

# Introduction

# Checking Knowledge and evaluation

- Third part of USI-Algo-Prog teaching unit (UE) - approx. 3ECTs
- 1 DS and 1 exam - 1 Test not planned in advance.
- Theoretical point of view of programming activities.
- Applications and implementations in Python

# History

Before availability of modern computer machine :

- Coming from works of a mathematician : Al Khwarismi (780-850).
- Algebraic calculus roots.
- **1834** : Charles Babbage publish the first description of a programmable machine.
- **1815-1852** : the first informatic program is written by Ada Lovelace.
- **1934** : Turing machine  
(<http://zanotti.univ-tln.fr/turing/>)

# Algorithms

- Donald Knuth, fundation of programmation “*The Art of Computer Programming*”.
- Author of TeX and Metafont.
- Algorithm of *Knuth-Morris-Pratt* (1977): pattern research in string - used in genome sequence processing.
- New architectures: recursive, parallel or quantum algorithmics.

# Computer Architecture

- At its core, two components :
  1. **C**entral **P**rocessing **U**nit
  2. Memory storage
- CPU allows you to read, to write and to modify the memory space.
- Data are 0 or 1 value - discrete/boolean.

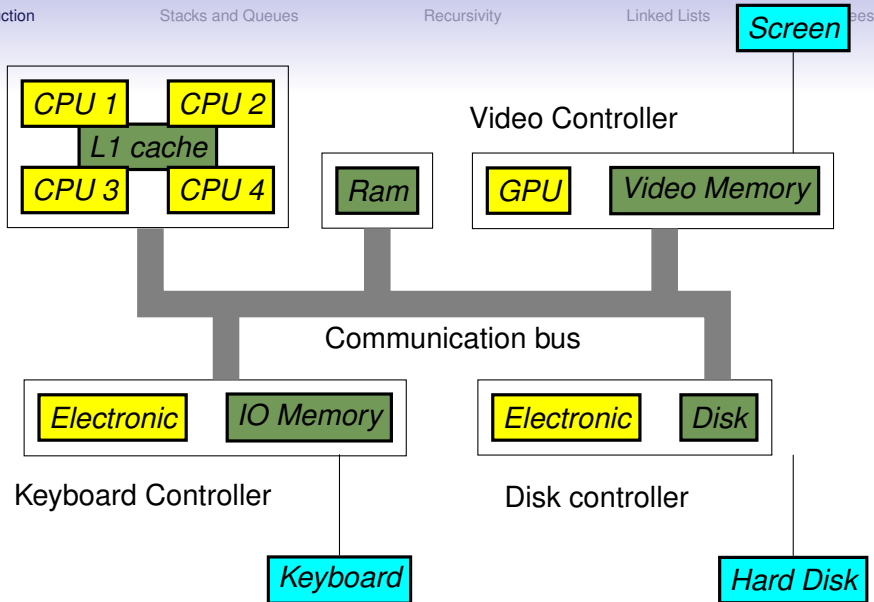


Figure : Computer Architecture

# Algorithm Principles

- An algorithm is step-by-step set of operations, or instructions, which solve a problem.
- Algorithmics is a set of rules and methods used to define and to design algorithms.

# Abstract Data Types

- An abstract data type is a data type along with all the operations which are allowed on the data of this type
- In other words, an abstract type is defined by:
  1. a name,
  2. some **data**, *i.e.* a set of values,
  3. a set of functions, named **primitives**, operating on this data.



# Stacks and Queues

# Stacks and Queues

## Outline :

1. Definitions.
2. Stack primitives and examples.
3. Queue primitives and examples.
4. Implementation of stacks.
5. Implementation of queues.

# Stack Definition

- Stacks and queues are containers where only one object is accessible at each time.
- **Stack**
  - In a stack, the accessible object is the last inserted one.
  - LIFO: last in - first out.
  - Example: a stack of plates.
- **Queue**
  - In a queue, the accessible object is the first inserted one.
  - FIFO: first in - first out.
  - Example: a queue to buy a cinema ticket.

