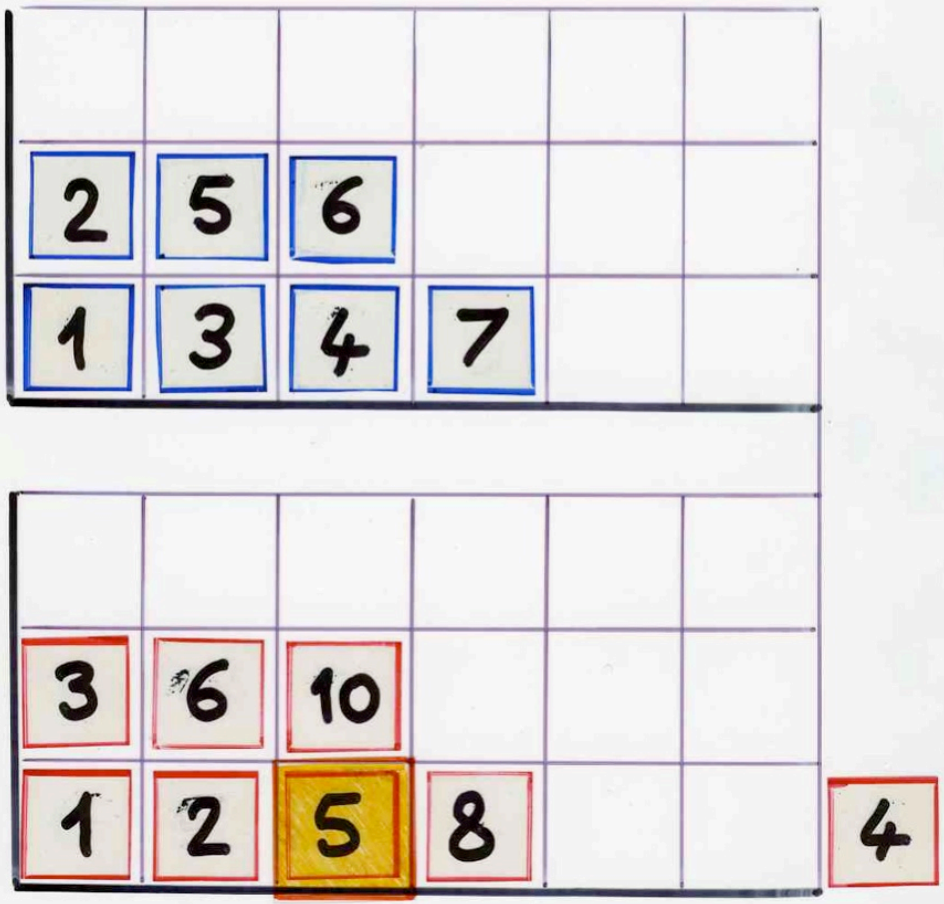
8	10			
2	5	6		
1	3	4	7	9

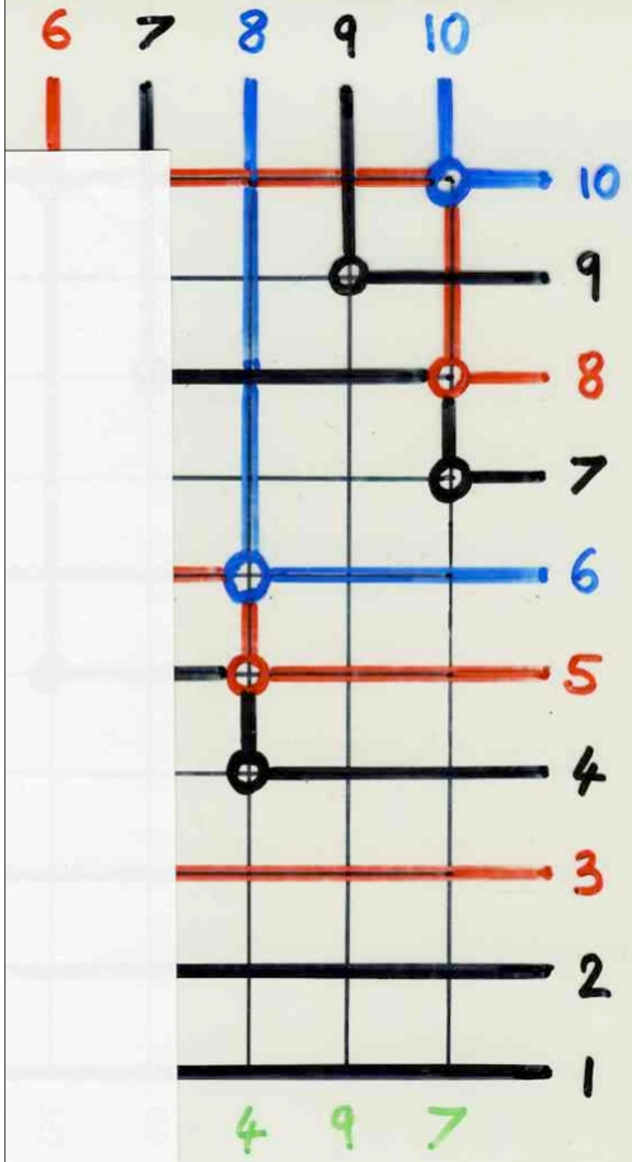
Q


1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
4 9 7

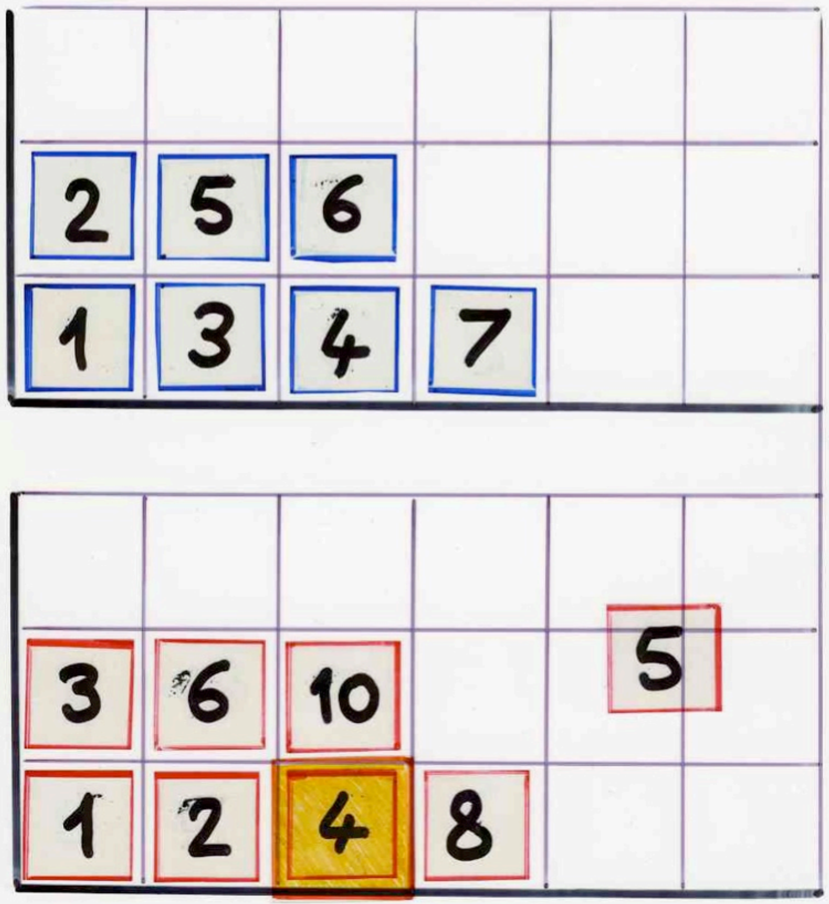


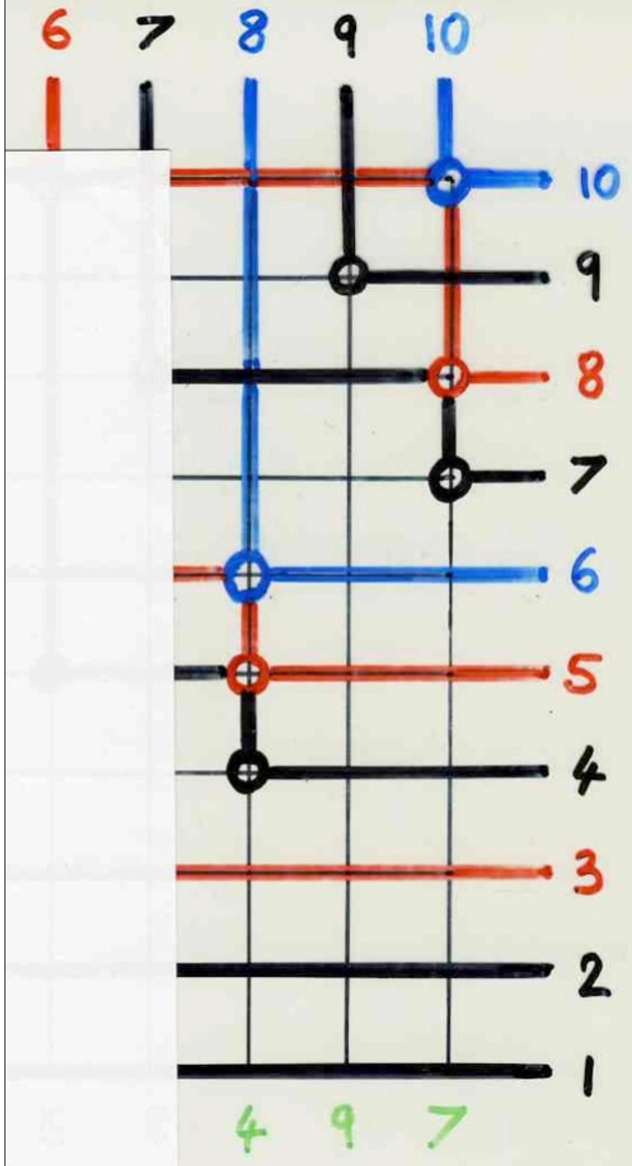
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



