



# Regular Model Checking Using Inference of Regular Languages

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# Introduction

- Regular Model Checking
- Inference (Learning) of Regular Languages
- Using inference for regular model checking
  - General model-checking algorithm
  - Experiments

## Regular Model Checking

[Kersten et al. 97, Fribourg et al. 97 (Infinity)]

- Configurations of systems are modeled as strings over a finite alphabet  $\Sigma$
- Finite automata  $A$  over  $\Sigma$  represent (infinite) regular sets of configurations.
  - *Init*: set of initial configurations
  - *Bad*: set of bad configurations
- Transitions are modeled by a transducer  $\tau$  (automata over  $\Sigma \times \Sigma$ ).
- Reachable configurations in  $n$  steps:  $\tau^n(\textit{Init})$
- $\tau^*(\textit{Init}) := \bigcup_{k=0}^{\infty} \tau^k(\textit{Init})$  (not necessarily regular)
- $\tau^* := \bigcup_{k=0}^{\infty} \tau^k$  (not necessarily regular)

## Regular Model Checking

- Model-checking problem 1:  $\tau^*(Init) \cap Bad = \emptyset ?$
- Model-checking problem 2:  $\tau^* \cap \tau_{Bad} = \emptyset ?$
- Several approaches exist  
[Abdulla, Boigelot, Bouajjani, Jonsson, Nilsson, Pnueli, Wolper, etc.]
- Calculating exact reachability sets or relations
  - Special classes where  $\tau^*$  can be calculated
  - Exact widening
- Calculating overapproximations (which are sometimes exact)
  - Abstract regular model-checking [Bouajjani et al. CAV 04]
  - **Inference of regular languages** (extending [Fribourg et al. 97])

## Regular Model Checking

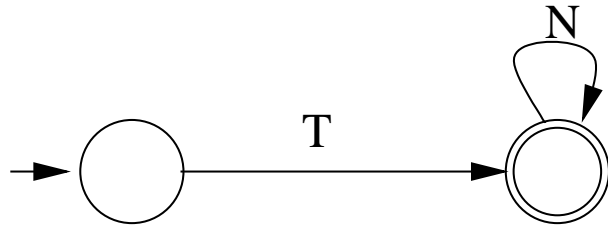
A lot of different systems can be encoded in this way, for example

- (lossy) FIFO-channel systems
- pushdown systems
- systems with counters
- parameterized systems (parameterized number of identical finite-state processes arranged in an array)

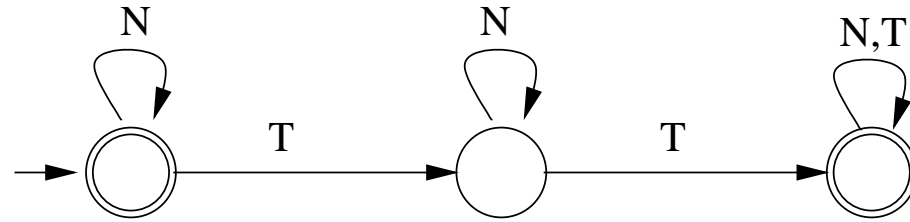
## Example

- Very simple token ring protocol
- Processes are arranged in a linear array.
- An individual process has ( $T$ ) or has not ( $N$ ) the token.
- A string of  $\{N, T\}^*$  of arbitrary length represents a configuration.
- The token can be passed from left to right ( $NTNN \rightarrow NNTN, NTN \rightarrow NNT$ ).