# Primitives of Stack

- Two operations are defined as stack **primitives**:
  - $\text{push}(P, e)$  to add an element  $e$  in  $P$ .
  - $\text{pop}(P)$  to suppress an element in  $P$ .
- *Property*: if an element  $f$  is pushed after an element  $e$  then the element  $f$  is accessible before  $e$ .

# Stack Implementation

```
def create_stack( ):  
    return []  
  
def push( theStack , element ):  
    theStack.append( element )  
  
def pop( theStack ):  
    return theStack.pop()
```

The primitive `pop()` takes time : have to go through the whole list.

# Stack Implementation

To make `pop()` faster, it is necessary to use the class `deque` belonging to the module `collections`:

```
import collections

def create_stack( ):
    return collections.deque()

def push( theStack , element ):
    theStack.append( element )

def pop( theStack ):
    return theStack.pop()
```

# Queue

- A Queue  $F$  is a data structures containing elements and having 2 operations :
  - `enqueue( $F, e$ )` which adds the element  $e$  into  $F$ ;
  - `dequeue( $F$ )` which suppresses an element from  $F$ ;
- *Property*: if an element  $f$  is added after an element  $e$ , then the element  $e$  is accessible before  $f$ .

# Queue Implementation

```
def create_queue( ):
    return []

def enqueue( theQueue, element ):
    theQueue.append( element )

def dequeue( theQueue ):
    return theQueue.pop(0)
```

The primitive `enqueue()` takes time : have to go through the whole list.



# Queue Implementation

- If we want to make things faster, we have to use the module *deque*:

```
import collections

def create_queue( ):
    return collections.deque()

def enqueue(theQueue, element ):
    theQueue.append( element )

def dequeue( theQueue ):
    return theQueue.popleft()
```

# Recursivity

# Recursivity and everyday-life

Something that repeats itself within itself.

- Art : *mise en abyme*, fractals :



(a)



(b)

Figure

- In computer science : when a function calls itself.

# Call stack

- Call stack:
  - The sequence of instructions in a function are stored in a specific memory place called the **call stack**.
  - The stack is dedicated to a program.
  - When a function is called, all the variables, instructions and return values are pushed in the stack.
  - At the end of the function, only the return value is kept on the stack, all the other elements are popped from the stack.
  - The program resumes from the last instruction in the main program (or function).

## Call stack

Consider the following program :

```
def g( a );  
    print("g(" + str(a) + ")")  
    return 1000  
  
def h( a );  
    print("h(" + str(a) + ")")  
    return 2000  
  
def f( a ):  
    print("f(" + str(a) + ")")  
    v = g(a+1)  
    print( v )  
    v = h(a+2)  
    print( v )  
  
f( 1 )  
print("fin")
```

The following result is displayed on the terminal :

```
f(1)  
g(2)  
1000  
h(3)  
2000  
fin
```

**Your turn :** Draw each step in the call stack resulting from the call to `f(1)`.

# Visualizing the stack in Python

Try this Python program

```
import inspect

def f( a, b ):
    g(a+1, b+1)

def g( c, d ):
    print( "the_stack_is:" )
    print( inspect.stack() )
    print( "" )

    print( "Data_of_the_function_situated_at_the_top_of_the_stack_are:" )
    print( inspect.stack()[0] )
    print( "" )

    print( "unction_variables_at_the_top_of_the_stack_are:" )
    print( inspect.getargvalues( inspect.stack()[0][0] ) )

f( 1, 2 )
```

