

Exercise 1

Determine the choosability of a cycle on n vertices.

Exercise 2

Prove that the only list assignment L of an odd cycle C with lists of size two such that C is not L -colorable is the one where all the lists are equal.

Exercise 3

Let k be an integer. Find an example of a bipartite graph that is not k -choosable.

Exercise 4

Derive the fact that every bipartite planar graph is 3-choosable from the following theorem :

Theorem 1 (Alon et al. 1992) *Let $\zeta(G) = \max(|E(H)|/|V(H)|)$ for all subgraphs H of G . Then every bipartite graph G is $(\zeta(G) + 1)$ -choosable.*

Exercise 5

1. Let G_0 be the plane graph with six vertices $v_1, \dots, v_6$ and 12 edges, obtained from a cycle $v_1v_2 \cdots v_6$ by adding three diagonals joining odd vertices inside the cycle and adding three diagonals joining even vertices outside the cycle.
Find a list assignment L_0 of G_0 such that $|L(v_i)| = 4$ for the three inner vertices (i odd) and $|L(v_i)| = 1$ for the three outer vertices (i even) such that G_0 is not L_0 -colorable.
2. Let G be a plane graph obtained from two copies of G_0 by identifying one edge incident to the outer face from each copy. Observe that G has 10 vertices and 23 edges ; its outer face is of size four.
Find a list assignment L_1 of G_1 such that the two vertices of degree four incident to the outer face receive lists of size one, whereas all the other vertices (including the two vertices of degree seven) receive lists of size four, such that G_1 is not L_1 -colorable.
3. Using several copies of the graph G_1 as gadgets, find a plane graph G_2 and a list assignment L_2 of G_2 such that two vertices incident to the outer face of G_2 receive lists of size three, all the other vertices receive lists of size four, and G_2 is not L_2 -colorable.
4. Use several copies of the graph G_2 as building blocks, find a plane graph G and a list assignment L of G with all the lists of size four, such that G is not L -colorable – hence find a planar graph that is not 4-choosable. What is its size ?