

Duchon le 2 février
(45€)

*Alternating sign matrices
and domino tilings*

Noam Elkies

Harvard College

Greg Kuperberg

University of California at Berkeley

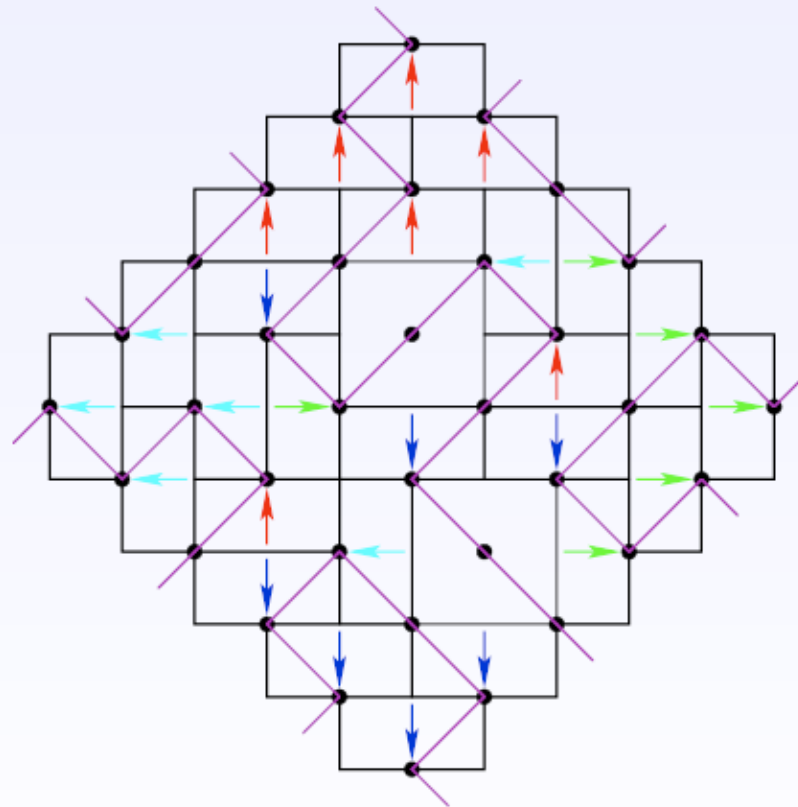
Michael Larsen

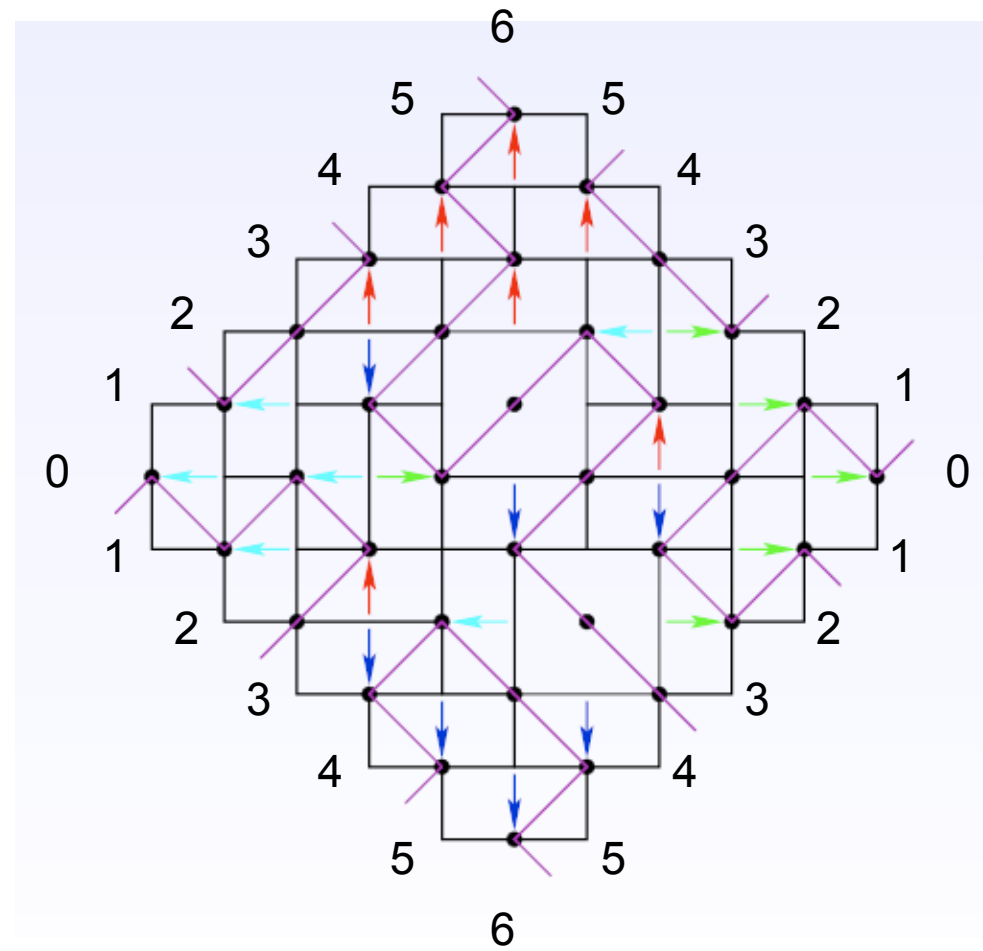
University of Pennsylvania