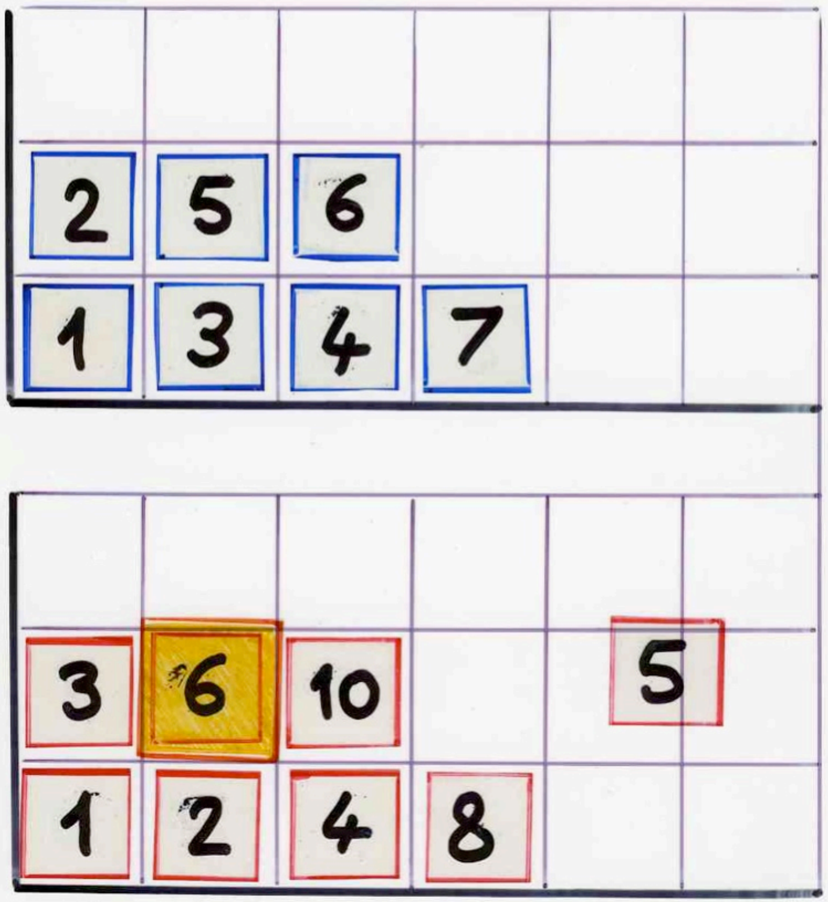


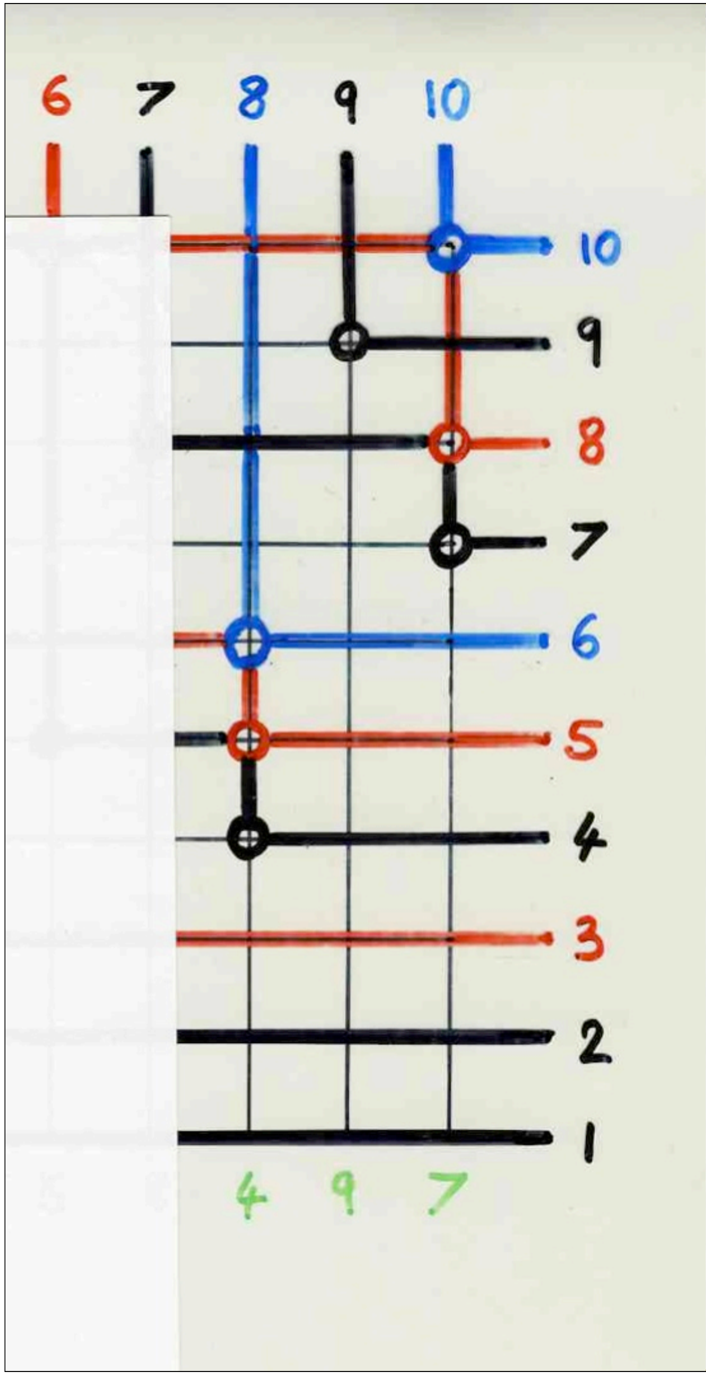
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7