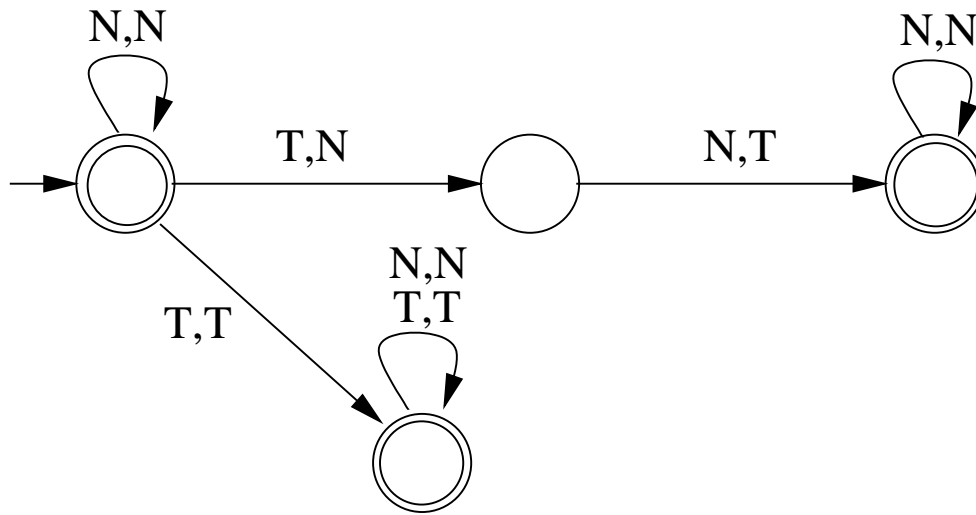
## Example: Token Ring



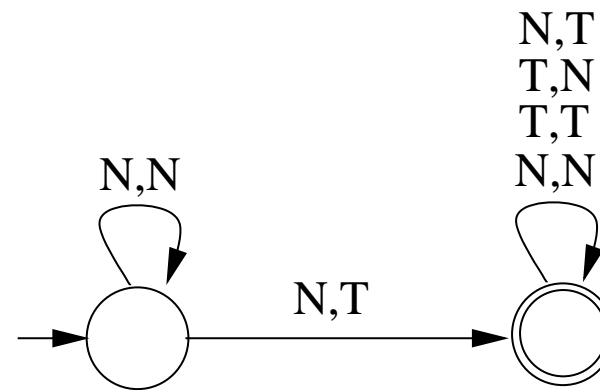
Initial configurations



Bad configurations



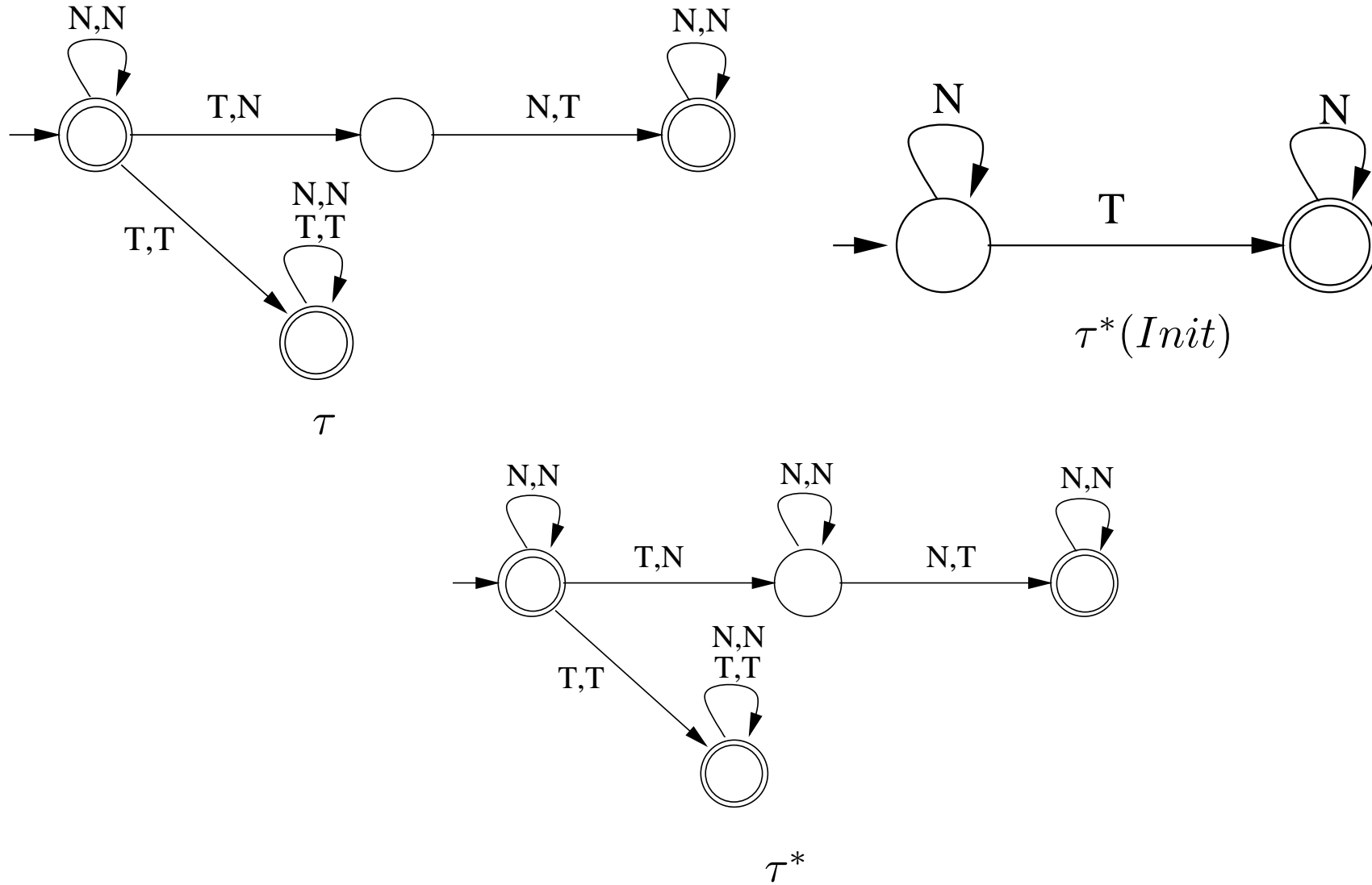
Transitions encoded by transducer  $\tau$



