

Exercise 1

Build a connected 5-degenerate graph that has all of its vertices but one of degree at least 6 and is not 5-colorable.

Exercise 2

Build by induction for all integer k a tree T_k , together with an ordering of the vertices of T_k , such that applying the greedy algorithm on T_k with the vertices in this order gives a k -coloring of T_k .

Exercise 3

Show that a graph is k -degenerate if and only if there exists an order of the vertices of the graph such that every vertex has at most k neighbors before it in the order.

Exercise 4

For each of the following statements, either prove it or give a counter-example.

1. For every graph G , if given the right order, one can find a $\chi(G)$ -coloring of G with a greedy algorithm.
2. For every graph class \mathcal{C} and every integer k , the complexity of deciding if a graph $G \in \mathcal{C}$ is k -colorable is the same as the complexity of finding a k -coloring of G .

Exercise 5

Give a class of graph where the coloring number can be arbitrarily large, but is bounded by a function of the clique number.

Exercise 6

Give a $O(n^2)$ -algorithm to 3-color an outerplanar graph.

Exercise 7

Show that for every triangle-free planar graph G , $V(G)$ can be partitioned into two sets, such that there is no cycle with every vertex in the same set.

Exercise 8

Give a $O(n^2)$ -algorithm to construct such a partition.