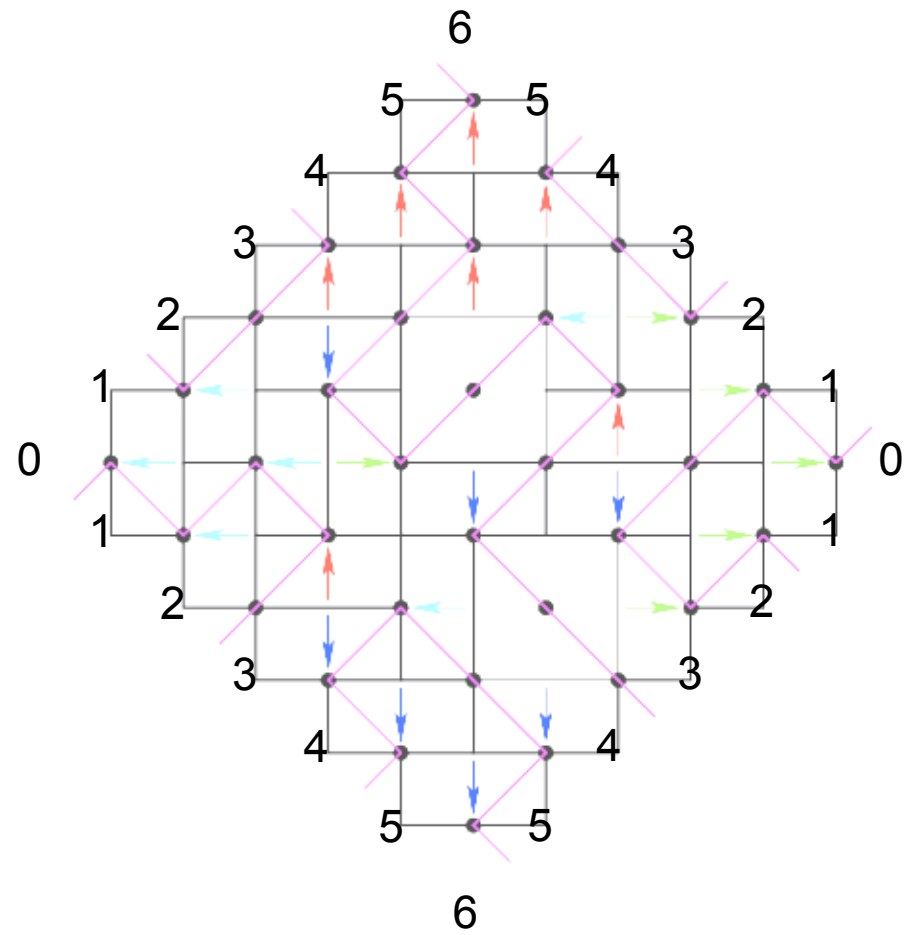
James Propp

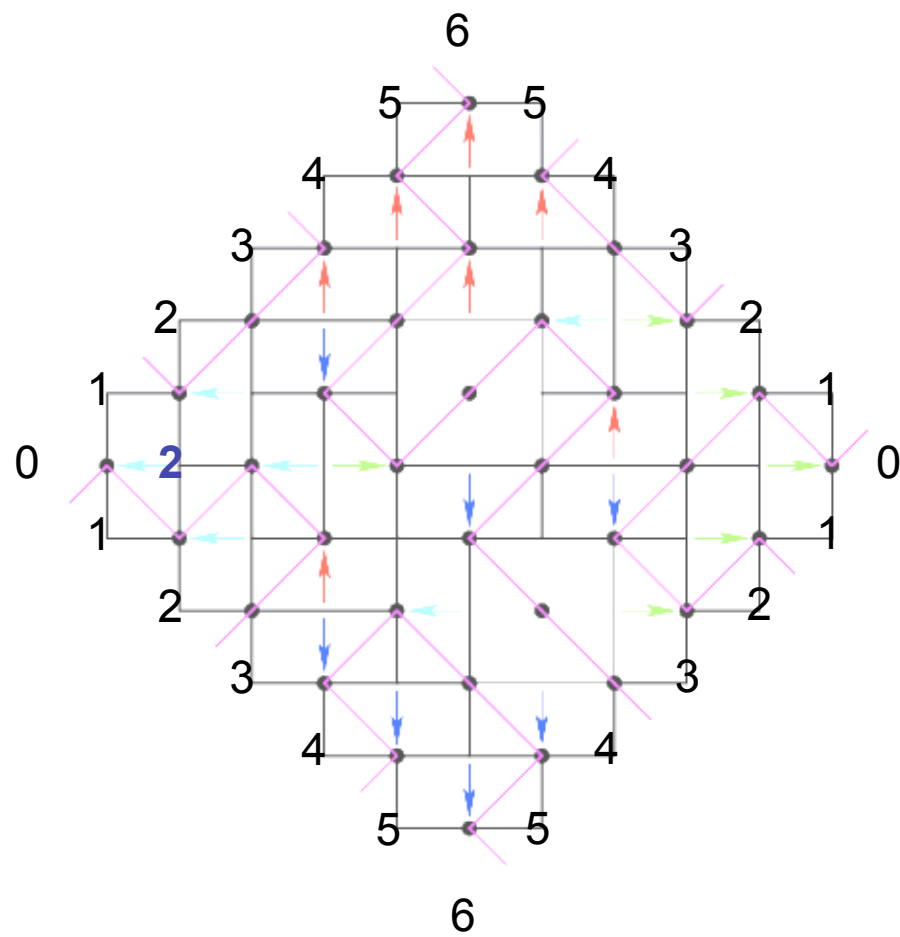
Massachusetts Institute of Technology

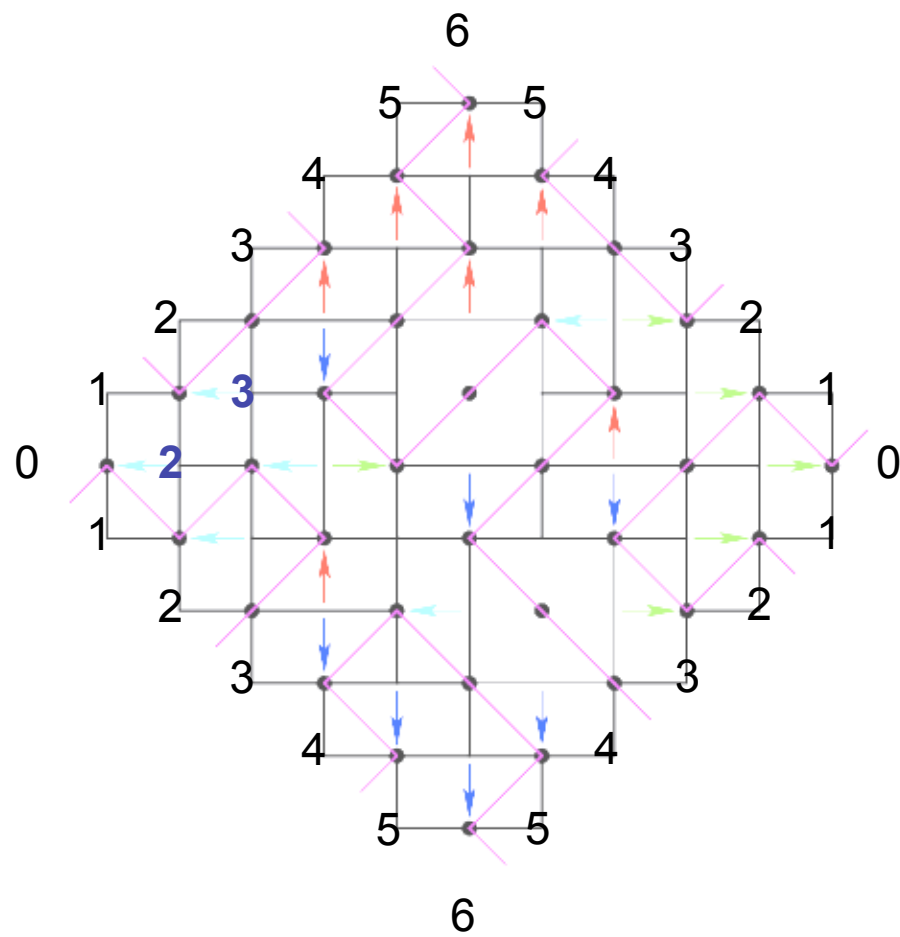
June 1991

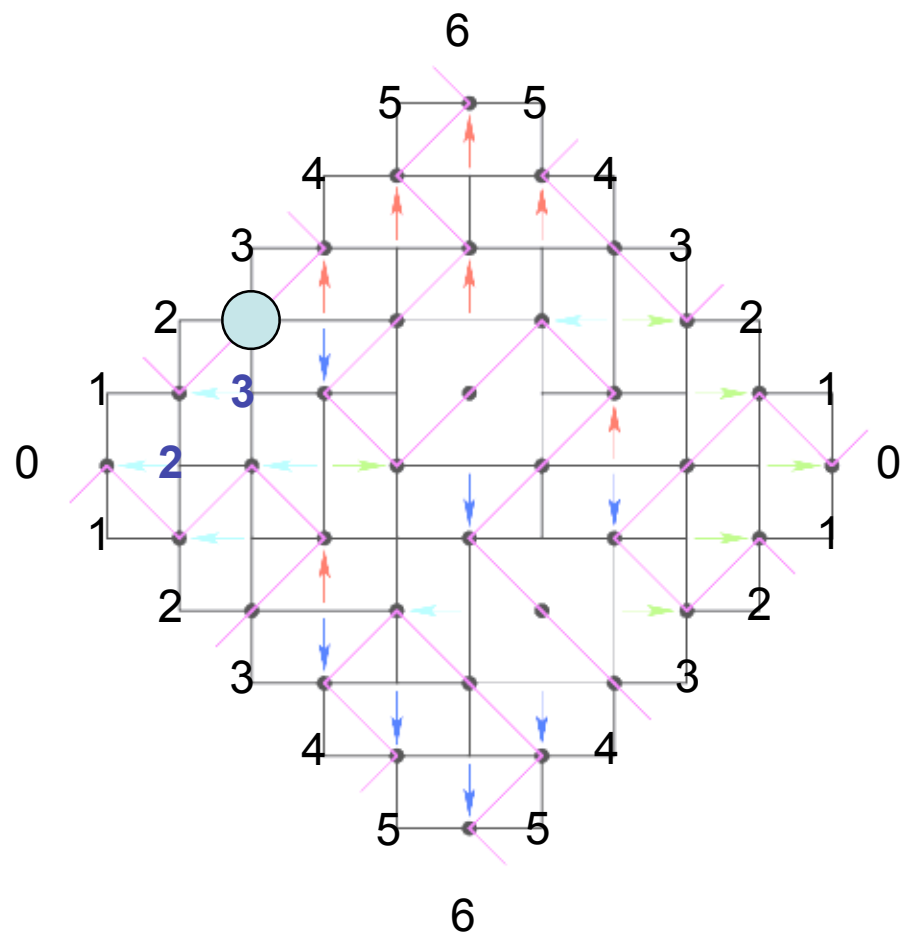


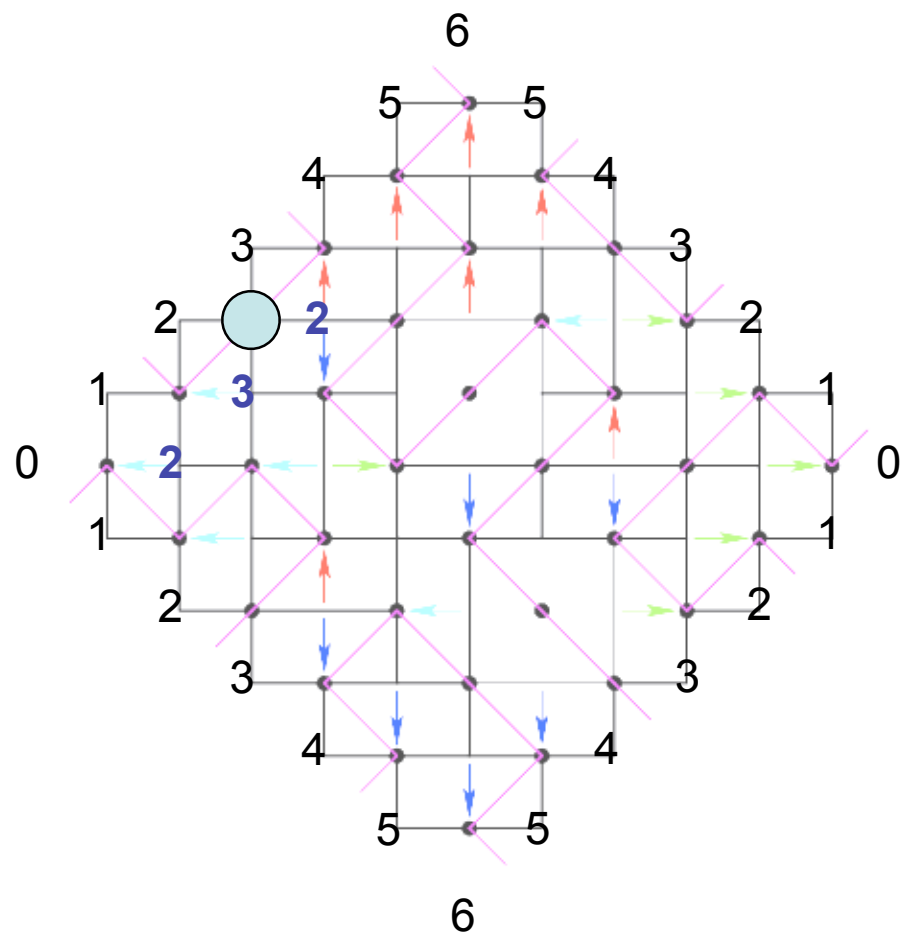


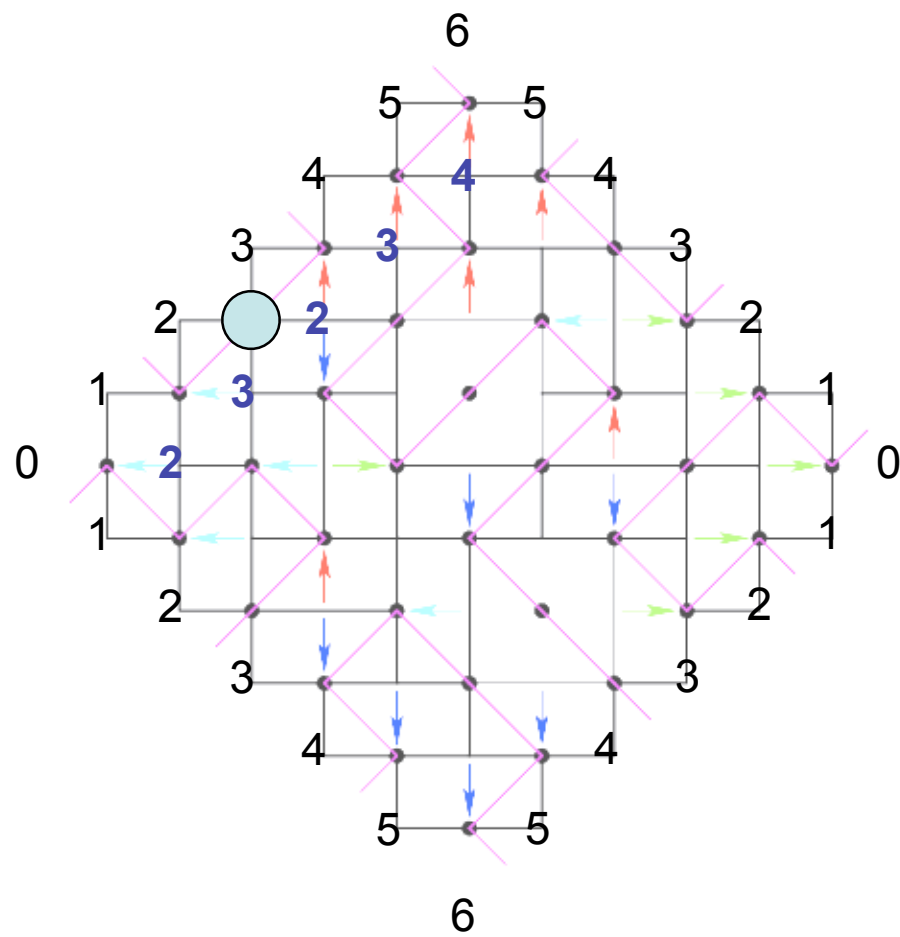