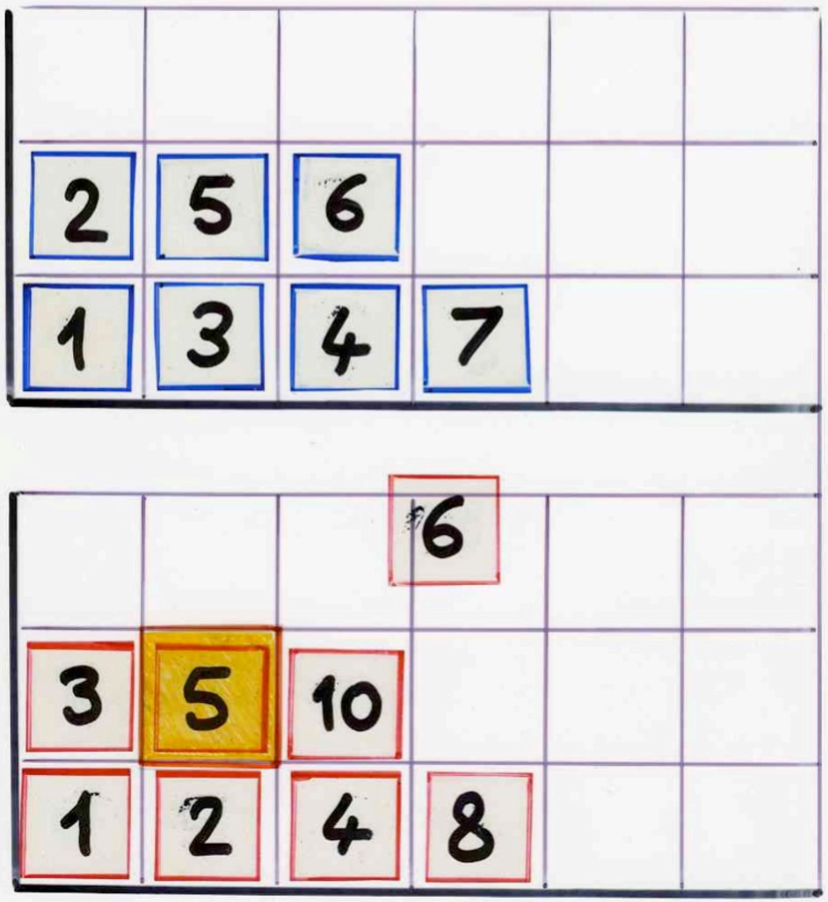


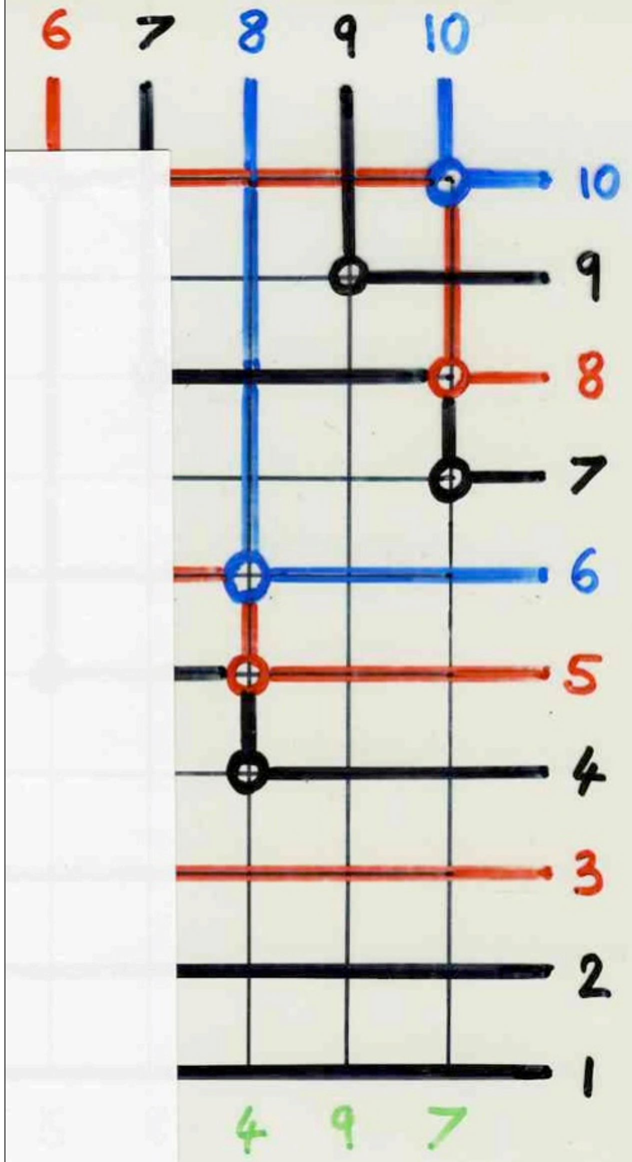
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7