$\tau_{Bad}$



# Example



## Length-preserving transducers

- Transducers do not contain  $\epsilon$ .
- For safety (reachability) properties this is not a restriction.
- Key observation for length-preserving transducers:
  - $\tau^*(Init)$  restricted to configurations of bounded size can be computed.
  - $\tau^*$  restricted to bounded lengths can be computed.
  - These finite sets can be considered complete samples of  $\tau^*(Init)$  and  $\tau^*$  resp.
  - gives rise to a special inference problem

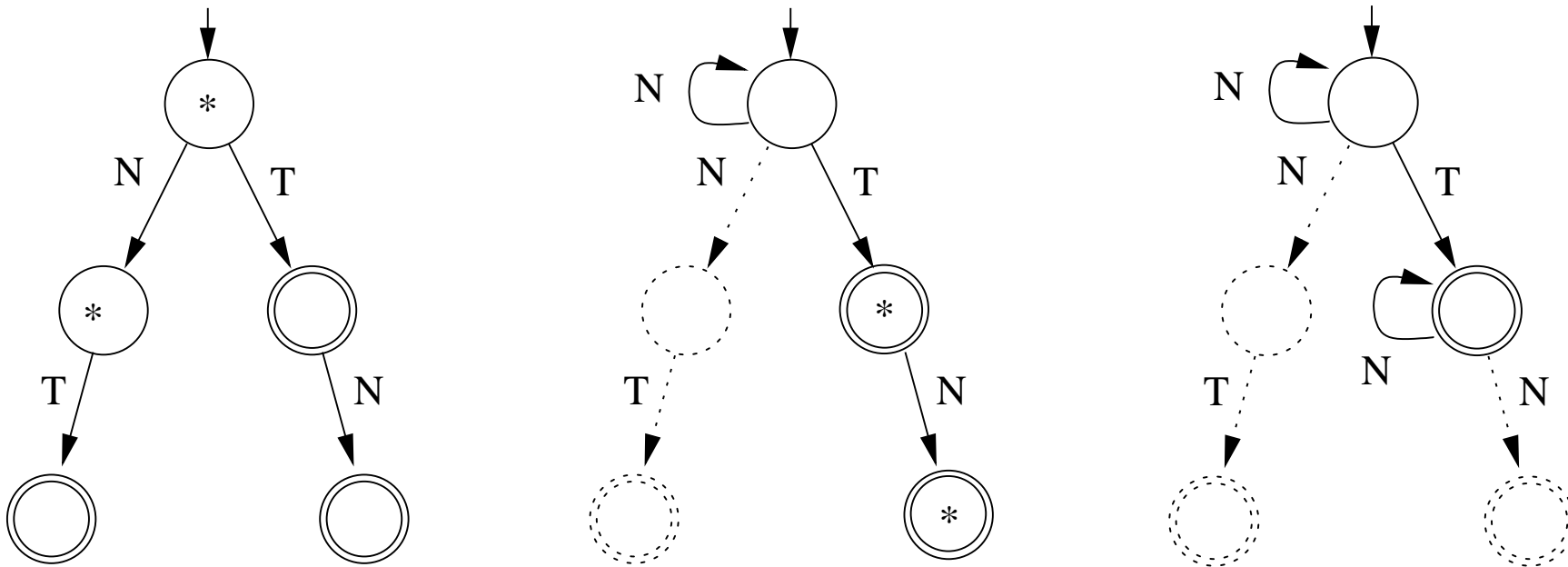
## Inference of regular languages from complete training sets

