Exercise 1

Build a 5-degenerate graph that has all of its vertices but one of degree 5 and is not 5-colorable.

Exercise 2

Build by induction for all integer k a tree T_k , together with an ordering of the vertices of T_k , such that applying the greedy algorithm on T_k with the vertices in this order gives a k-coloring of T_k .

Exercise 3

For each of the following statements, either prove it or give a counter-example.

- 1. For every graph G, one can find a $\chi(G)$ -coloring of G with a greedy algorithm.
- 2. For every graph class C and every integer k, the complexity of deciding if a graph $G \in C$ is k-colorable is the same as the complexity of finding a k-coloring of G.

Exercise 4

Give a $O(n^2)$ -algorithm to 3-color an outerplanar graph.