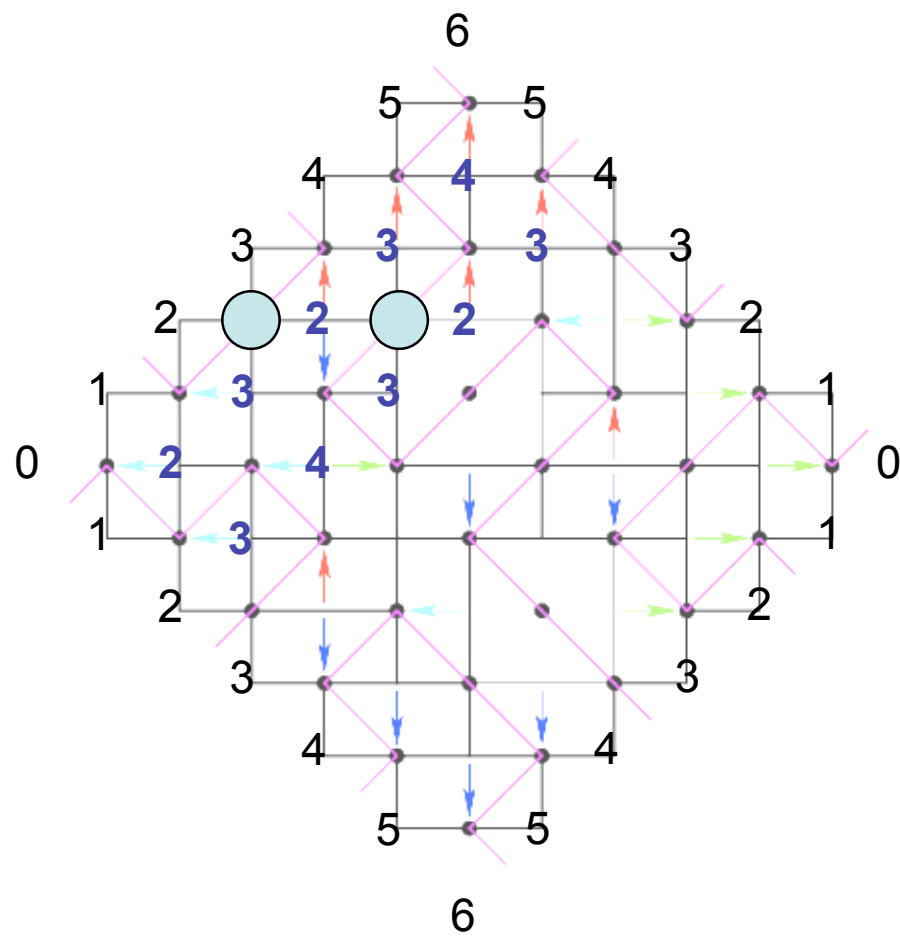


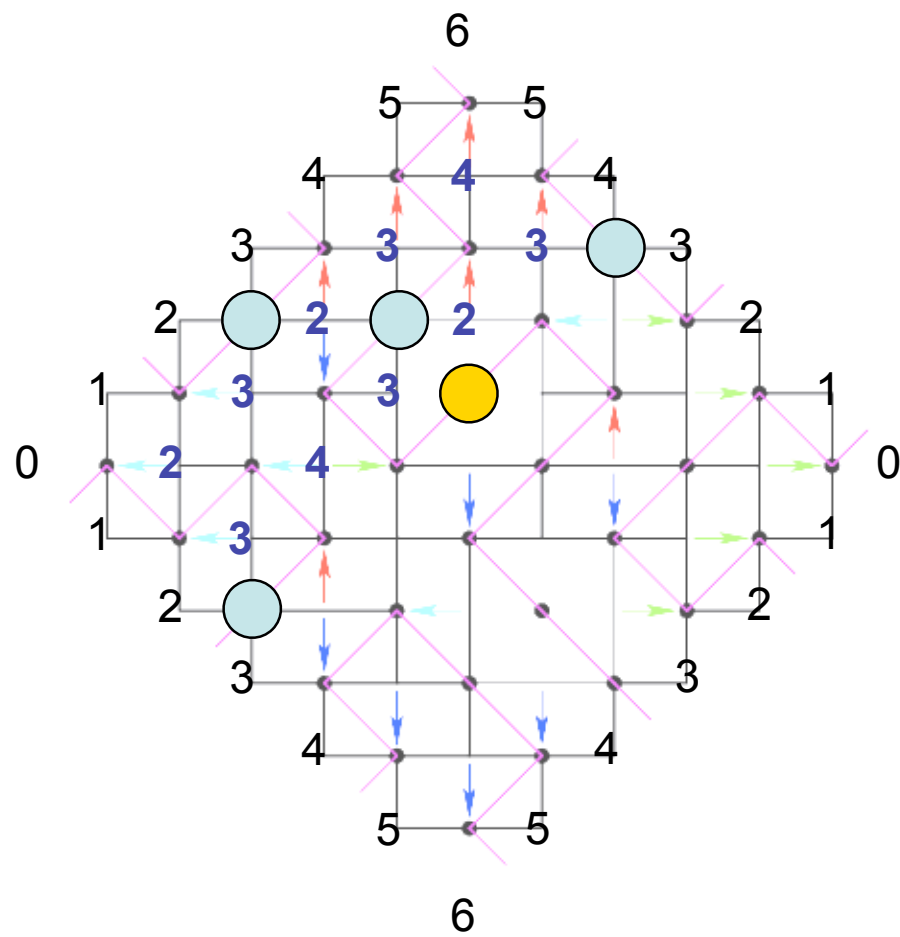


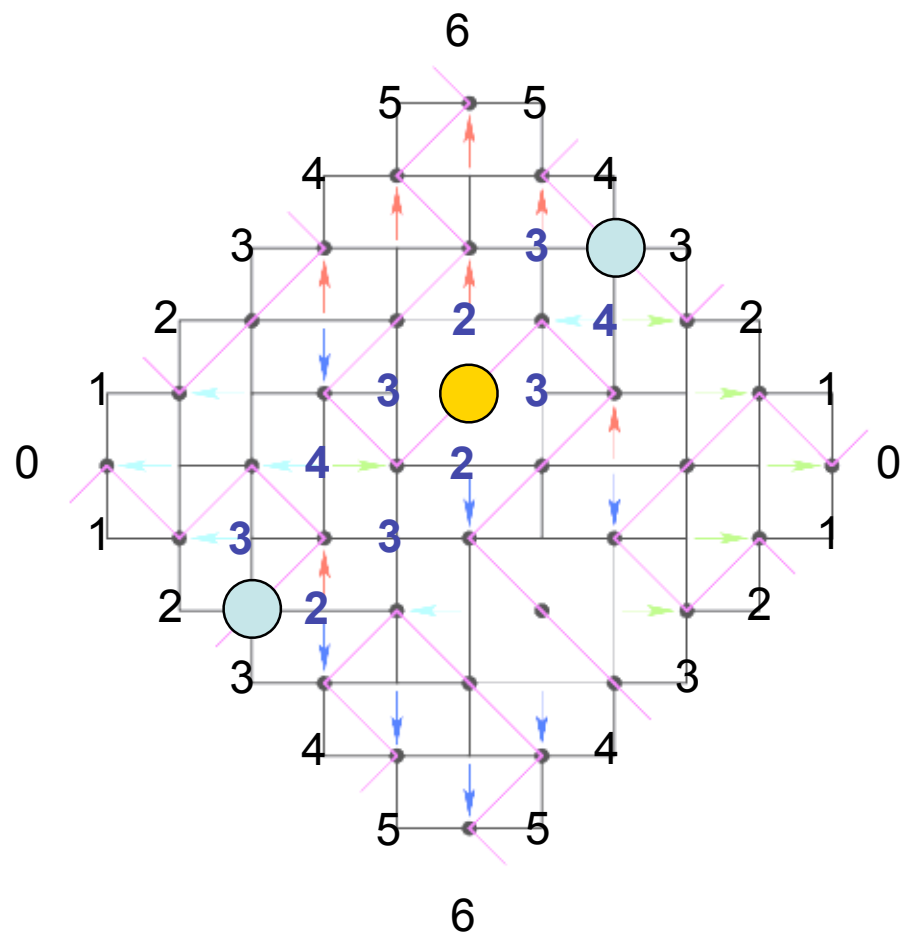






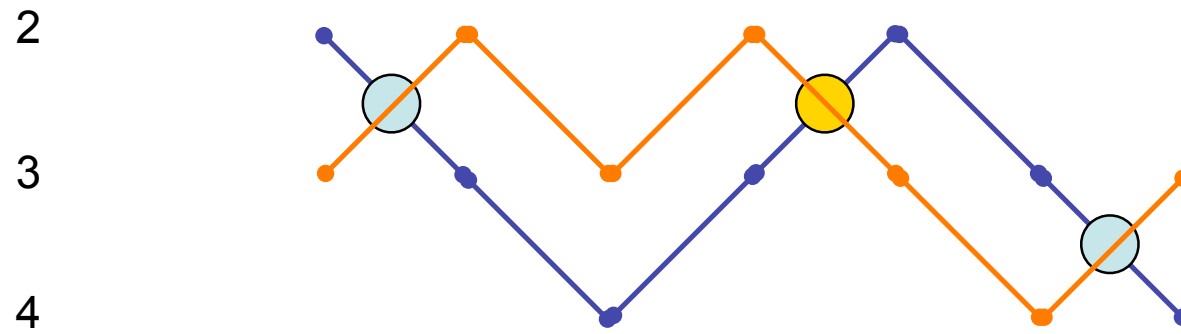


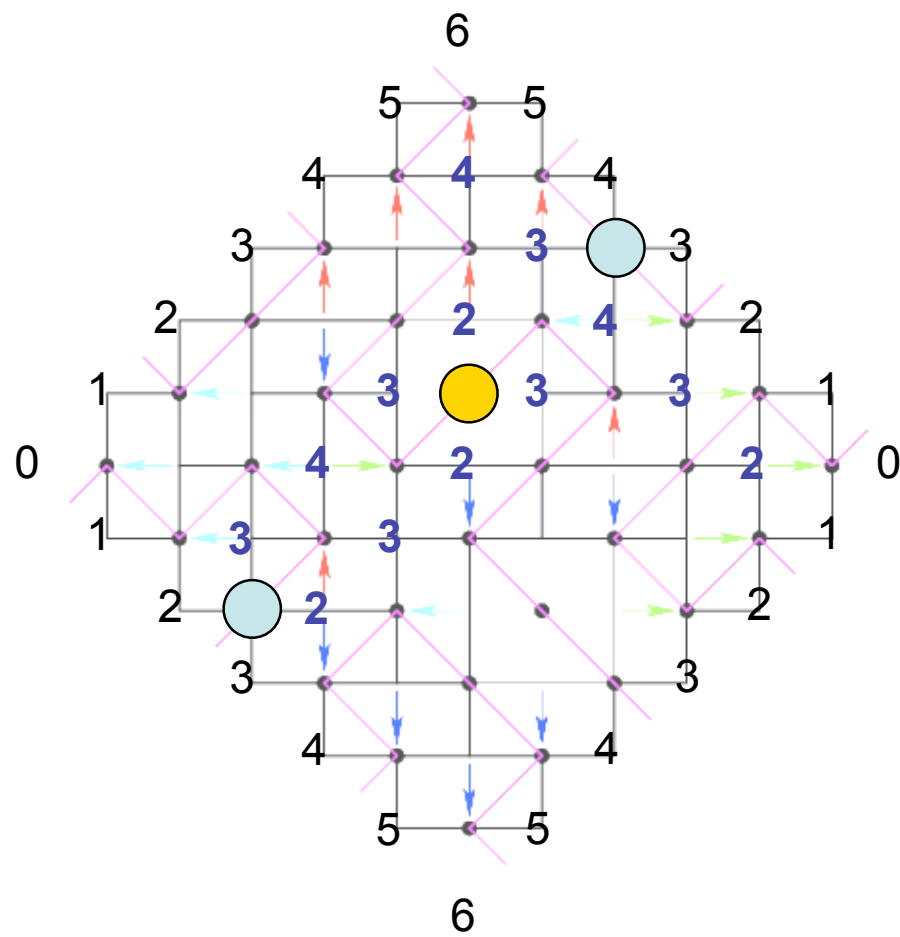


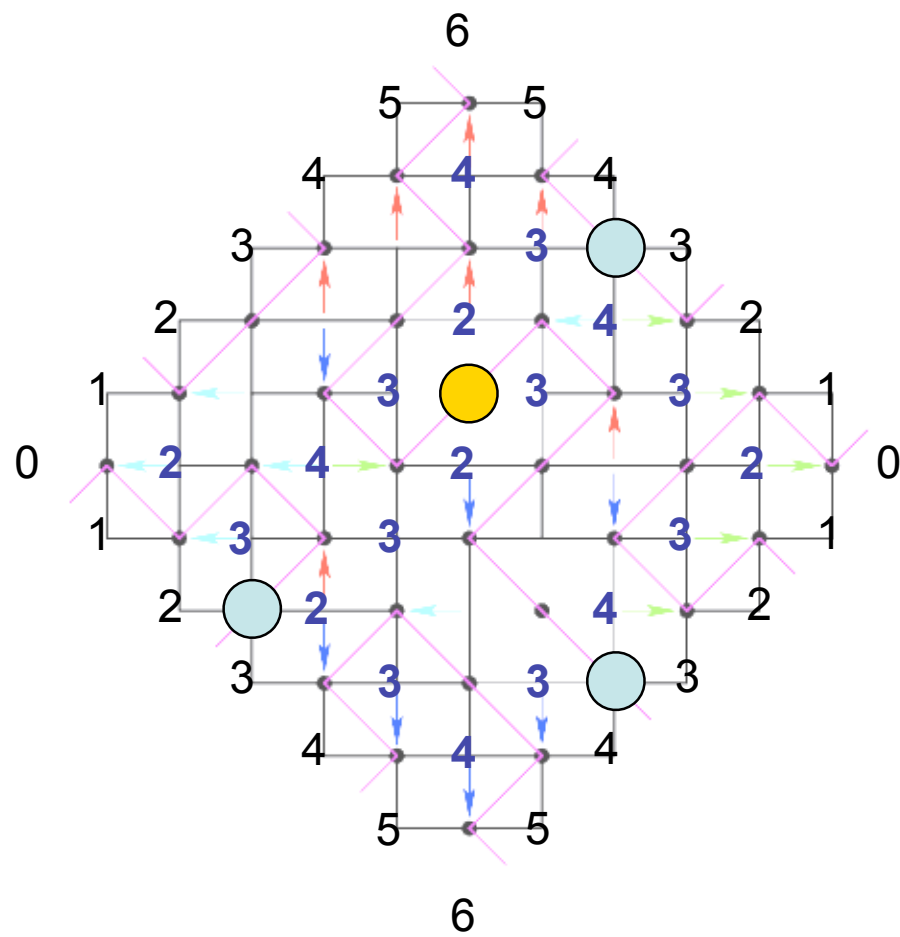