1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7





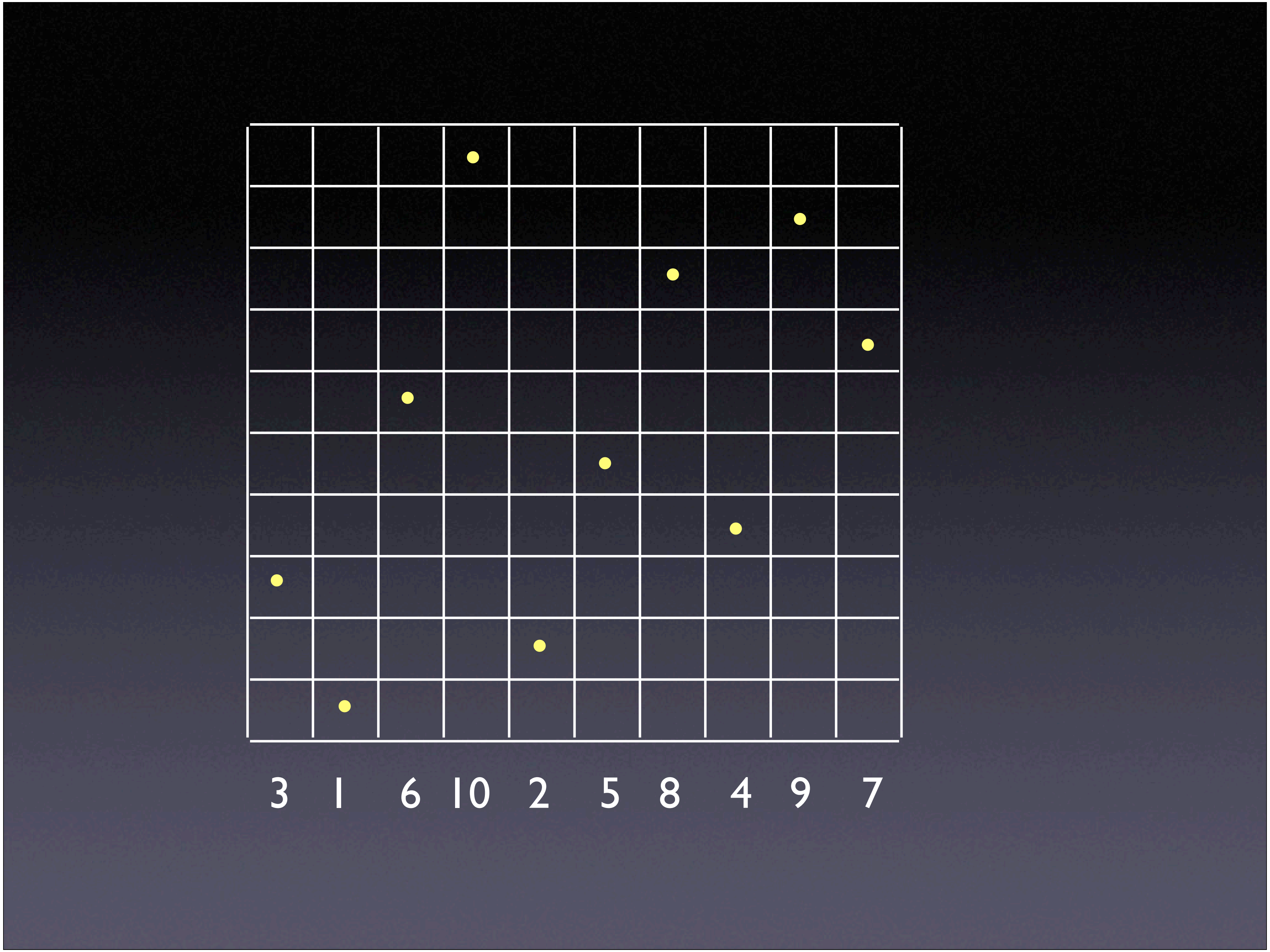
1	2	3	4	5	6	7	8	9	10
3	1	6	10	2	5	8	4	9	7

