Exercise 1

Determine the chromatic index of the Petersen graph.

Exercise 2

Determine the chromatic index of the complete graph on n vertices.

Exercise 3

Let G be a $\Delta(G)$ -regular graph on an odd number of vertices. Show that G is class II.

Exercise 4

Let G be a $\Delta(G)$ -regular graph with a cut vertex. Show that G is of class II.

Exercise 5

Let G be a $\Delta(G)$ -regular $\Delta(G)$ -edge-colorable graph. Let e be an edge of G and let G' be the graph obtained from G by subdividing the edge e. Prove that G' is class II, i.e., $\chi'(G') = \Delta(G') + 1$.

Exercise 6

Find a subcubic planar graph that is not 3-edge-colorable. Is there a cubic planar graph of class II?

Exercise 7

Prove that every hamiltonian cubic graph is class I. (A graph is hamiltonian if it admits a Hamilton cycle – a cycle passing through every vertex exactly once). Is it also true that every connected class I cubic graph is hamiltonian?

Exercise 8

Two colorings φ_1 and φ_2 are equivalent if there is a sequence of Kempe swaps to transform φ_1 into φ_2 . Find two non equivalent 3-edge colorings of $K_{3,3}$.

Exercise 9

Let G be a Δ -edge-colorable Δ -regular graph such that G has a unique Δ -edge-coloring (up to renaming the colors). Show that G is hamiltonian.