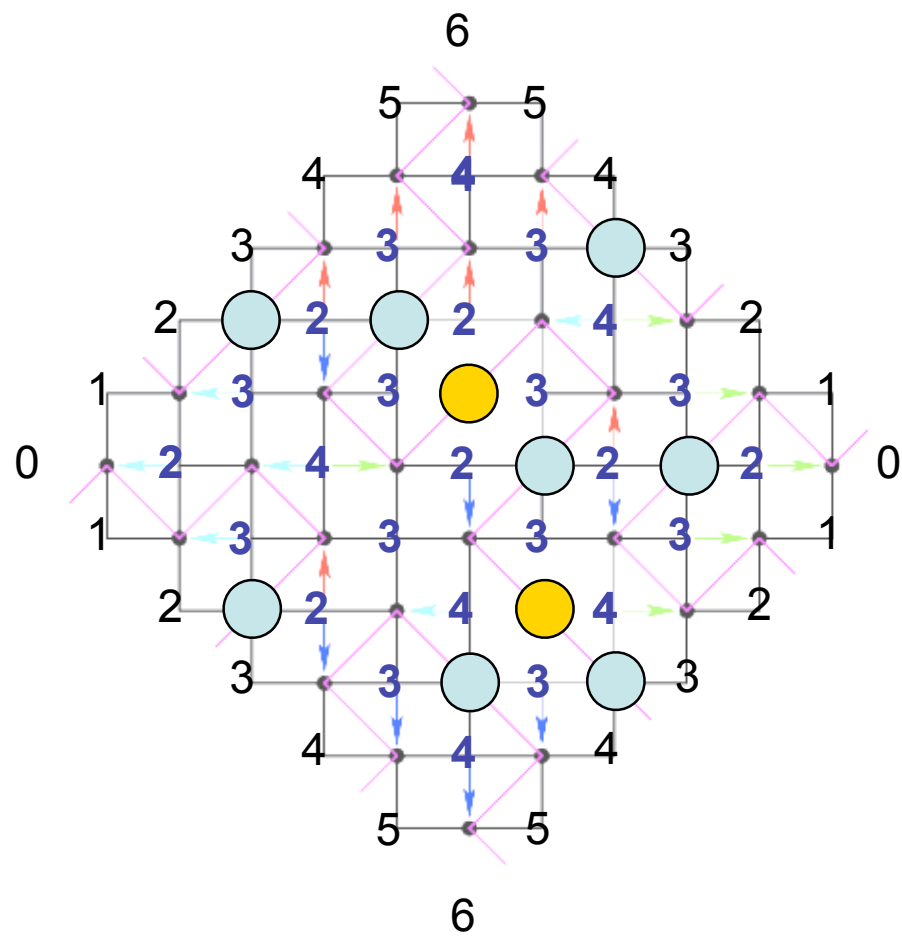
2 3 4 3 2 3 4

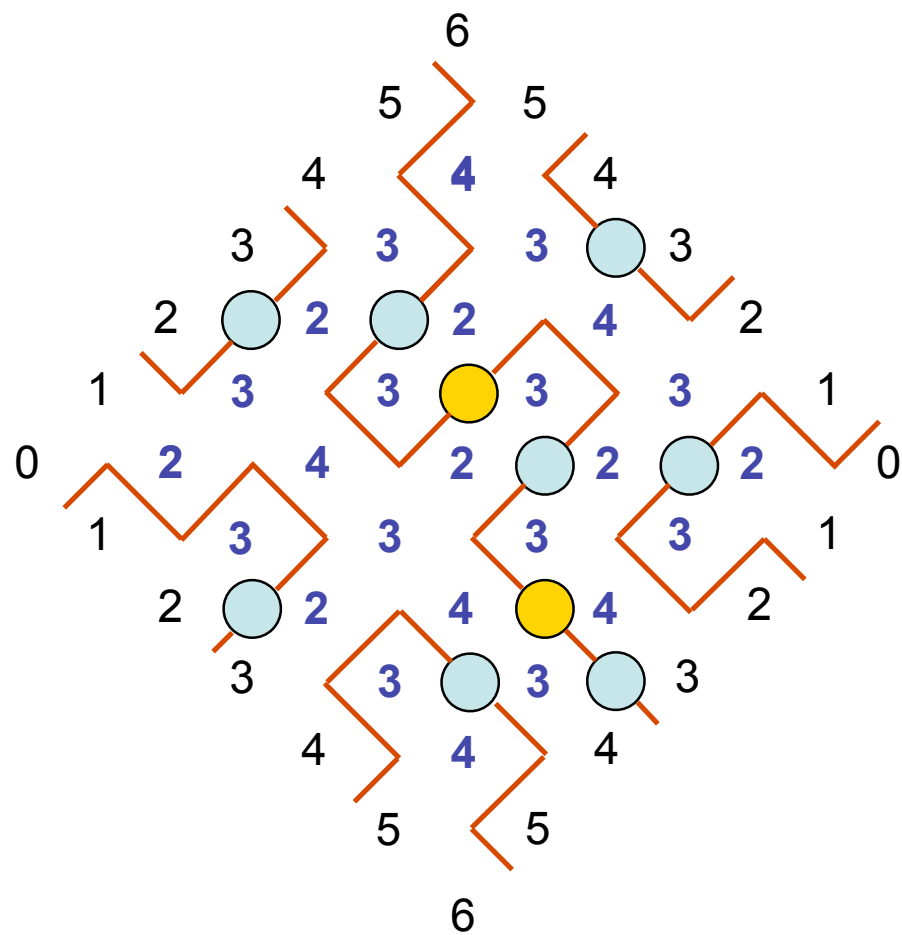
3 2 3 2 3 4 3





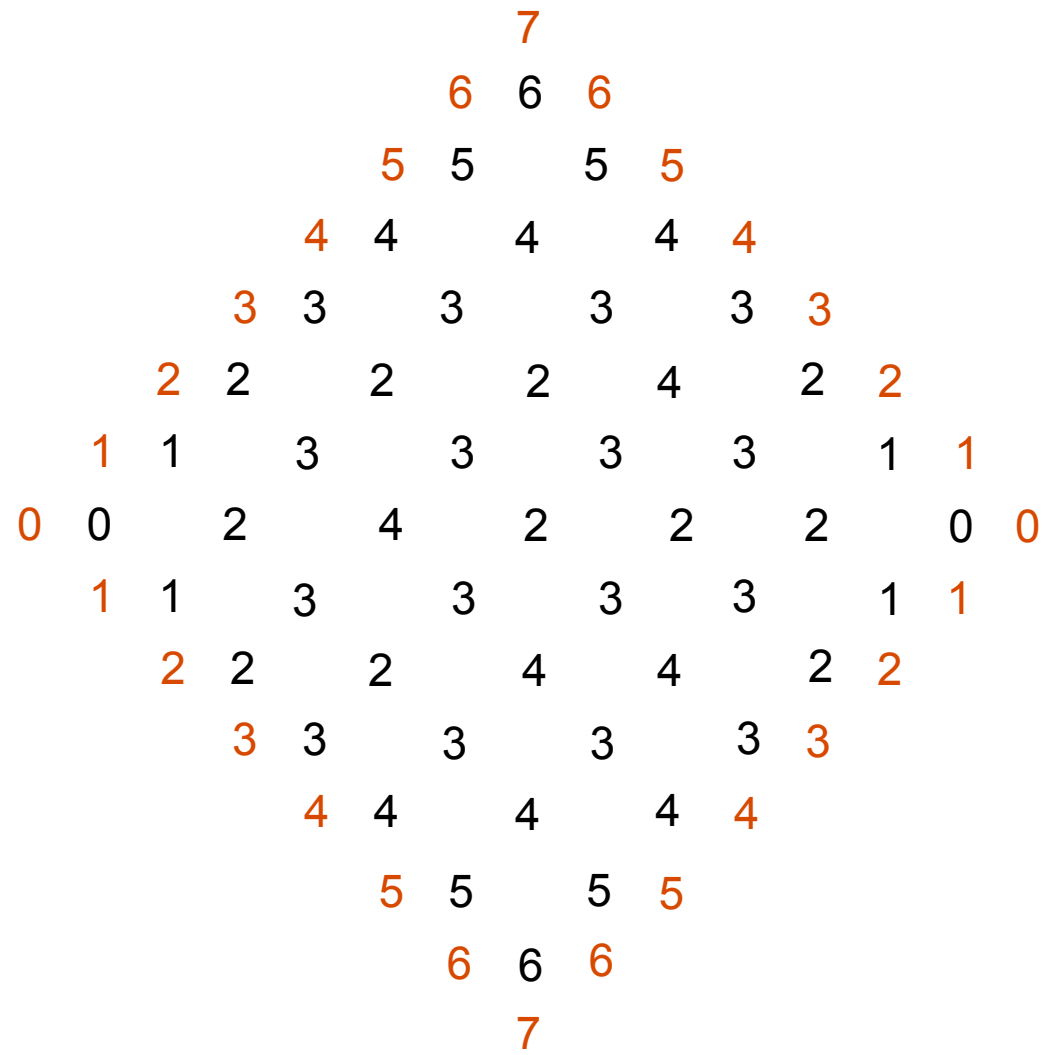


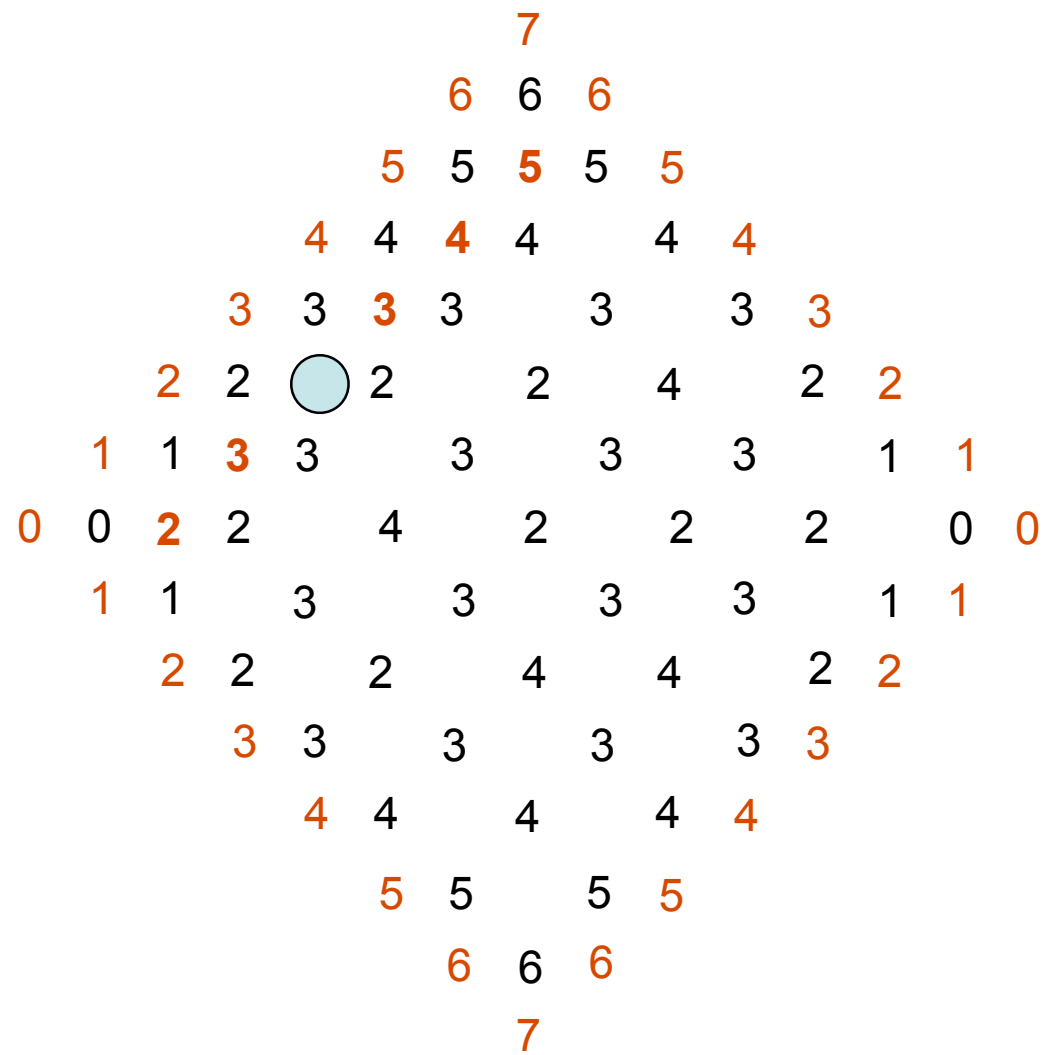


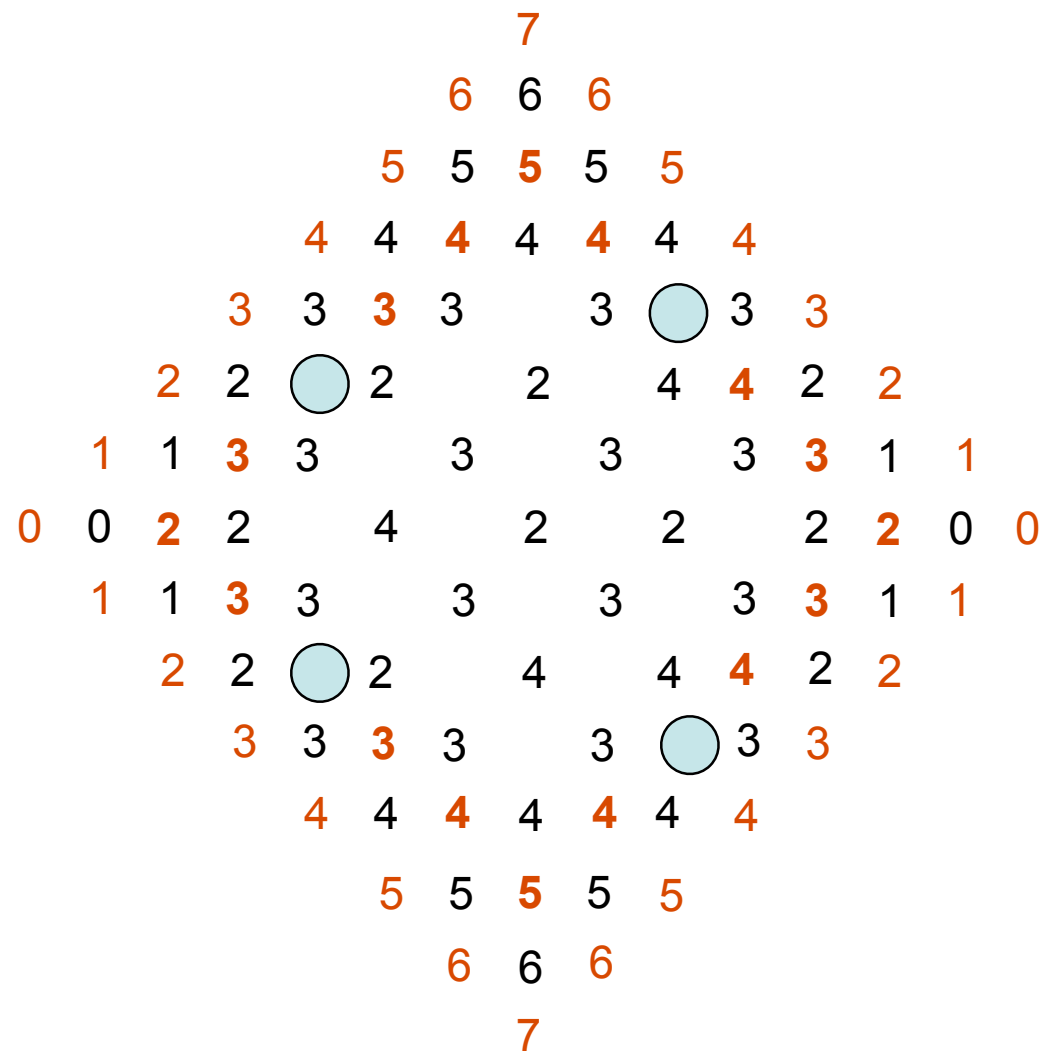


ü matrice à signes alternants ->
matrice des hauteurs (rappel)