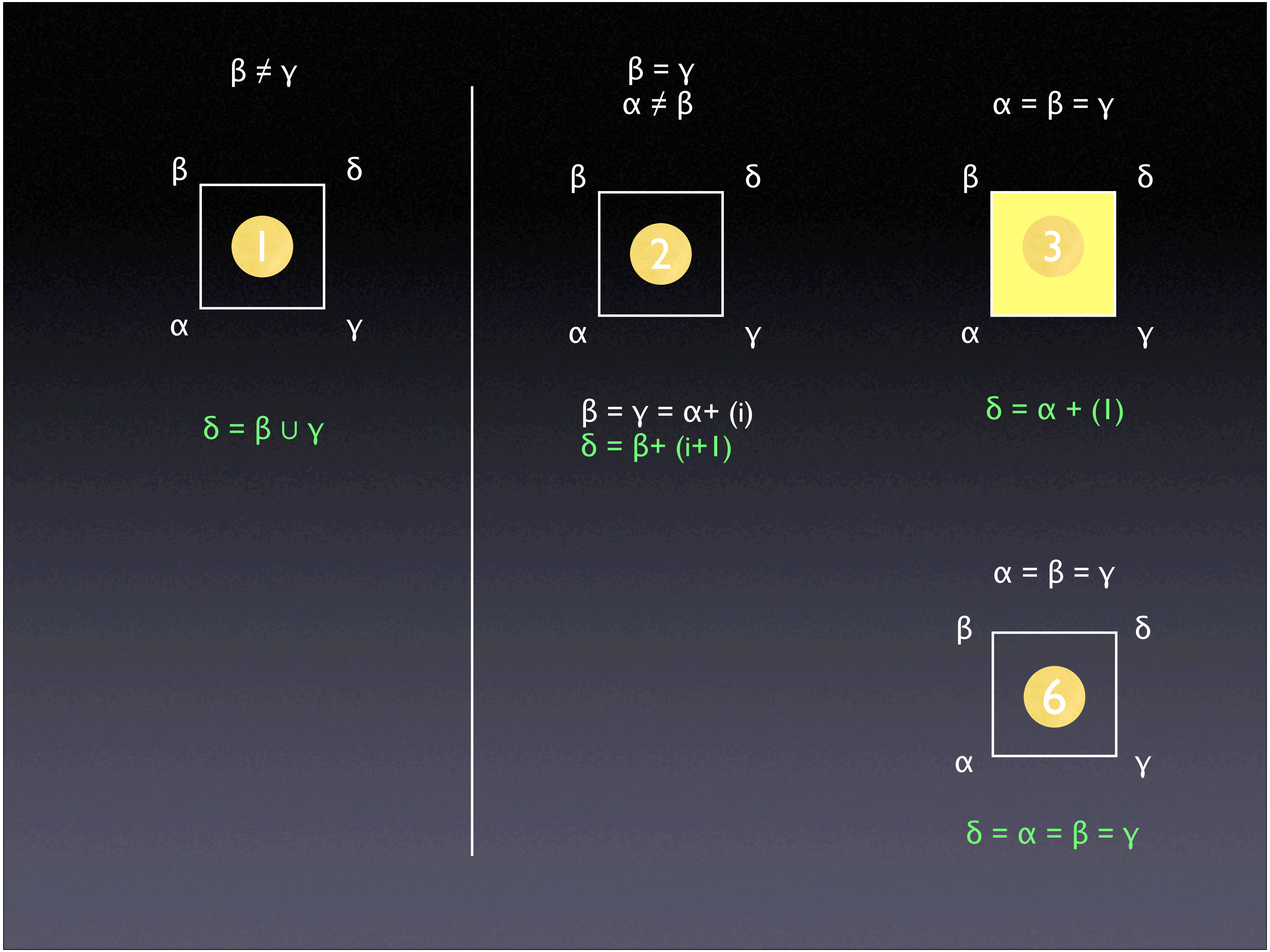
8					
2	5	6			
1	3	4	7		

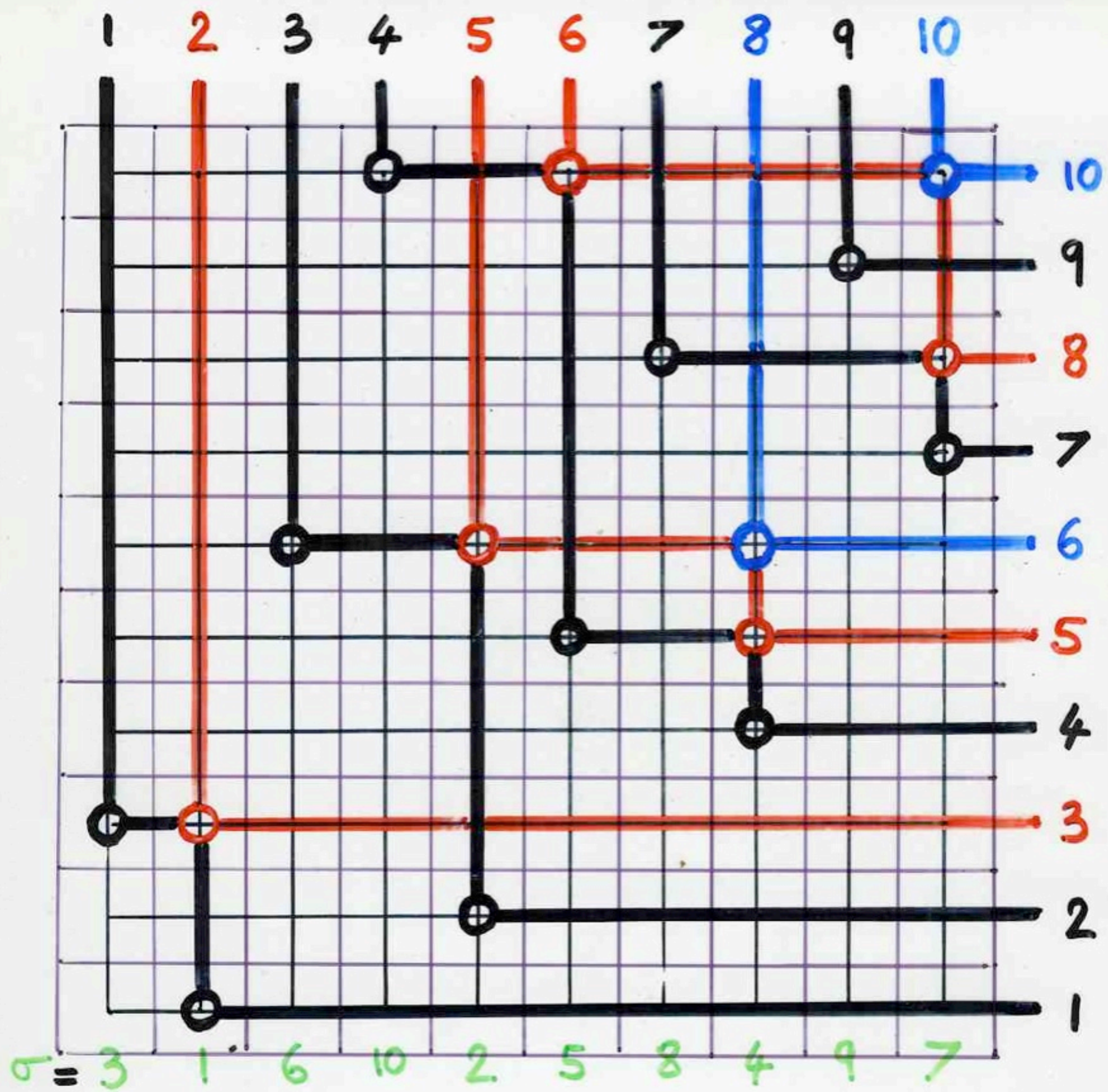
6					
3	5	10			
1	2	4	8		

Fin de la preuve :

