Asymptotics for the Mandelbrot's Measure

Alain Rouault (joint work with) Quansheng LIU

UVSQ (email : rouault@math.uvsq.fr) et IRMAR, Université de Rennes I, (Email: liu@univ-rennes1.fr)

Let $W \geq 0$ be a random variable with $EW = 1$, and let μ_r be the classical Mandelbrot measure on $[0,1]$, built with the r-ary tree and products of independent copies of W. We study asymptotic properties of μ_r as $r \to \infty$: laws of large numbers, invariance principles, and large deviations theorems. We use limit theorems for the mass $Z^{(r)}$ of μ_r. This random variable is the limit of the Mandelbrot's martingale and satisfies the equation (in distribution)

$$Z^{(r)} = \frac{1}{r} \sum_{k=1}^{r} W_k Z_k^{(r)}$$

where $Z_k^{(r)}$ are independent copies of $Z^{(r)}$. We use also an approximation of μ_r by μ_r^1, the smoothed partial-sum process with fixed locations defined by

$$\mu_r^1(f) = \sum_{k=1}^{r} W_k \int_{\frac{k-1}{r}}^{\frac{k}{r}} f(x)\ dx.$$

Convergence simultanée de certaines martingales et continuité du spectre multifractal d'une mesure statistiquement auto-similaire

Julien BARRAL, Université Paris XI

Marches aléatoires de branchement

Nina GANTERT
Université de Berlin.

We consider an infinite Galton-Watson tree Γ and label the vertices v with a collection of i.i.d. random variables $(Y_v)_{v\in\Gamma}$. In the case where the upper tail of the distribution of Y_v is semiexponential, we then determine the speed of the corresponding tree-indexed random walk. In contrast to the classical case where the random variables Y_v have finite exponential moments, the normalization in the definition of the speed depends on the distribution of Y_v. Interpreting the random variables Y_v as displacements of the offspring from the parent, $(Y_v)_{v\in\Gamma}$ describes a branching random walk. The result on the speed gives a limit theorem for the maximum of the branching random walk, i.e. for the position of the rightmost particle. In our case, this maximum grows faster than linear in time.

Arbres et modèle d'Ising

Claire KENYON LRI, Université de Paris XI

Consider a process in which information is transmitted from a given root node on a noisy tree network T. We start with an unbiased random bit R at the root of the tree, and send it down the edges of T. On every edge the bit can be reversed with probability ϵ, and these errors occur independently. The goal is to reconstruct R from the values which arrive at level n of the tree. This model has been studied in information theory, genetics and statistical mechanics. We bound the reconstruction probability from above using the maximum flow on T viewed as a capacitated network, and from below using the electrical conductance of T. For general infinite trees, we establish a sharp threshold: The probability of correct reconstruction tends to $1/2$ as $n \to \infty$ if $(1-2\epsilon)^2 < p_c(T)$, but the reconstruction probability stays bounded away from $1/2$ if the opposite inequality holds. Here $p_c(T)$ is the critical probability for percolation on T; in particular $p_c(T) = 1/b$ for the $b+1$ regular tree. The asymptotic reconstruction problem is equivalent to purity of the "free boundary" Gibbs state for the Ising model on a tree. Thus our results extend the work of Bleher, Ruiz and Zagrebnov (1995), who considered the case of regular trees.

Processus stables sur la frontière d'un arbre

Philippe MARCHAL Laboratoire de probabilités, Université de Paris VI