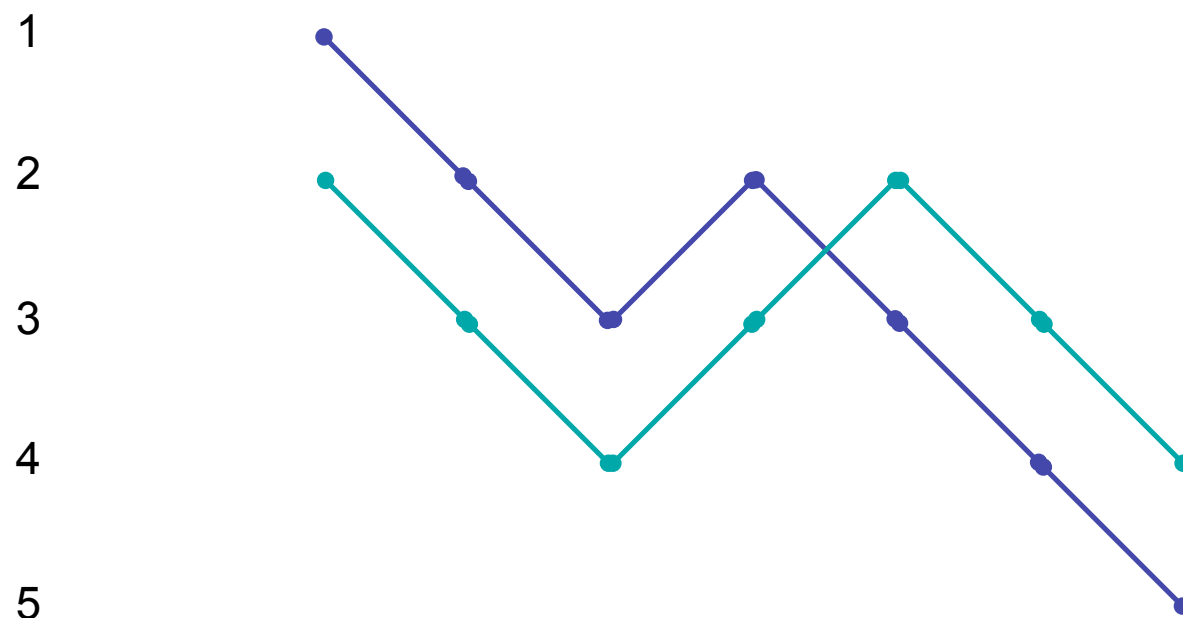
- gonfler une matrice de hauteurs



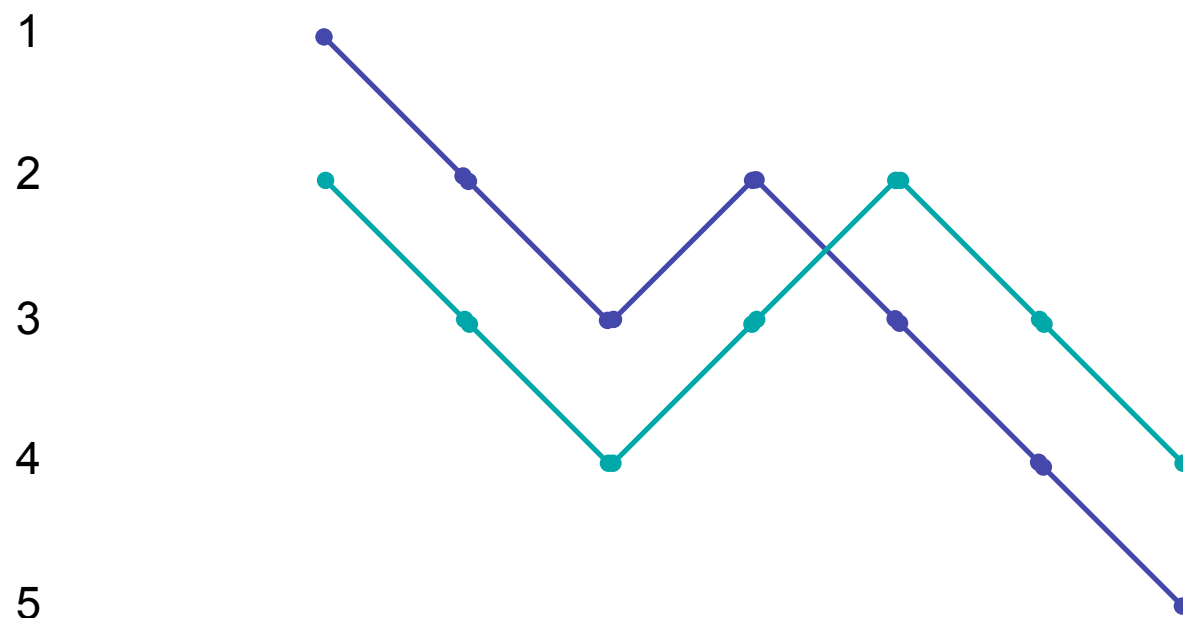




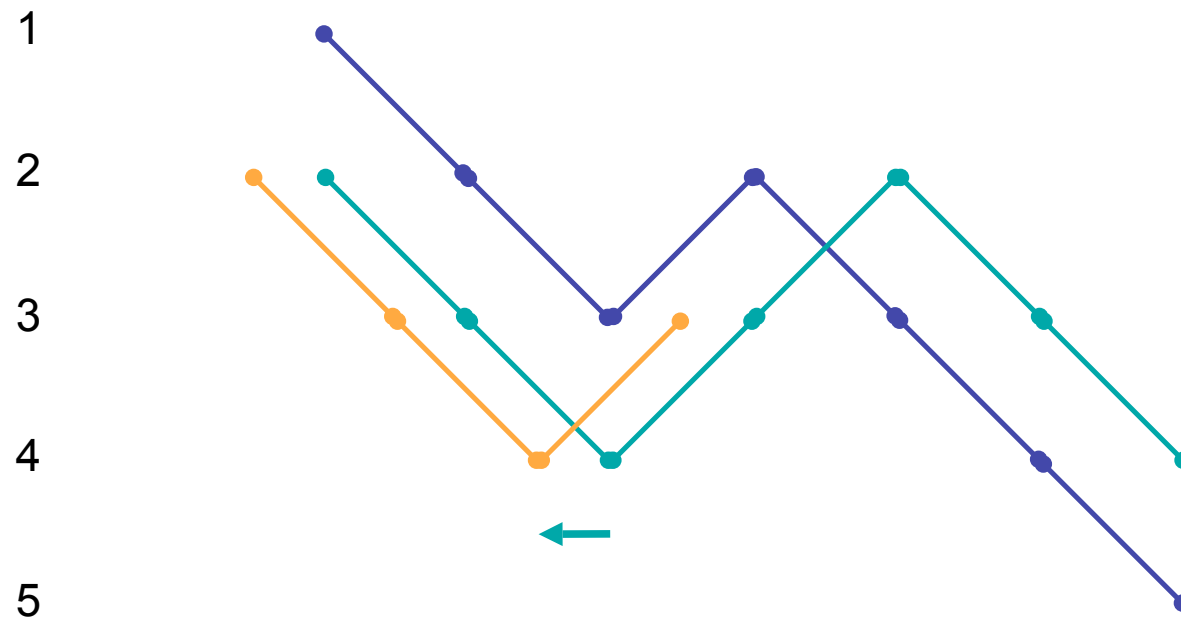
1 2 3 2 3 4 5
2 3 4 3 2 3 4



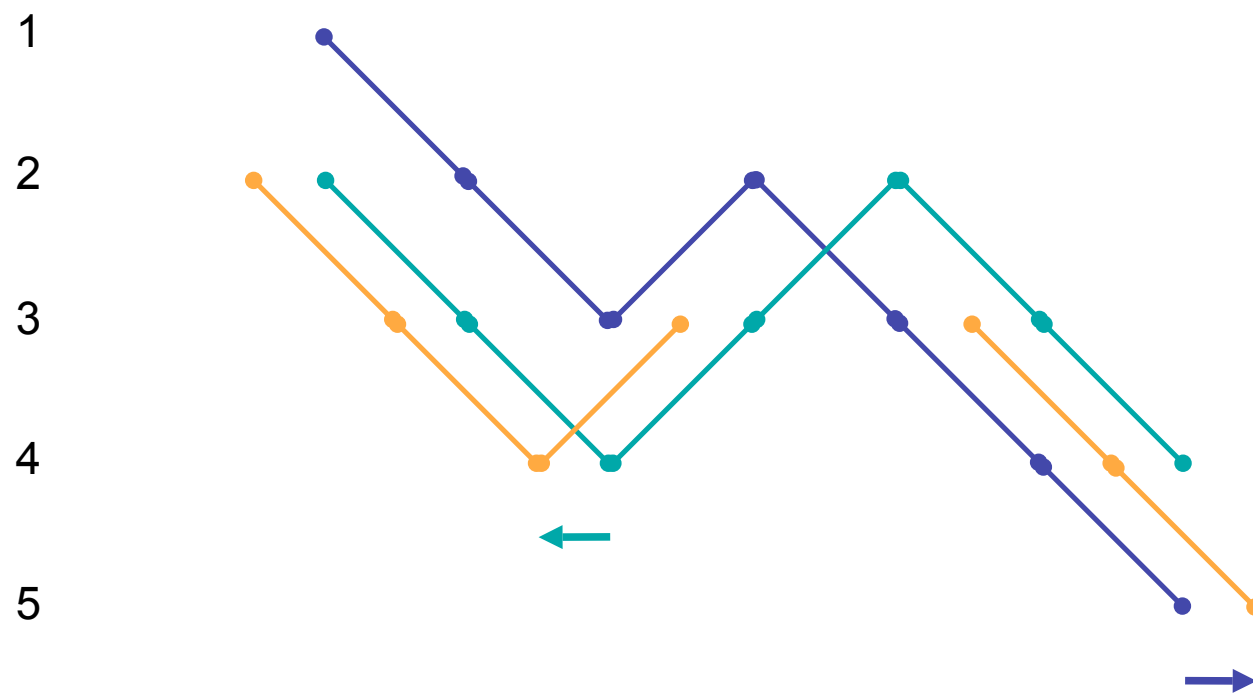
1 2 3 2 3 4 5
2 5
2 3 4 3 2 3 4



1 2 3 2 3 4 5
2 3 4 3 5
2 3 4 3 2 3 4

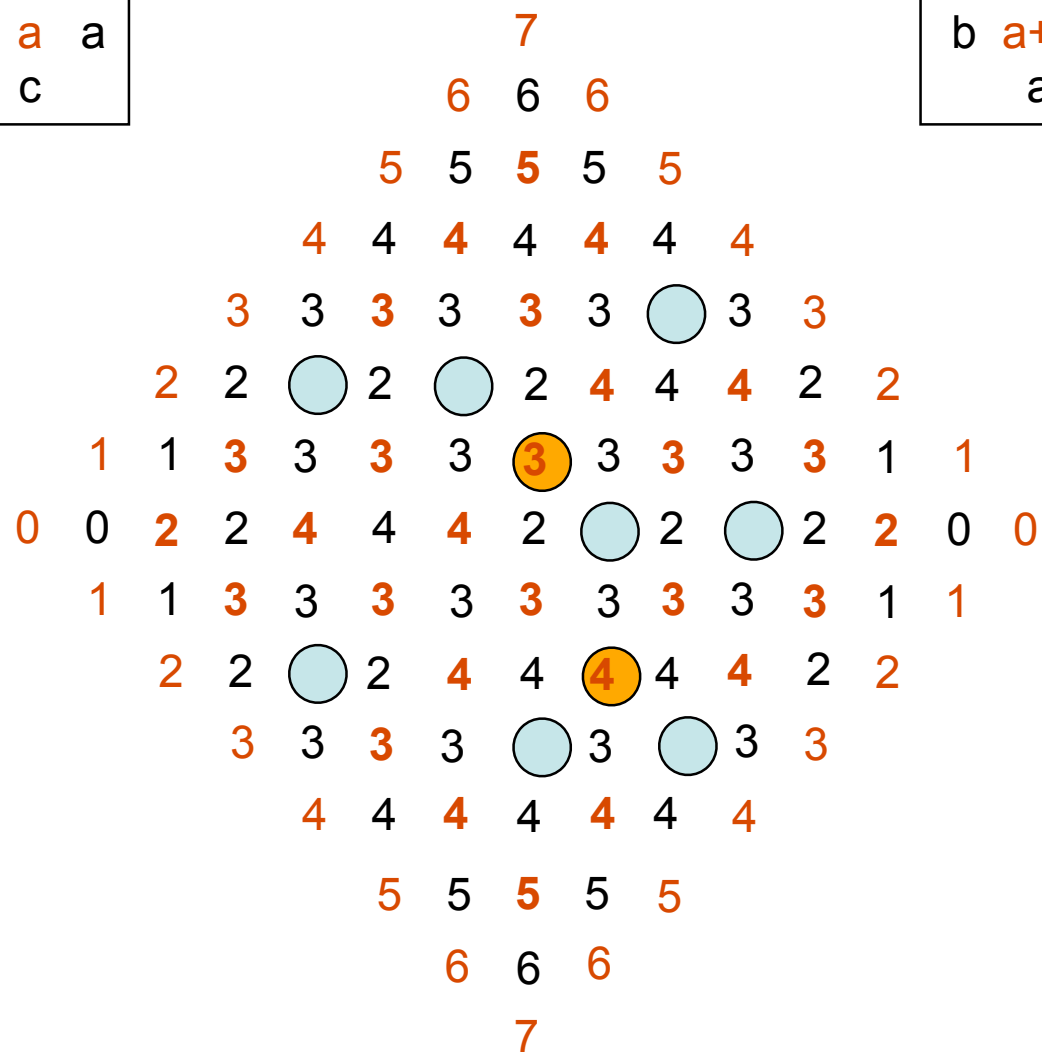


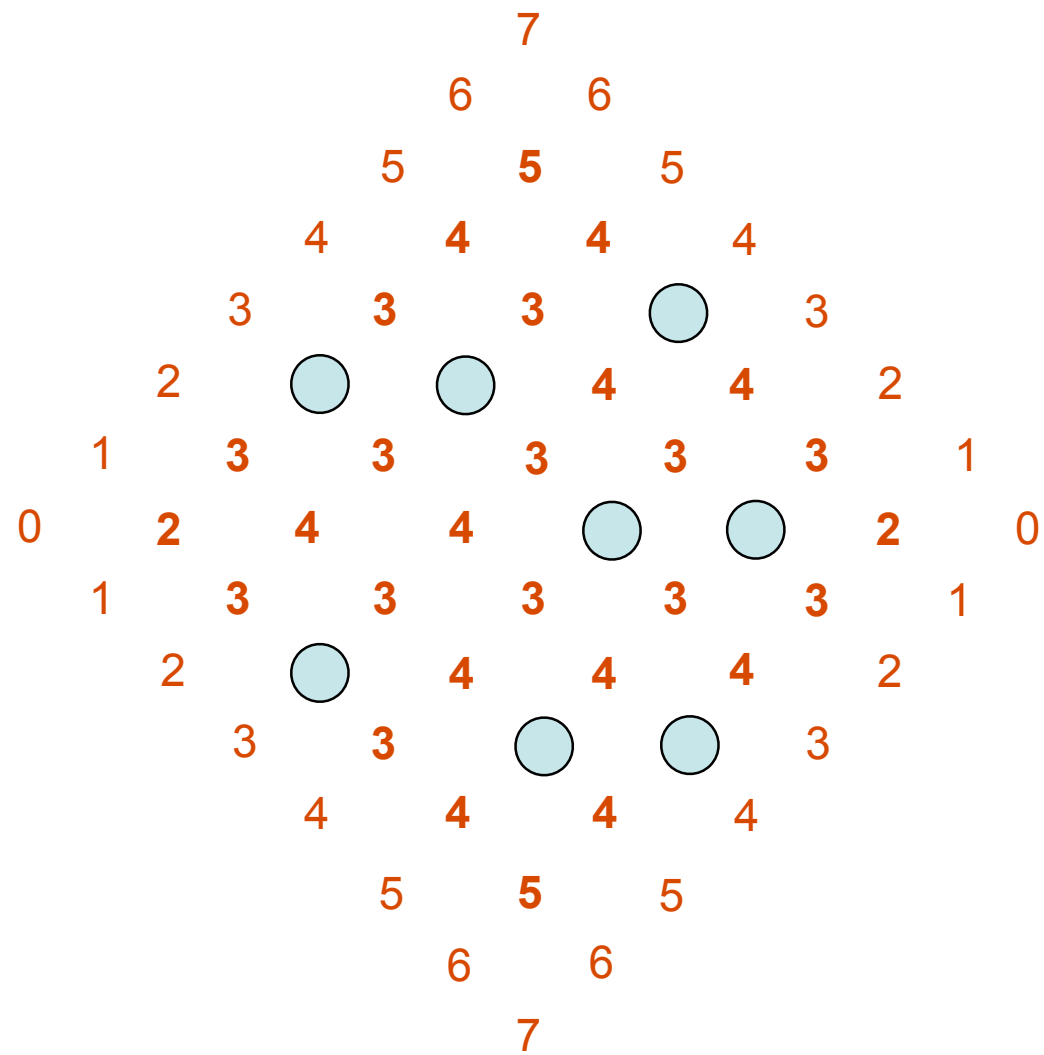
1 2 3 2 3 4 5
2 3 4 3 3 4 5
2 3 4 3 2 3 4



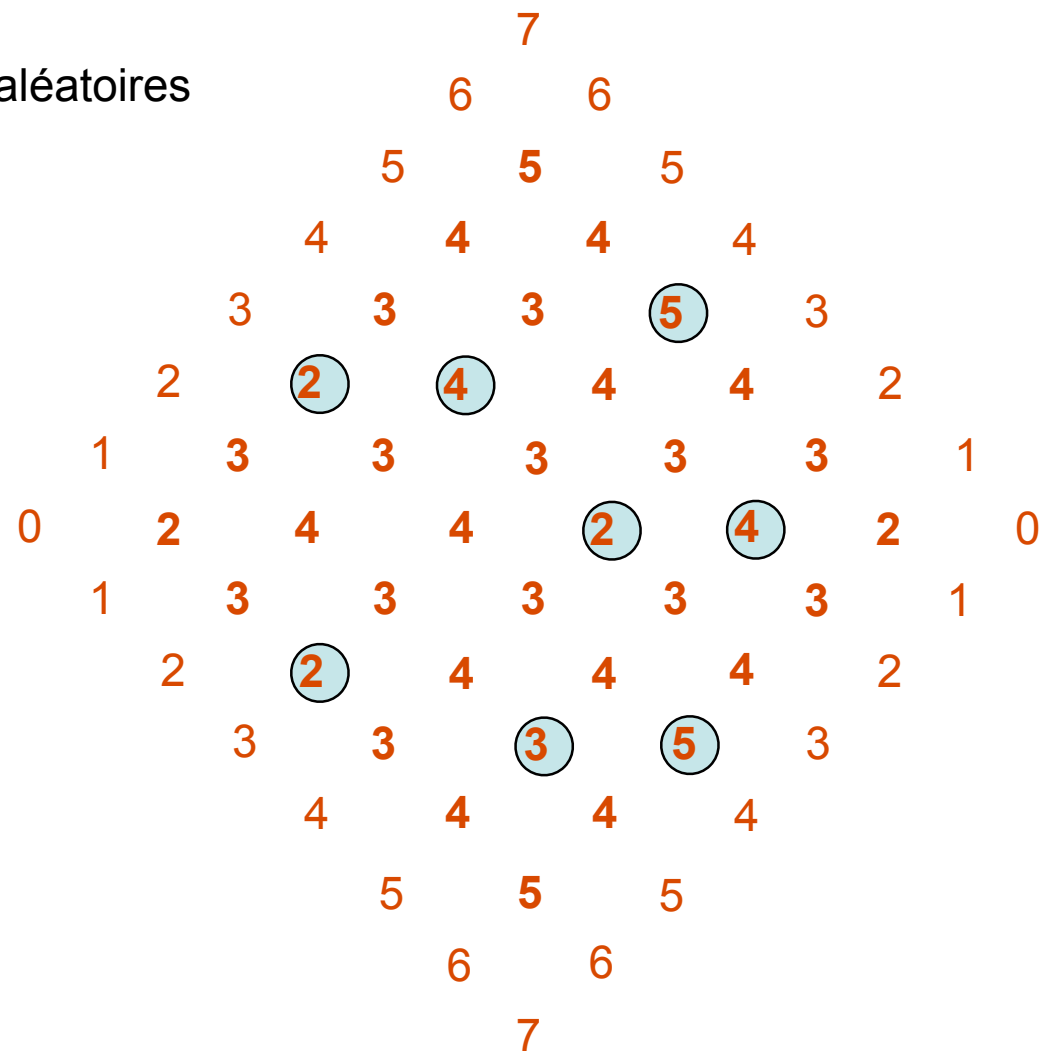
	b	
a	a	a
	c	

	a	
b	a+1	c
	a	





2⁸ choix aléatoires



Reste à vérifier la cohérence des règles magiques :

$$\begin{array}{ccc} & b & \\ a & \textcolor{brown}{a} & a \\ & c & \end{array}$$

$$\begin{array}{ccc} & a & \\ b & \textcolor{brown}{a+1} & c \\ & a & \end{array}$$

y compris les cas particuliers ($b = c$) :

$$\begin{array}{ccc} & a-1 & \\ a & \textcolor{brown}{a} & a \\ & a-1 & \end{array}$$

$$\begin{array}{ccc} & a+1 & \\ a & \textcolor{brown}{a \text{ ou } a+2} & a \\ & a+1 & \end{array}$$