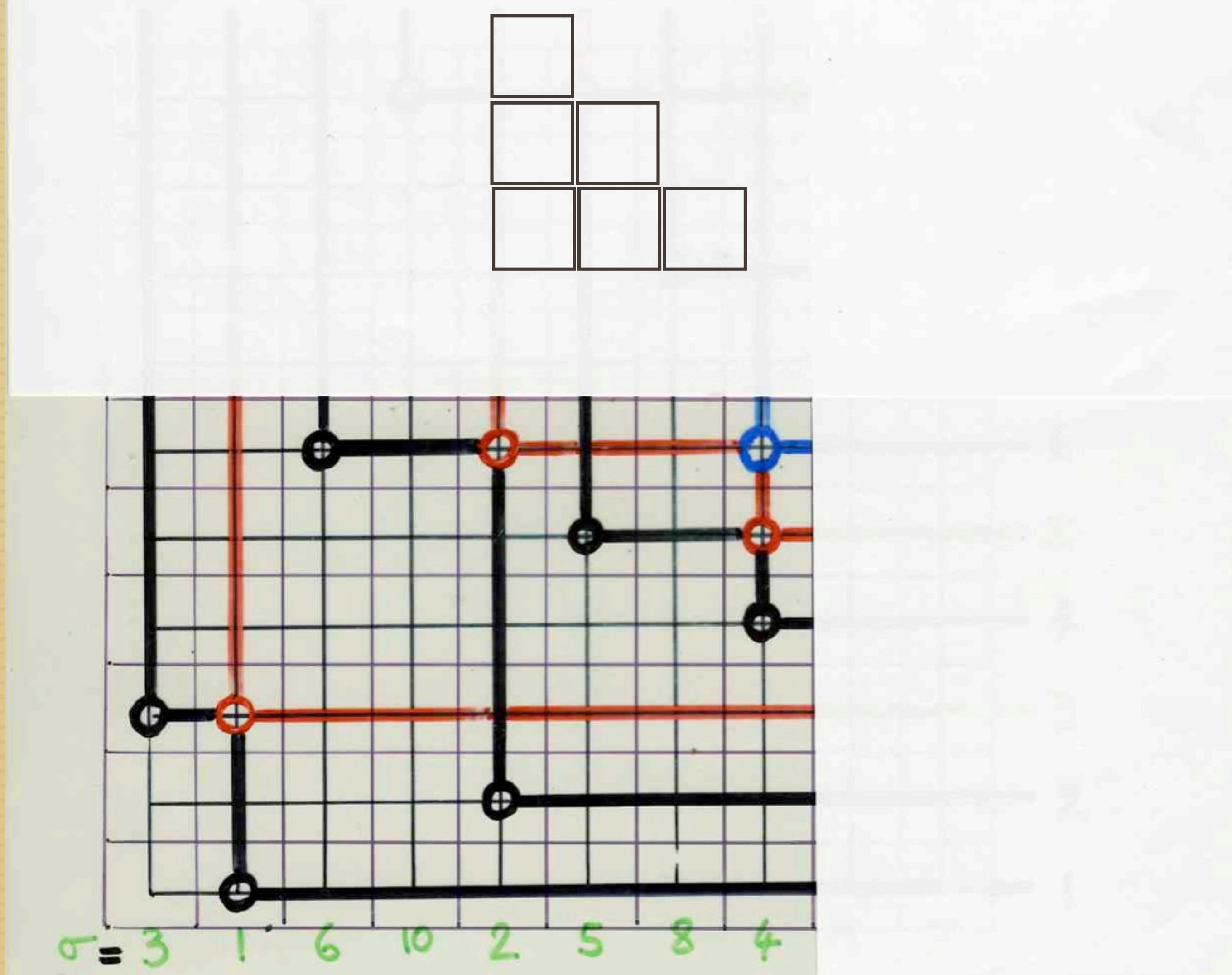
RSK “géométrique” = l’algorithme “local”

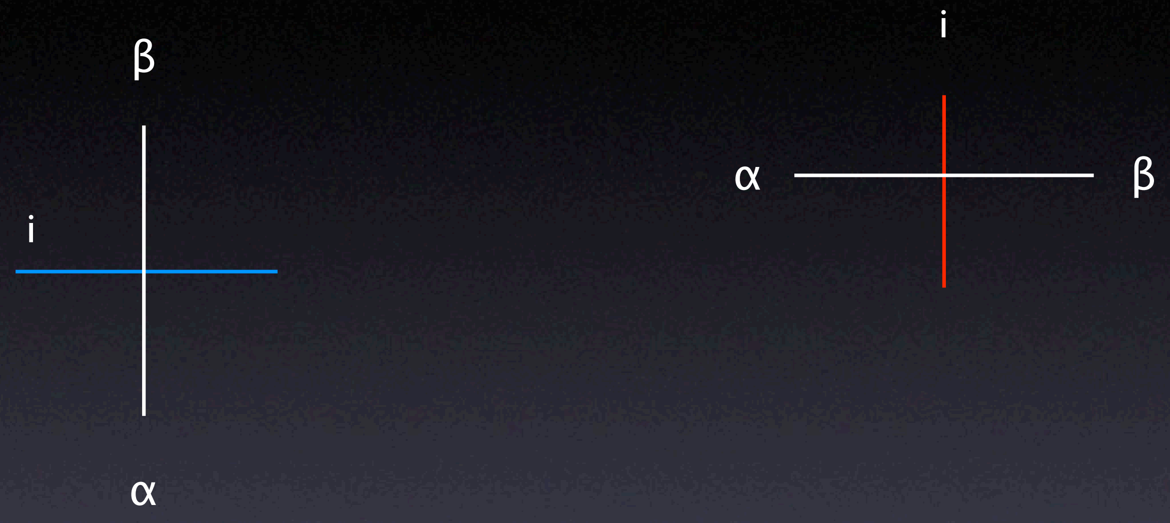

3 1 6 10 2 5 8 4 9 7

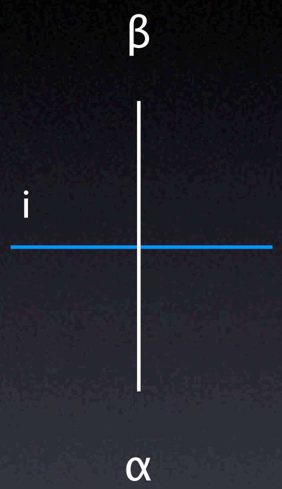

β ≠ γ
β
δ
1
α
γ
δ = β ∪ γ
β = γ
α ≠ β
β
δ
2
α
γ
β = γ = α+ (i)
δ = β+ (i+1)
α = β = γ
β
δ
3
α
γ
δ = α + (1)
α = β = γ
β
δ
6
α
γ
δ = α = β = γ



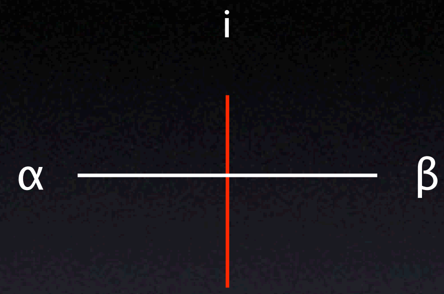
1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
σ = 3 1 6 10 2 5 8 4 9 7