[Trakhtenbrot, Barzdin 72]

- Automata to be inferred:  $A$
- Complete training set  $T_k = (T_k^+, T_k^-)$ , where  $T_k^+$  contains all words of  $L(A)$  up to length  $k$  and  $T_k^-$  all others.
- Construct prefix-tree automaton from  $T_k^+$ .
- Collapse compatible states (which do not introduce inconsistencies)
- Theorem: given a sufficiently big (depends on the structure of the automaton) complete training set the Trakhtenbrot-Barzdin algorithm infers  $A$ .  
In the **worst case** all words of  $L(A)$  up to length  $2|A| - 1$  are needed.

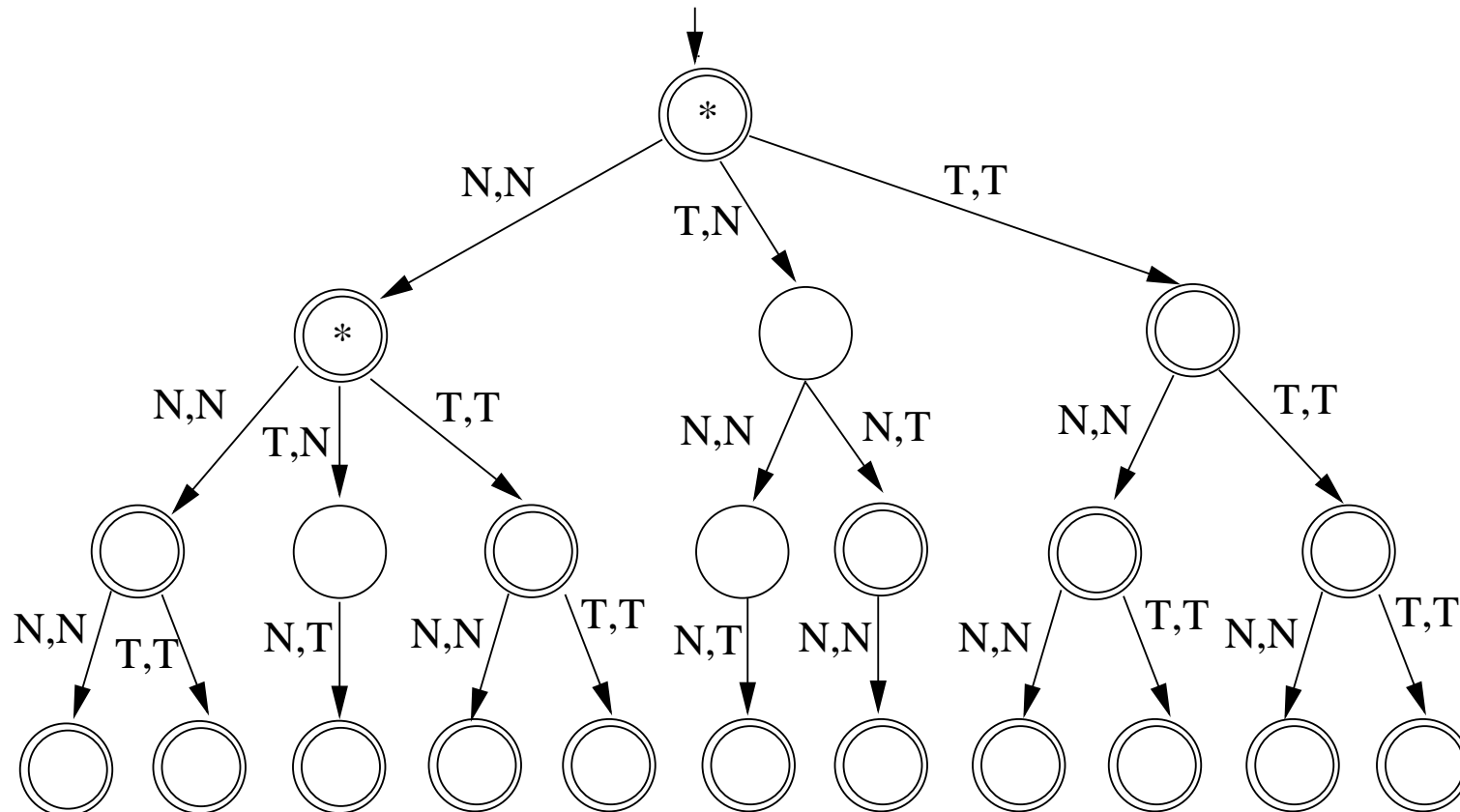
## Example

Inference of  $\tau^*(Init)$  from a complete sample of size 2.



