Exercice 1. Theorems. Read this part of the manual about theorem [https://tinyurl.com/huh7u3f](https://tinyurl.com/huh7u3f) as well as this summary of the EventB language [https://tinyurl.com/j3zby9q](https://tinyurl.com/j3zby9q)

Exercice 2. Arithmetics Create a context and prove as a theorem that:
1. $1 + 1 = 2$.
2. $0 \neq 1$.
3. A rectangle with edges 3meters and 4meters long has a diagonal 5meters long.
4. There are infinitely many integers.
5. There are finitely many integers between 3 and 7.
6. If $0 = 1$ then there are infinitely many integers between 3 and 7.

Exercice 3. Set theory Show that
1. There is no element in the empty set.
2. Every set created in a context is non-empty.
3. If $(A, B)$ is a partition of $\mathbb{Z}$ then there is no element which is both in $A$ and in $B$.
4. If $(A, B)$ is a partition of $\mathbb{Z}$ then every integer is either in $A$ or in $B$.

Exercice 4. At the bar We consider a bar with customers, some of them drink, some of them do not drink. Is it true that
1. there exists one customer such that if that customer drinks, everybody drinks.
2. if there exists one customer who drinks then all customers drink.

Exercice 5. Interactive proofs 1 Use the interactive proof tool in order to show that
— There exists a partition of $\mathbb{N}$.
— There exists a partition $(A, B)$ of $\mathbb{N}$ such that both $A$ and $B$ are non-empty.
— There exists a positive integer $n$ such that $n \mod 8 = 0$ and $n \mod 7 = 0$.

Exercice 6. Interactive proofs 2 We want Rodin to prove that
$$\exists n \in \mathbb{N}, 2 \cdot n = 57$$

is not true. The provers used by Rodin are not able to prove automatically this property. We need to provide Rodin three extra theorems.
— \( 57 \mod 2 = 1 \)
— \( \forall n \in \mathbb{N}, (2 \ast n \mod 2 = 0) \)
— \( \forall n \in \mathbb{N}, \forall m \in \mathbb{N}, (n = m) \implies (n \mod 2 = m \mod 2) \)

Add the three hypotheses as new axioms (marked as theorems). They should be proved automatically by Rodin.

Terminate the proof by modifying the goal (click on the red elements) and providing the right parameters to the last theorem.

Exercice 7. Interactive proofs 3
We denote \( E = \{2 \ast n \mid n \in \mathbb{N}\} \) and \( O = \{2 \ast n + 1 \mid n \in \mathbb{N}\} \). We consider the following theorems.
— \( \text{mod} \) : for every integer \( n \), \( n \mod 2 \in \{0, 1\} \).
— \( \text{even} \) : for every integer \( n \), if \( n \mod 2 = 0 \) then there exists an integer \( x \) such that \( n = 2 \ast x \).
— \( \text{odd} \) : for every integer \( n \), if \( n \mod 2 = 1 \) then there exists an integer \( x \) such that \( n = 2 \ast x + 1 \).
— \( \text{notoddandeven}1 \) : for every integers \( x \) and \( y \), \( 2 \ast x \neq 2 \ast y + 1 \),
— \( \text{notoddandeven}2 \) : \( E \cap O = \emptyset \),
— \( \text{oddoreven} \) : \( \mathbb{N} \subseteq E \cup O \),
— \( \text{partition} \) : \( (E, O) \) is a partition of \( \mathbb{N} \).

1. Create a new context in Rodin and prove the previous theorems in this new context by interacting with the auto provers. Some hints:
— Sometimes, you can guide the autoprover by clicking on the red elements.
— If there are too many hypotheses, then Rodin auto provers may get confused In this case, remove the useless hypotheses.
— Theorem \( \text{even} \) : provide a witness
— Theorem \( \text{odd} \) : provide a witness
— Theorem \( \text{notoddandeven}1 \) : transform the equation using "ah" (add hypothesis).
— Theorem \( \text{notoddandeven}2 \) : click on the various red elements in the goal in order to guide the provers.
— Theorem \( \text{oddoreven} \) : do a case distinction on \( (x \mod 2 = 0) \) using "dc", then add the right hypotheses using "ah" and reuse the previous theorems.
— Theorem \( \text{partition} \) : reuse the previous theorem.

2. Proved directly theorem \( \text{partition} \) as a single theorem in a new context.

1. For each proof obligation appearing in DecimalCounter and Min-MaxAlgorithm, describe the corresponding rule of the proof obligation generator.
2. If some proof obligations of the DecimalCounter or the MinMaxAlgorithm machines are not green, terminate the proof using the interactive mode.

3. Prove the correctness of the ConcreteROMController.