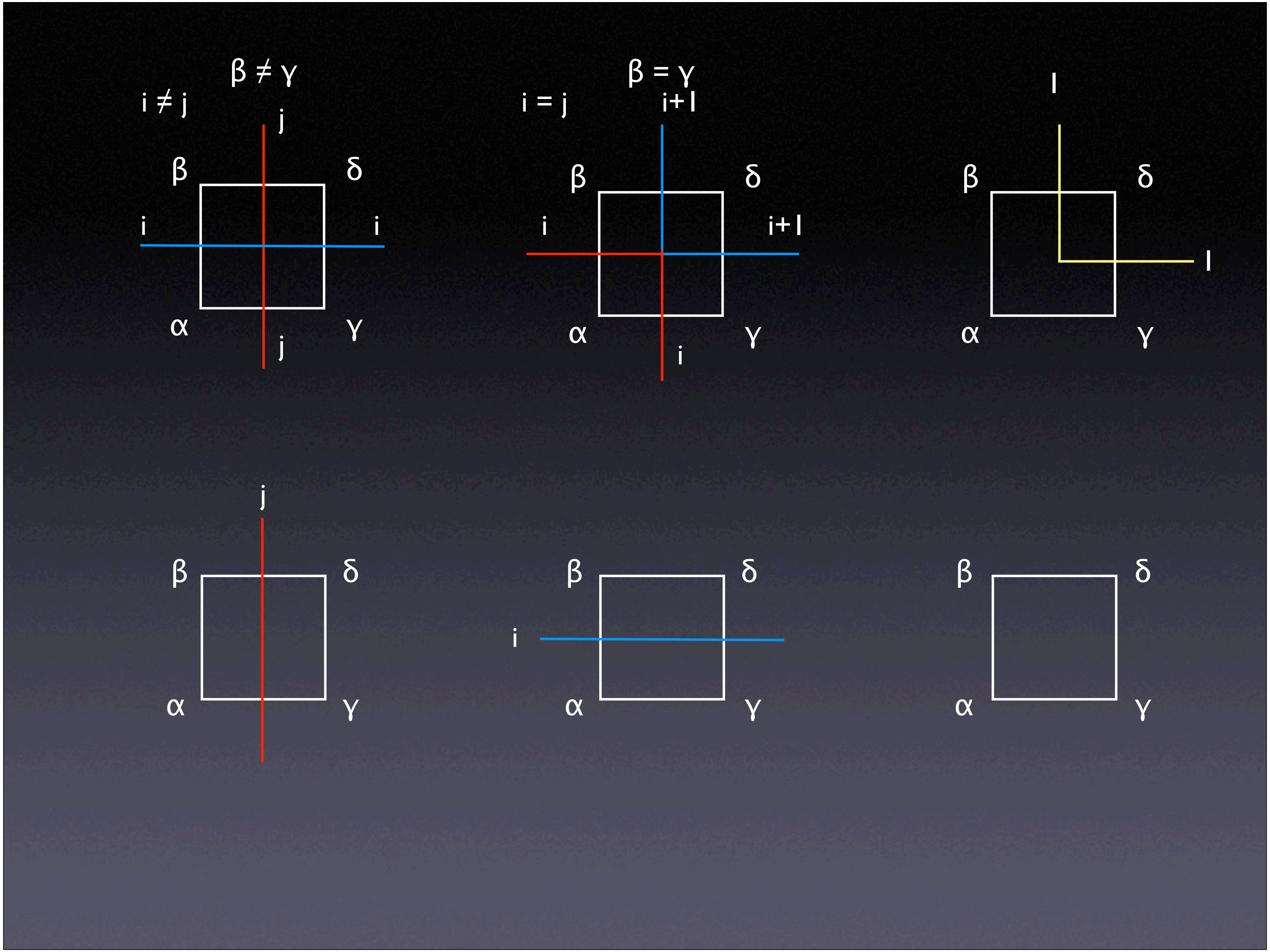

σ = 3 1 6 10 2 5 8 4


β
i
α
i
α
β

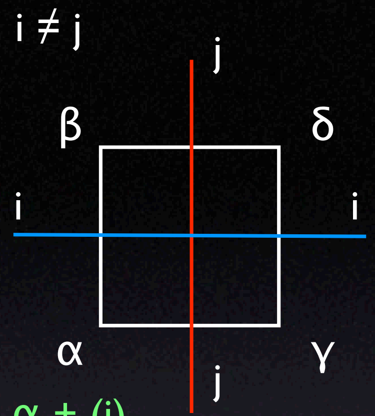


$\beta = \alpha + (i)$

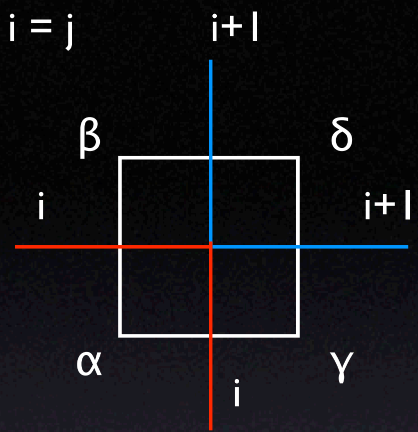




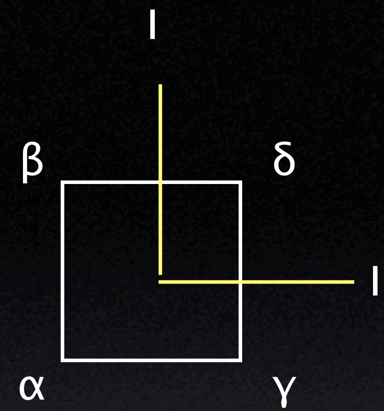
β ≠ γ
i ≠ j
j
β
δ
i
i
α
j
γ
β = γ
i = j
i+1
β
δ
i
i+1
α
i
γ
1
β
δ
1
α
γ
j
β
δ
α
γ
β
δ
i
α
γ
β
δ
α
γ



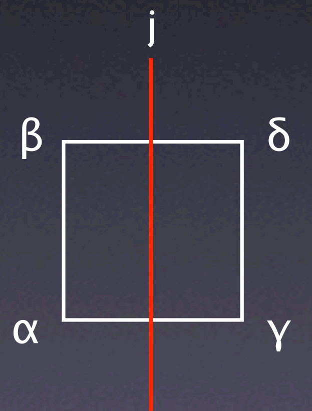
$\beta = \alpha + (i)$
$\gamma = \alpha + (j)$
$\delta = \alpha + (i) + (j)$



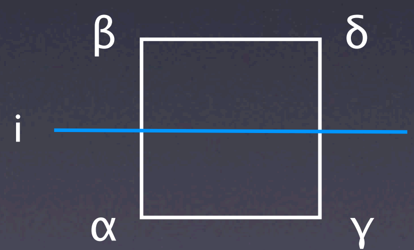
$\beta = \gamma = \alpha + (i)$
$\delta = \alpha + (i) + (i+1)$