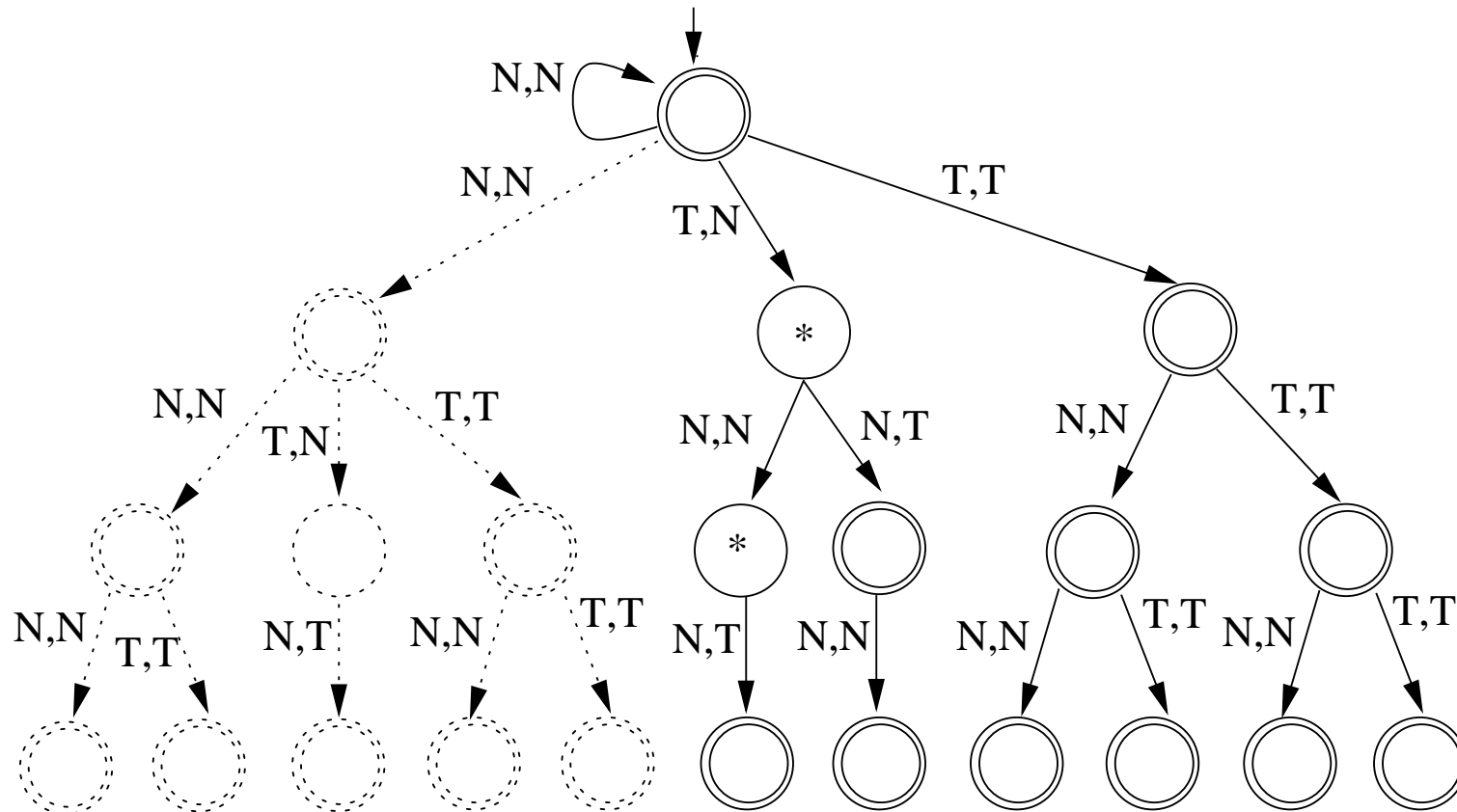
## Example

Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3