# Visualizing the stack in Python

The following result is obtained when you run the program :

```

1 the stack is :
2 [
3     (<frame object at 0x7f59363a6938>, 'prog.py', 8, 'g', ['__print(__inspect.stack().__)\n'], 0)
4     (<frame object at 0x7f59363a93f0>, 'prog.py', 4, 'f', ['__g(a+1,__b+1)\n'], 0),
5     (<frame object at 0x7f59364f6050>, 'prog.py', 18, '<module>', ['f(__1,__2__)\n'], 0)
6 ]
7
8 Data of the function situated at the top of the stack are :
9 (<frame object at 0x7f59363a6938>, 'prog.py', 12, 'g', ['__print(__inspect.stack()[0]__)\n'], 0)
10
11 Function variables at the top of the stack are :
12 ArgInfo(args=['c', 'd'], varargs=None, keywords=None, locals={'c': 2, 'd': 3})

```

# Recursivity

- **Definition**

- If an algorithm - a function - calls itself, it is called **recursive**.
- In other words, a function is recursive if it is found **at least twice** in the call stack during its execution.

- **Complexity**

- A recursive algorithm needs to store the calling context of each call.
- Recursivity can lead to a memory complexity bigger than iterative algorithm.



# Recursivity : example

- It is possible to write a recursive power function :  
 $pow(x, n) = x^n$ .
- In Python :

```
def power(x, n):  
    if (n == 0):  
        return 1  
    if (n == 1):  
        return x  
    return power(x, n-1) * x
```

# Linked Lists

# Linked Lists

## Description

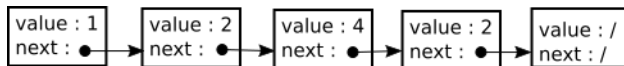
- **Linked list** = Data structure where each element is nested into a `cell`.
- A `cell` contains :
  1. a value;
  2. a way to access the next cell

## Definition

- In the list `l` the element sequence  $l_1, l_2, \dots, l_n$  are nested into  $c_1, c_2, \dots, c_n$  cells.
- One more `cell`,  $c_{n+1}$ , is used to specify the end of the list.
- Even if two values are similar, the cells nesting them are different.
- A reference to the `next` cell is given for all  $i \leq n$ .
- The last cell points to `None`.

# Linked Lists

- The list is empty if it contains only one `cell`.
- Representation of the list `[1,2,4,2]` :



# Primitives of Linked Lists

- The linked list type provides the following functions.

```
create_list(l)
push_front(l, e)
first( l )
end( l )
next( l, cell )
value( cell )
```

where *l* is a list, *e* an element of the list and *cell* a cell.

## Primitives of Linked Lists

- *create\_list* create an empty list with a single end cell.;
- *push\_front* adds the element  $e$  at the beginning of the list (and increases the size by one);
- Primitives *first*, *end* and *next* all return cells;
- The primitive *end*( $l$ ) returns the cell  $c_{n+1}$  at the end of the list ;
- The primitive *first*( $l$ ) returns the first cell  $c_1$  belonging to the list;
- For each integer  $i \in [1, n]$  the function *next*( $l, c_i$ ) returns the successor of  $c_i$  which is  $c_{i+1}$  ;
- For each integer  $i \in [1, n]$ , the function *value*( $c_i$ ) returns the value of  $c_i$ , denoted  $l_i$ ;

# Primitives of Linked Lists

- In Python

```
def create_list():  
    return { 'first':None, 'end': None }  
def add_to_beginning(l, e):  
    l['first'] = { 'value':e, 'next':l['first'] }  
def next(l, e):  
    return e['next']  
def first(l):  
    return l['first']  
def end(l):  
    return l['end']  
def value(l, e):  
    return e['value']
```

# Primitives of Linked Lists

- List can be used as in the following example :

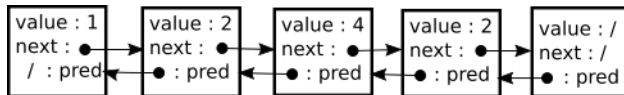
```
l = create_list()
add_to_beginning(l, 3)
add_to_beginning(l, 5)
add_to_beginning(l, 2)

it = first(l)
while( it != end(l) ):
    print( value(l, it) )
    it = next(l, it)
```



