Actions of the Symmetric Group on Polynomials

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The symmetric group acts in different ways on the ring of polynomials. Instead of only permuting indeterminates, one can follow Newton and use *divided differences*, and even deform them and obtain an *Hecke algebra* of operators. We shall show how Newton and Lagrange interpolation, the representation theory of the symmetric group, the coinvariant ring of the symmetric group, the ring of polynomials as a free module over symmetric ones and Yang-Baxter bases occur naturally from this divided difference approach.

1 Symmetric functions.
Generating functions of complete or elementary symmetric functions.
Matrix generating functions.
Schur functions.
Cauchy formula.
The Resultant. Application: Padé approximants

2 Symmetrizing Operators.
Divided Differences and Isobaric Divided Differences
Braid Relations
Yang-Baxter Graph
Special Yang-Baxter Elements and Idempotents
Maximal Symmetrizers
Lagrange Interpolation
Newton Interpolation

3 The Coinvariant ring of the Symmetric Group
Scalar Product
Yang-Baxter Bases
Irreducible Representations of the Symmetric Group