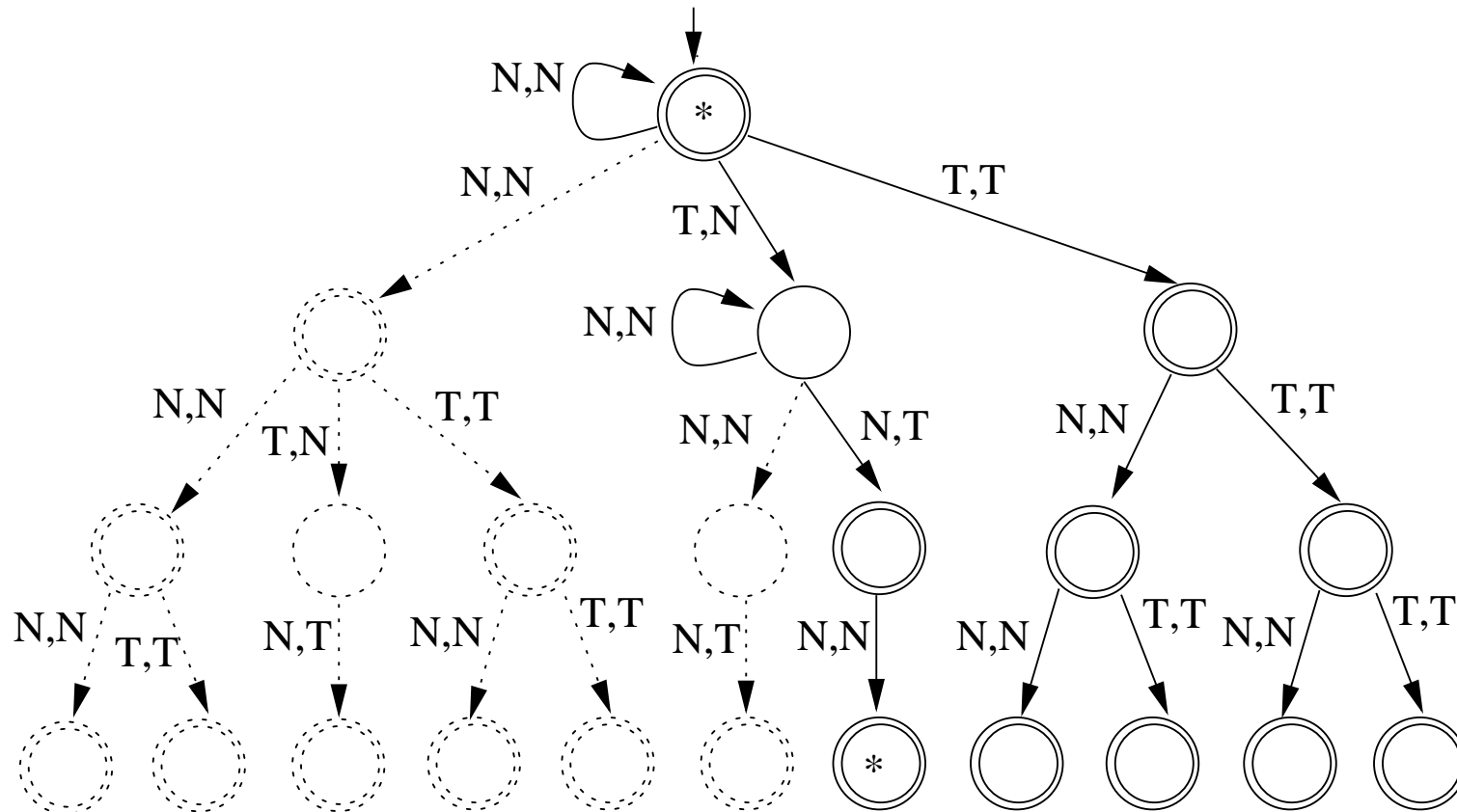
## Example

Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3