## Double Linked List

- A double linked list is a linked list with an additional information : the precursor.
- Precursor is a reference to :
  1.  $c_{i-1}$  if  $2 \leq i \leq n+1$
  2. to the end list cell  $c_{n+1}$  if  $i = 1$
  3. else to None
- The list [1, 2, 4, 2] could be represented by :



## Double Linked List

- Two primitives functions are defined for the list  $l$  and the cell  $cell$ :

```
push_back(l, e)  
prev(l, cell)
```

- $push\_back$  adds the element  $e$  at the end of the list (and increases the size by one).
- For all integers  $i \in [2, n + 1]$  the function  $prev(l, c_i)$  returns the precursor of  $c_i$ .
- For all integers  $i \in [1, n]$ , the function  $value(c_i)$  returns the value of  $c_i$  which is  $l_i$ ;

# Complexity

- Evaluation of time and memory space to execute a program.
- Mathematic formula to obtain it from initial data and executing time (and memory).
- **Caution** : difficult to obtain in practice.
- Suppose the time of each operation is constant and counting how many operations are done for each program.
- **Notation** : complexity of the function  $fct$  is noticed  $\mathcal{C}(fct)$

# Complexity - Example

- The following program :

```
def fct( n ) :           # initialisation de n : 1 opération
    r = 0                # 1 opération
    for i in range(n):   # n*( incrément + corps de boucle ) = n*( 1 + 2 )
        r = r + 1        # | 2 operations
    return r             # 1 opération
```

realize  $3 + 3n$  operations, its complexity is  $\mathcal{C}(fct) = 3 + 3n$ .

# Complexity - Example

```
def max( T ):                # init. : 1 opération
    res = T[0]               # 1 opération
    for i in range( len(T) ): # C1
        if res < T[i] :      # | C2
            res = T[i]       # | | 2 opérations
    return res               # 1 opération
```

- Let  $n$  the size of the array  $T$ .
- Complexity  $C2$  of the conditional jump `if` is equal to 3 operations plus the operations of the `if` core.
- The core of `if` adds 2 or 0 supplementary operations depending on the test value.
- Result :  $3 \leq C2 \leq 5$ .

## Complexity - Example

- Complexity of  $C1$  - `for` loop - is

$$n \times (\text{incrément} + \text{loop core})$$

Then :

$$C1 = \begin{cases} n(1 + 3) & \text{if false;} \\ n(1 + 5) & \text{if true.} \end{cases}$$

Program complexity is :

$$3 + 4n \leq C(\text{max}) \leq 3 + 6n.$$

- The lower bound is reached if the element is at the first position in the array.
- The upper bound is reached if all element are distinct and sorted by ascendant order in the array.

## Complexity - Example

- Another stupid implementation of the program :

<b>def</b> max2( T ):	<i># init. : 1 opération — on pose <math>n = \text{len}(T)</math></i>
T = list( T )	<i># = + Copie de tableau : 1+n opérations</i>
<b>for</b> i <b>in</b> range( len(T) ):	<i># C1</i>
<b>for</b> j <b>in</b> range( len(T) ):	<i>#   C2</i>
<b>if</b> T[i] < T[j]:	<i>#     C3</i>
T[i] = T[j]	<i>#       3 opérations</i>
<b>return</b> T[ len(T)-1 ]	<i># 3 opérations</i>

By analogy to the previous one, one can compute :

$$\begin{aligned}
 3 &\leq C3 \leq 6, \\
 n(1 + 3) &\leq C2 \leq n(1 + 6), \\
 n(1 + 4n) &\leq C1 \leq n(1 + 7n).
 \end{aligned}$$

The complexity is :

$$5 + 2n + 4n^2 \leq \mathcal{C}(\text{max2}) \leq 5 + 2n + 7n^2.$$

# Trees



# Trees

- A tree is a data structure  $A$  containing elements named `vertex`.
- Primitive functions are :
  - `Child(A, p)` return the list of vertex accessible from  $p$ .
  - `parent(A, f)` return the vertex of  $A$  which is the **ascendant** of  $f$  or `None`.
  - `root(A)` return the vertex of  $A$  which is the root of the tree.