$\textcolor{brown}{(-1)}$

$\textcolor{brown}{(1)}$

(dans la matrice à signes alternants)

		b			
	a	a	a		
		c			
			a		

		b			
	a	a	a		
		c		b	
			a		

		b			
	a	a	a		
		c	a+1	b	
			a		

		b			
	a	a	a		
		c	a+1	b	
	d		a		
		c			

		b			
	a	a	a		
		c	a+1	b	
	d	c+1	a		
		c			

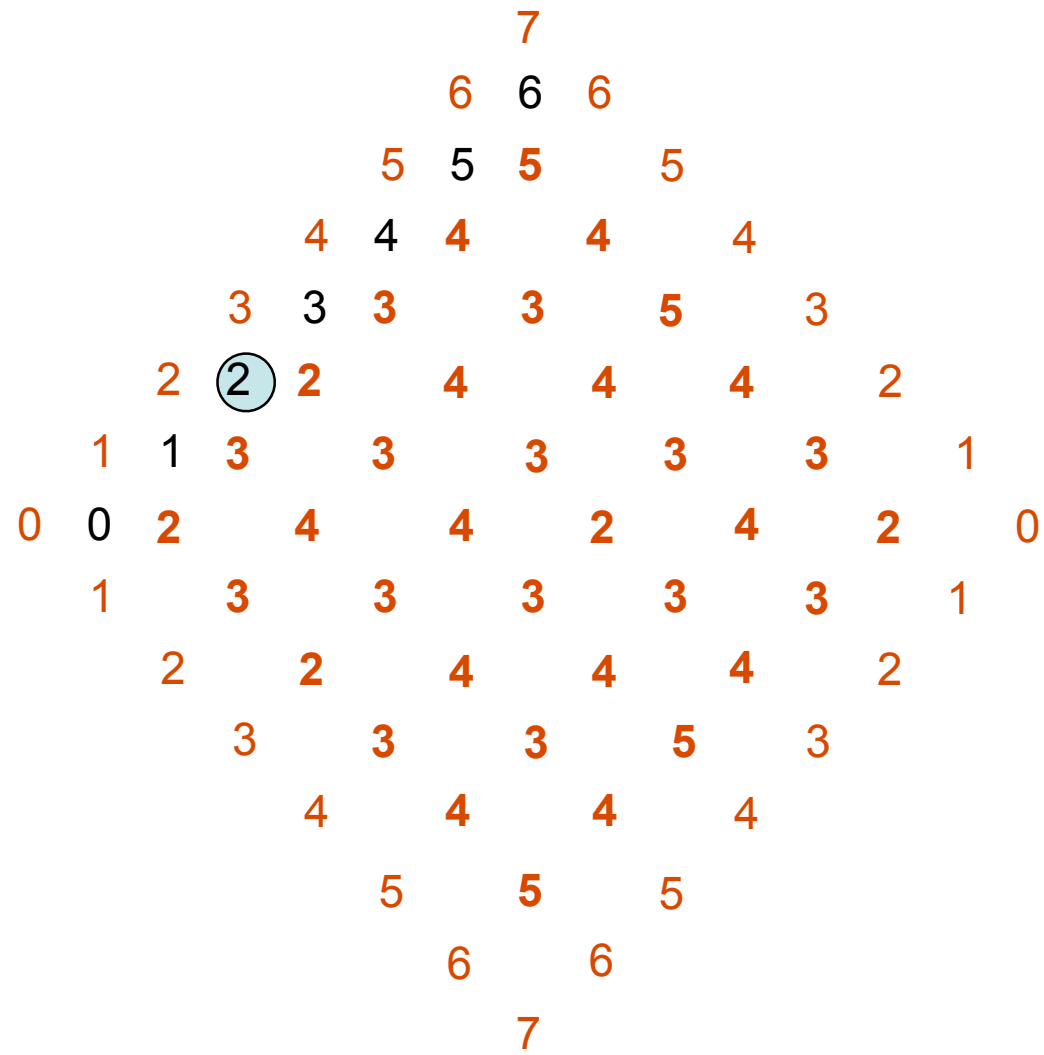
		b			
	a	a	a		
c		c	a+1	b	
	d	c+1	a		
		c			

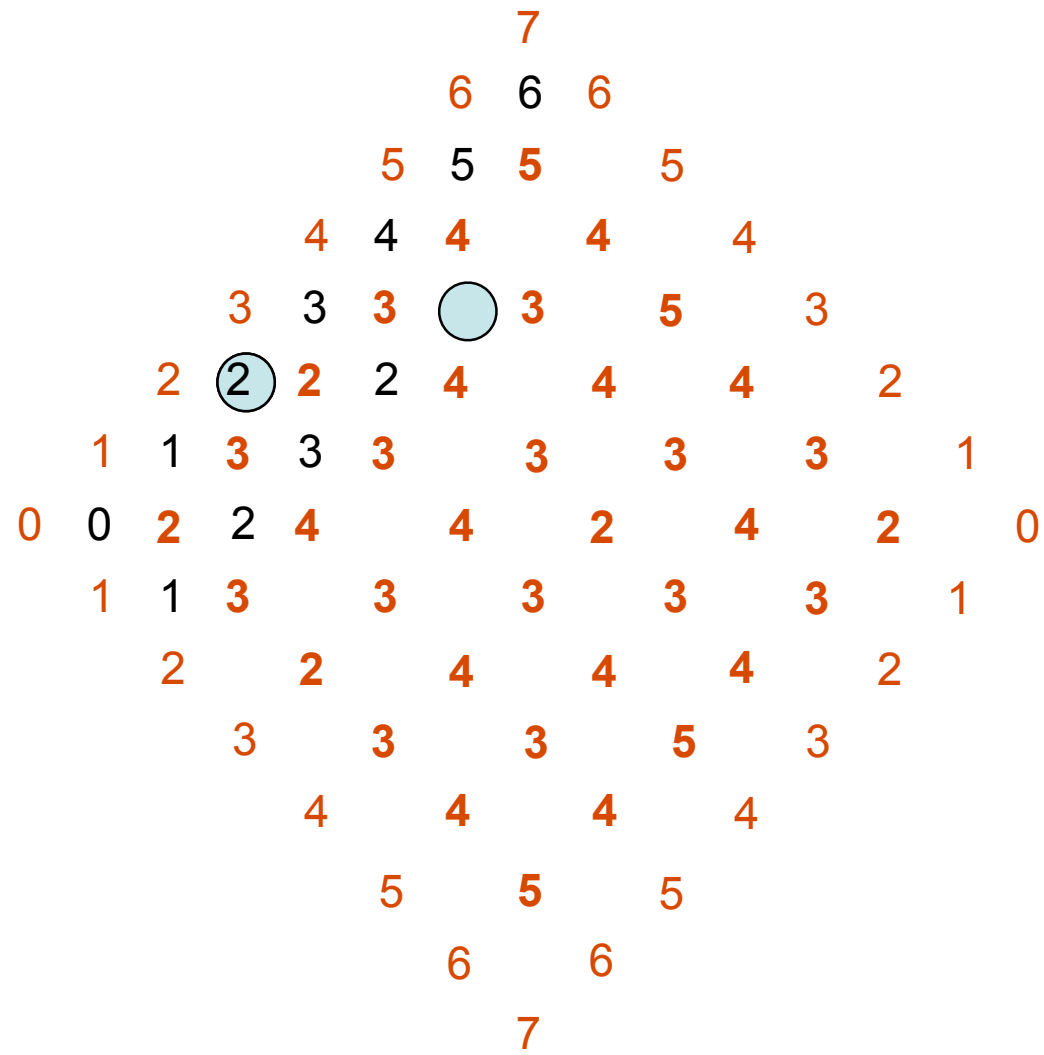
		b			
	a	a	a		
c	c	c	a+1	b	
	d	c+1	a		
		c			

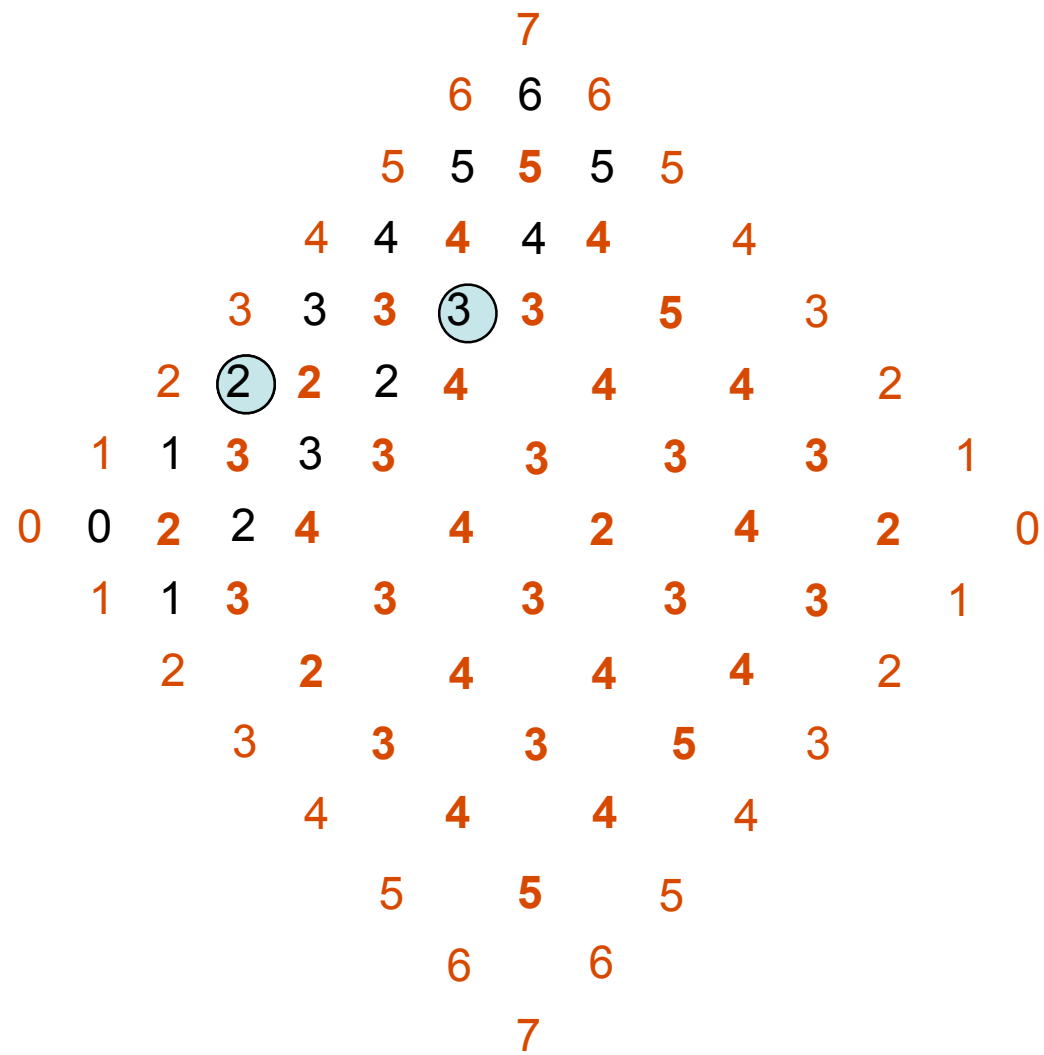
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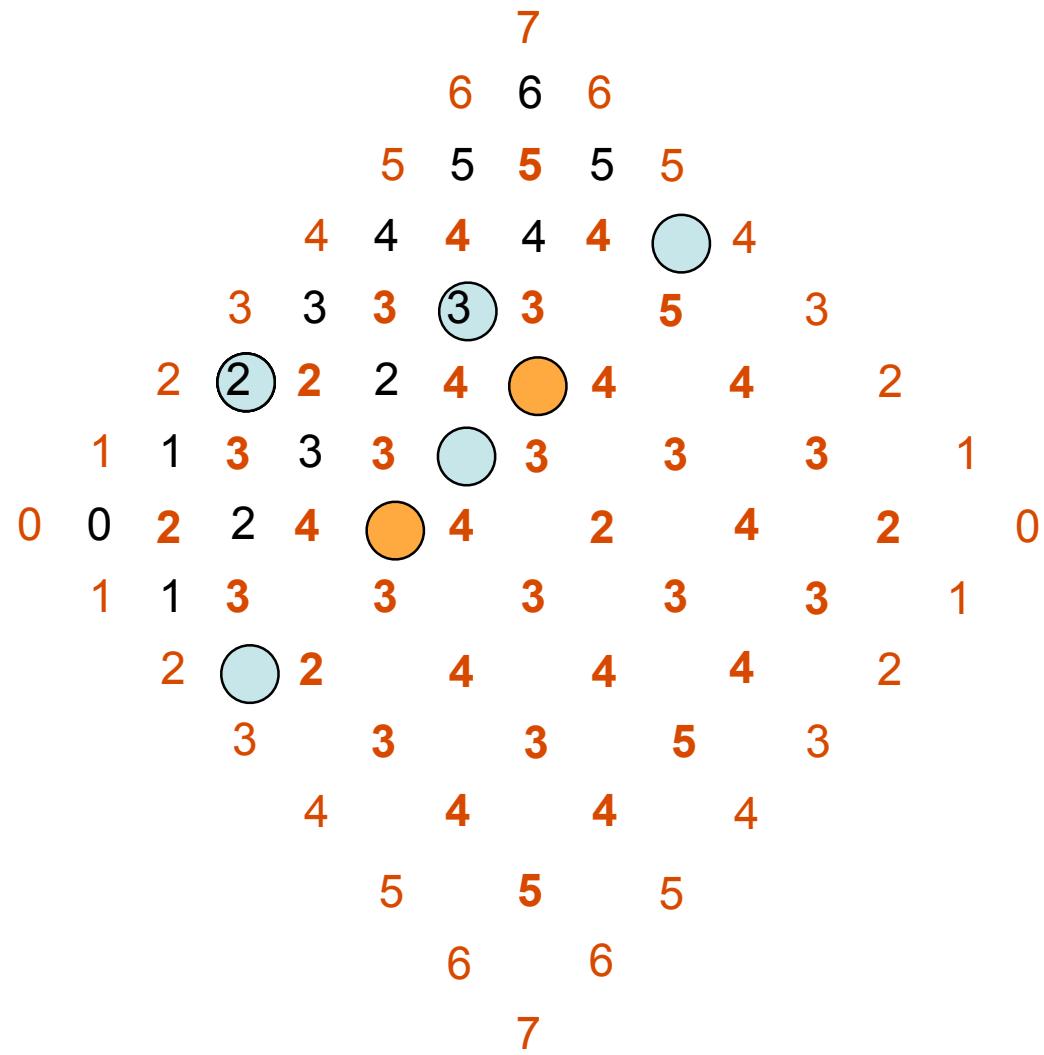
ü gonfler une matrice de hauteurs