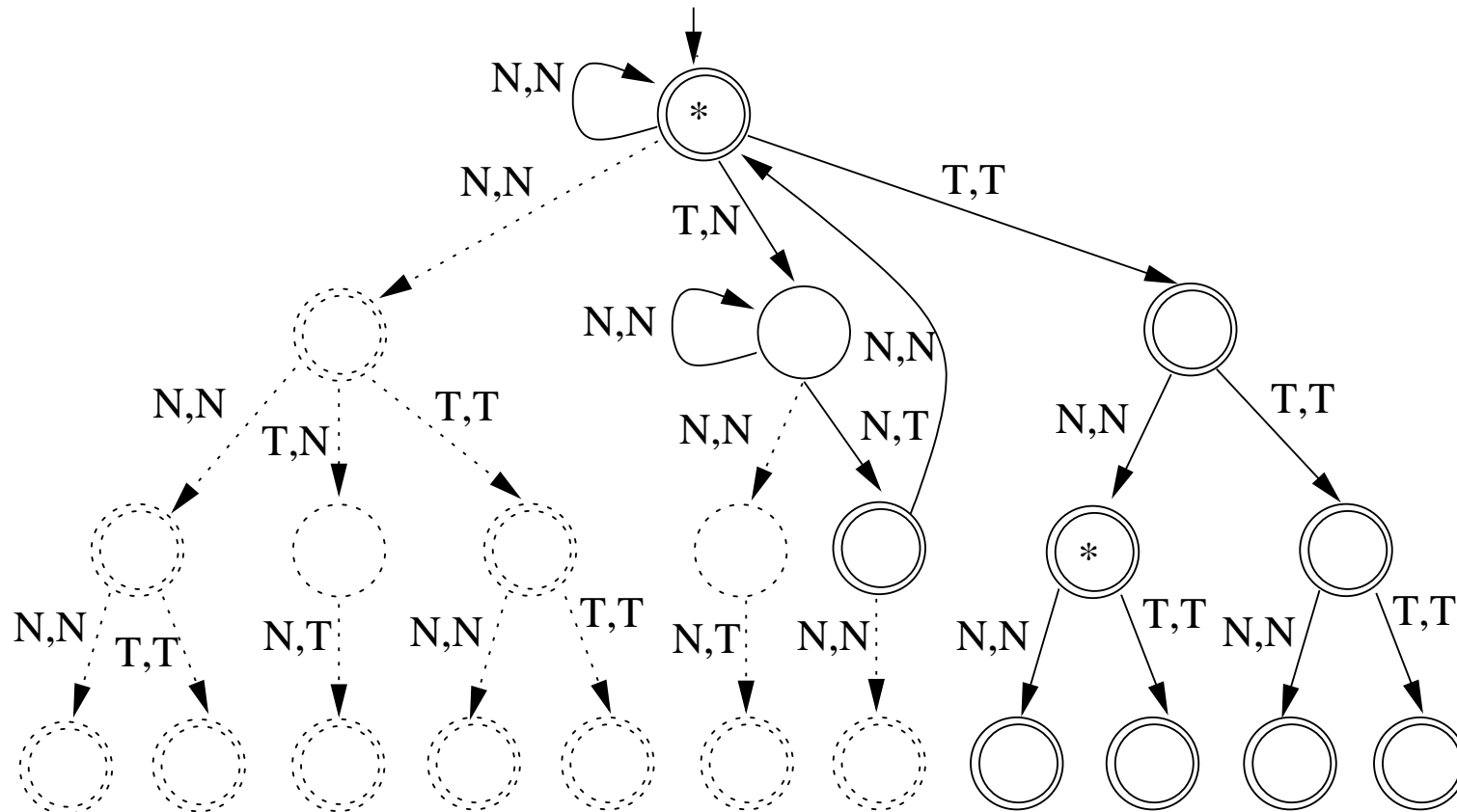
## Example

Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3