# Tree Properties

- A root has no parent :  $parent(A, root(A)) = None$
- All vertex, except the root, have a parent (and only one) :

$$\forall s \in A \subset \{root(A)\}, parent(A, s) \in A$$

- From each vertex, root is accessible by using parent relationship.

$$\forall s \in A, \exists k \in \mathbb{N}, parent^k(A, s) = None;$$

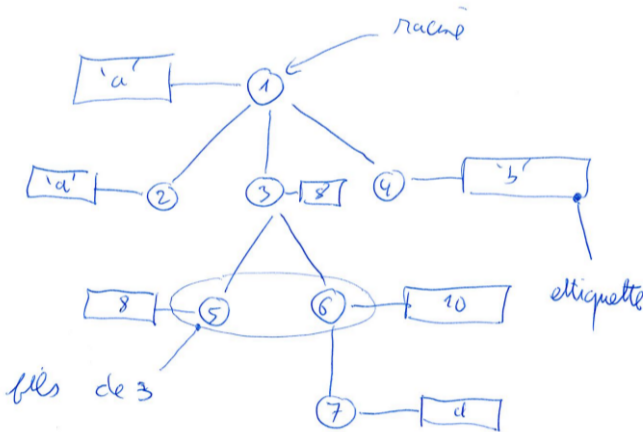
where  $parent^k$  is recursively defined by :  $parent^0(A, s) = s$  and  $parent^{k+1}(A, s) = parent(A, parent^k(A, s))$  for  $k \geq 1$ .

- The childs of a vertex  $s$  have as  $s$  as parent:

$$\forall s \in A, \forall f \in child(A, s), parent(A, f) = s.$$

## Labeled Trees

- A labeled tree is a tree  $A$  with one operation  $label(A, s)$  which returns an element named label of  $s$  for each vertex.
- It is possible to draw a tree like that :



# Binary Trees

- A binary tree is a data structure  $A$  containing elements named *vertex*.
- The primitive functions are :
  - $left\_child(A, p)$  takes as parameters a tree  $A$ , a vertex  $p$  and returns a vertex of  $A$  named left child of  $p$  or *None* if  $p$  has no left child.
  - $right\_child(A, p)$  takes as parameters a tree  $A$ , a vertex  $p$  and returns a vertex of  $A$  named right child of  $p$  or *None* if  $p$  has no right child.
  - $parent(A, f)$  takes as parameter a tree  $A$ , a vertex  $f$  and returns a vertex of  $A$  or *None*; this vertex is called the parent of  $f$ ;
  - $root(A)$  takes as parameter a tree  $A$  and returns a vertex of  $A$  named the root of the tree.

# Primitives of Binary Trees - Properties

- a root has no parent :  $parent(A, root(A)) = None$ ;
- From any vertex, it is possible to reach the root by using parent relationship :

$$\forall s \in A, \exists k \in \mathbb{N}, root(parent^k(A, s));$$

where  $parent^k$  is recursively defined by :  $parent^0(A, s) = s$  and  $parent^{k+1}(A, s) = parent(A, parent^k(A, s))$  for  $k \geq 1$ .

- The left child of a vertex  $s$  have as parent  $s$  :

$$\forall s \in A, \text{if } left\_child(A, s) \neq None \text{ then } parent(A, left\_child(A, s)) = s;$$

- The right child of a vertex  $s$  have as parent  $s$  :

$$\forall s \in A, \text{if } right\_child(A, s) \neq None \text{ then } parent(A, right\_child(A, s)) = s;$$

## Labeled Binary Trees

- A labeled binary tree is a tree  $A$  with one operation  $label(A, s)$  which returns an element named label of  $s$  for each vertex.
- It is possible to draw a tree like that :