- dégonfler ...



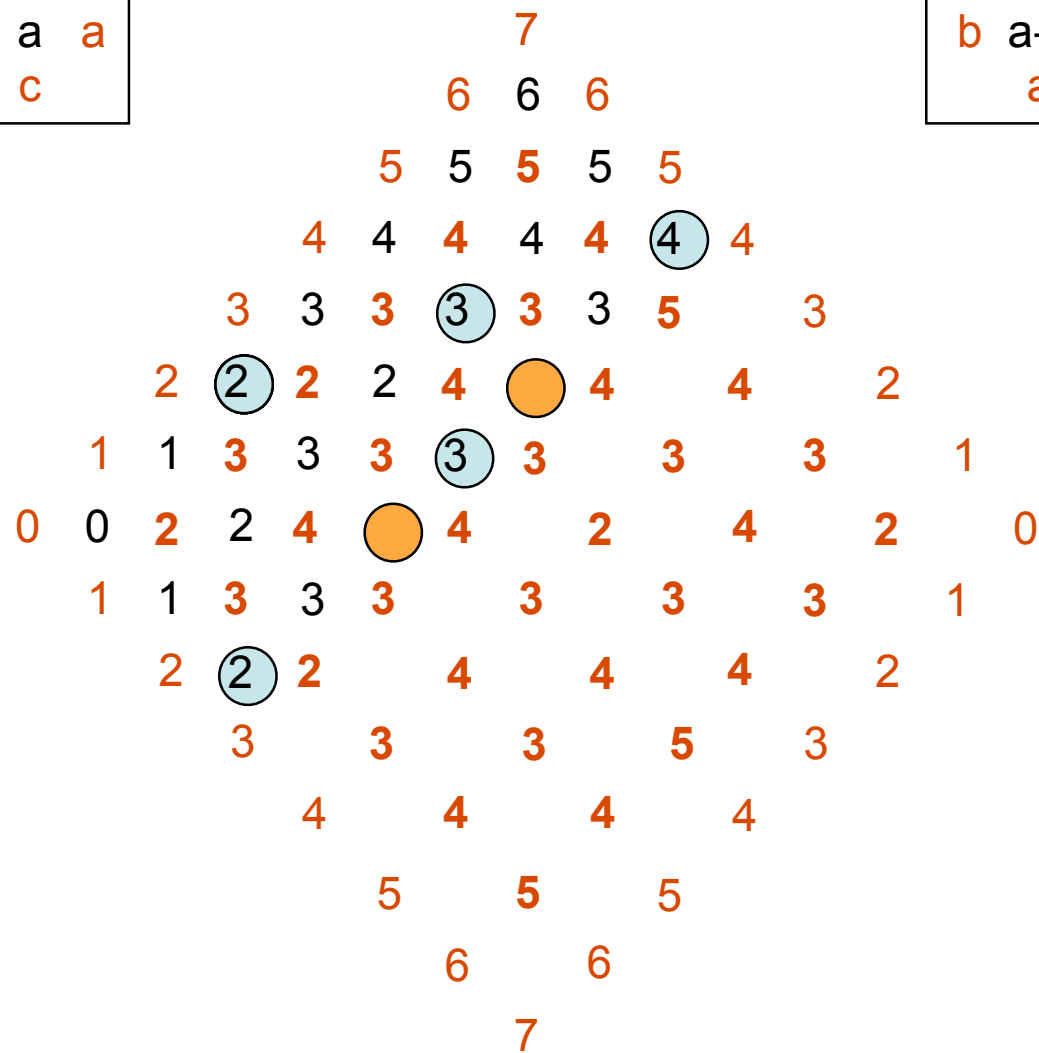


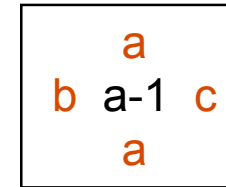


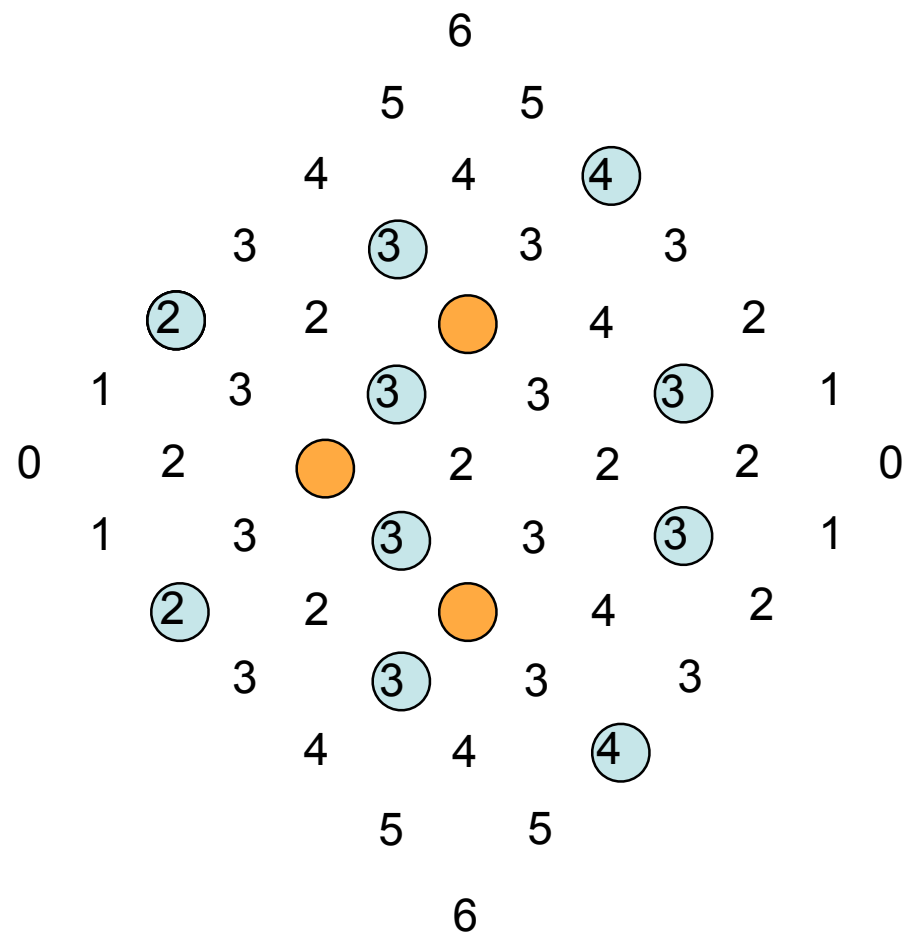


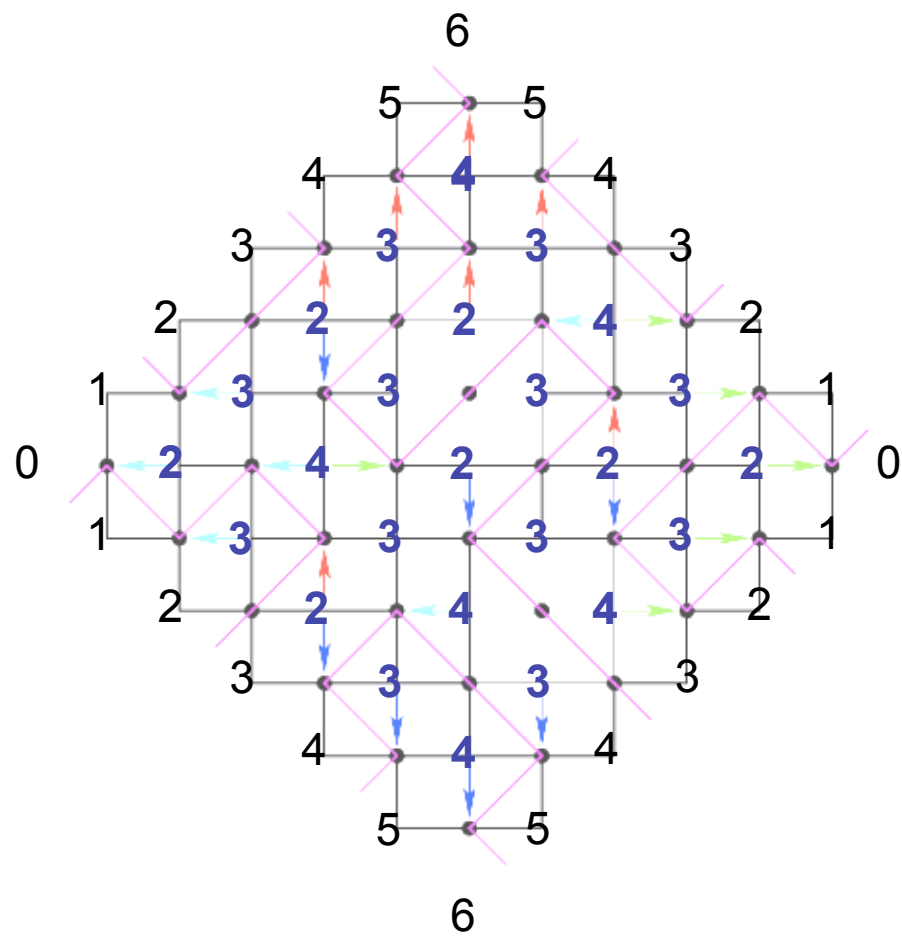
	b	
a	a	a
	c	

	a	
b	a-1	c
	a	









- Reste à vérifier :

$$B \in g_{\text{onfle}}(A) \Leftrightarrow A \in d_{\text{égonfle}}(B)$$

- D'où le dénombrement des ASM pondérées par $2^{#(-1)}$
- Cercle arctique ?
- Opérateurs Up / Down ?