$\beta = \gamma = \alpha$
$\delta = \alpha + (1)$

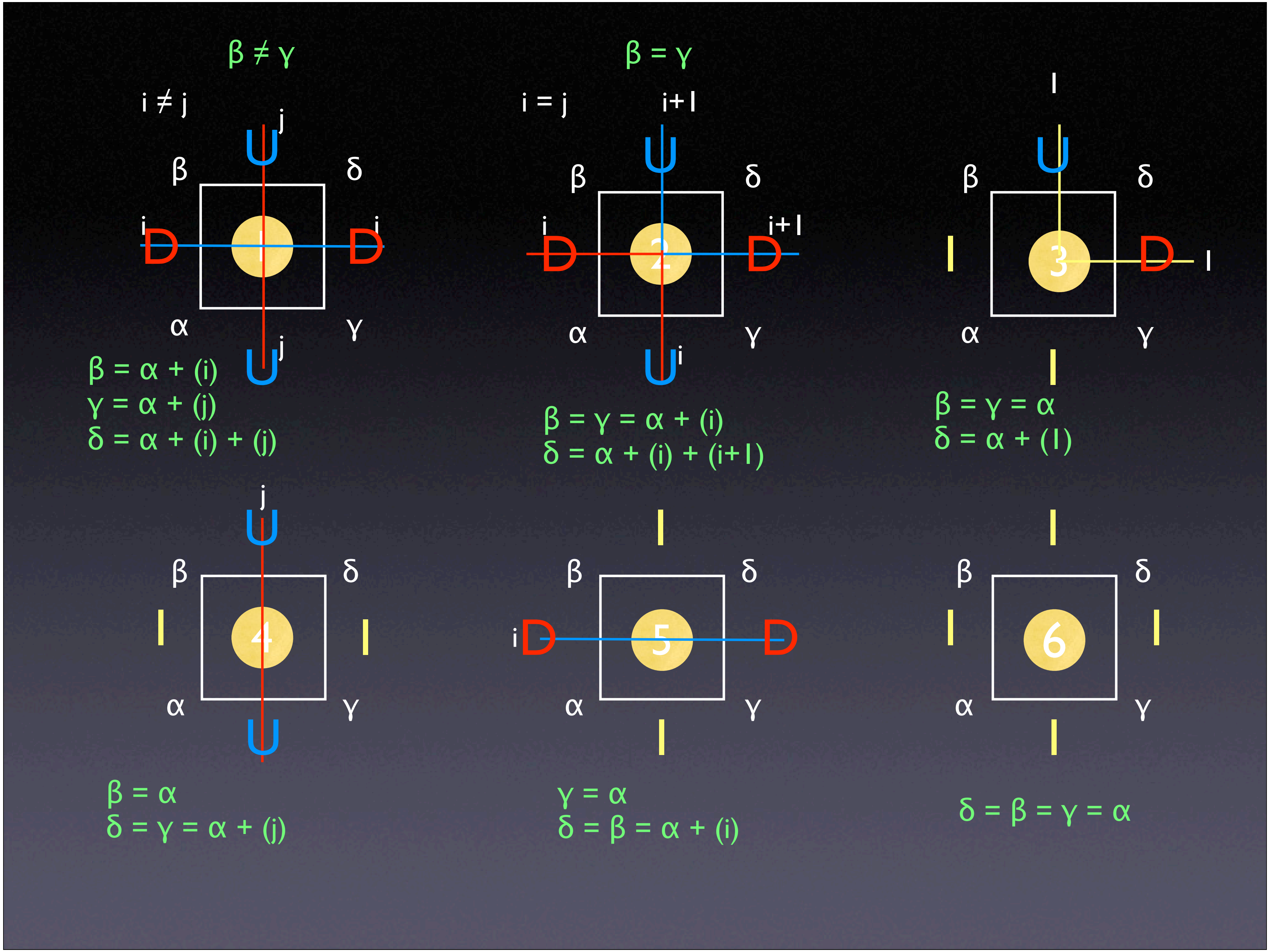


$\beta = \alpha$
$\delta = \gamma = \alpha + (j)$

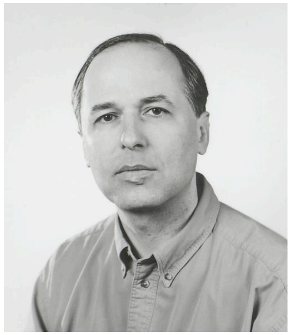


$\gamma = \alpha$
$\delta = \beta = \alpha + (i)$

$\delta = \beta = \gamma = \alpha$


β ≠ γ
i ≠ j
β = α + (i)
γ = α + (j)
δ = α + (i) + (j)
β = γ
i = j
i+1
β = γ = α + (i)
δ = α + (i) + (i+1)
β = γ = α
δ = α + (1)
β = α
δ = γ = α + (j)
γ = α
δ = β = α + (i)
δ = β = γ = α

Sergey Fomin



- **Schur operators and Knuth correspondences**, *Journal of Combinatorial Theory, Ser.A* **72** (1995), 277-292.
- **Duality of graded graphs**, *Journal of Algebraic Combinatorics* **3** (1994), 357-404.
- **Schensted algorithms for dual graded graphs**, *Journal of Algebraic Combinatorics* **4** (1995), 5-45.
- **Dual graphs and Schensted correspondences**, *Series formelles et combinatoire algebrique*, P.Leroux and C.Reutenauer, Ed., Montreal, LACIM, UQAM, 1992, 221-236.

- **Finite posets and Ferrers shapes** (with T.Britz, 41 pages)

Advances in Mathematics **158** (2000), 86-127.
A survey on the Greene-Kleitman correspondence; many proofs are new.

- **Knuth equivalence, jeu de taquin, and the Littlewood-Richardson rule** (30 pages)

Appendix 1 to Chapter 7 in: R.P.Stanley, *Enumerative Combinatorics, vol.2*,
Cambridge University Press, 1999.

Richard P. Stanley

- **Differential posets, *J. Amer. Math. Soc.* 1 (1988), 919-961.**
- **Variations on differential posets, in *Invariant Theory and Tableaux* (D. Stanton, ed.),**

The IMA Volumes in Mathematics and Its Applications, vol. 19, Springer-Verlag, New York, 1990, pp. 145-165.

Xavier Gérard Viennot

- Une forme géométrique de la correspondane de Robinson-Schensted, in "Combinatoire et Représentation du groupe symétrique" (D. Foata ed.) Lecture Notes in Mathematics n° 579, pp 29-68, 1976

Marc van Leeuwen

- The Robinson-Schensted and Schützenberger algorithms, an elementary approach

(a 272 Kb dvi file) Electronic Journal of Combinatorics, Foata Festschrift, Vol 3(no.2), R15 (1996)

Guoniu Han

http://math.u-strasbg.fr/~guoniu/software/rsk/index.html
Autour de la correspondance de Robinson-Schensted
Exposé au SLC 52 et LascouxFest, 29/03/2004

merci !