## Example

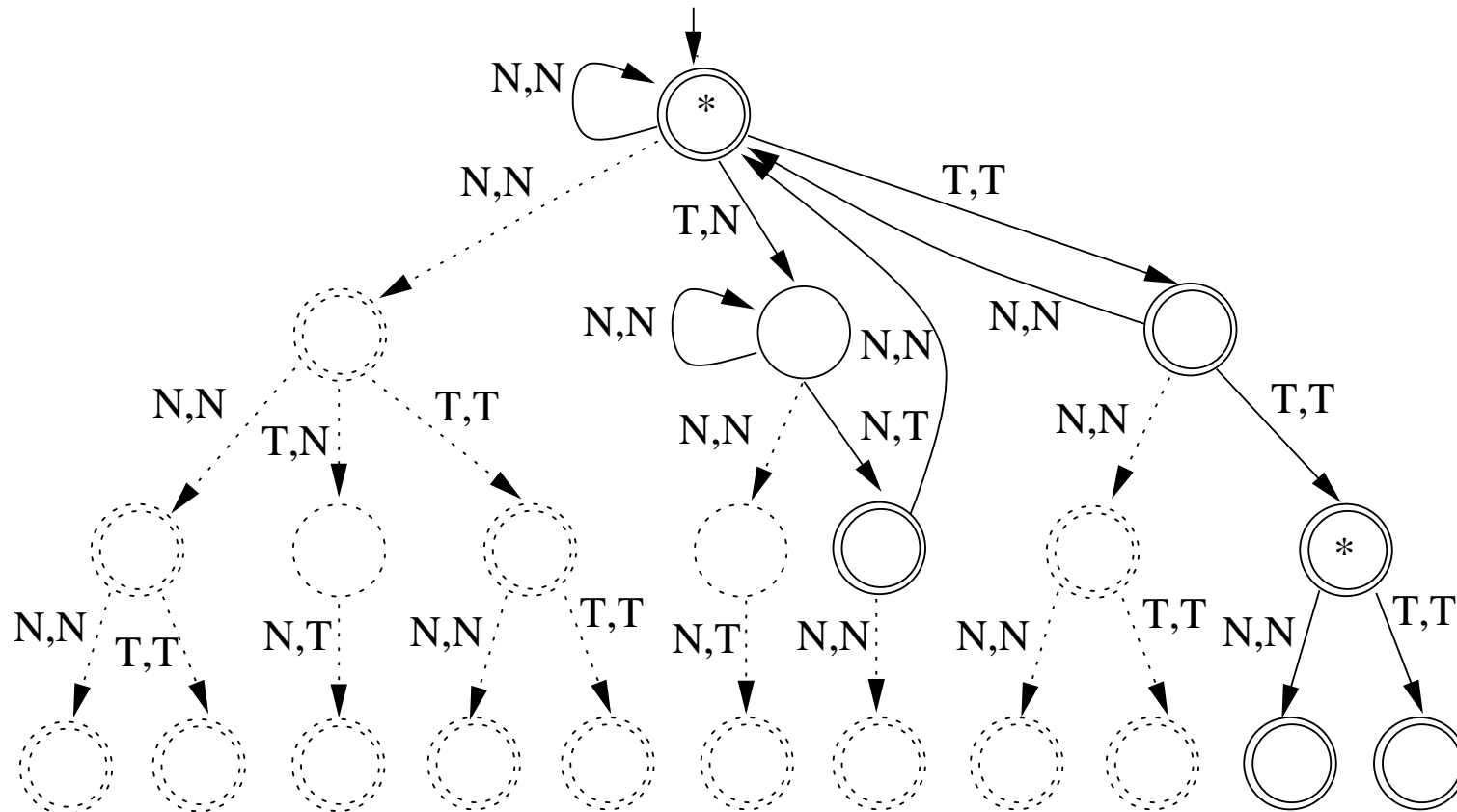
Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3





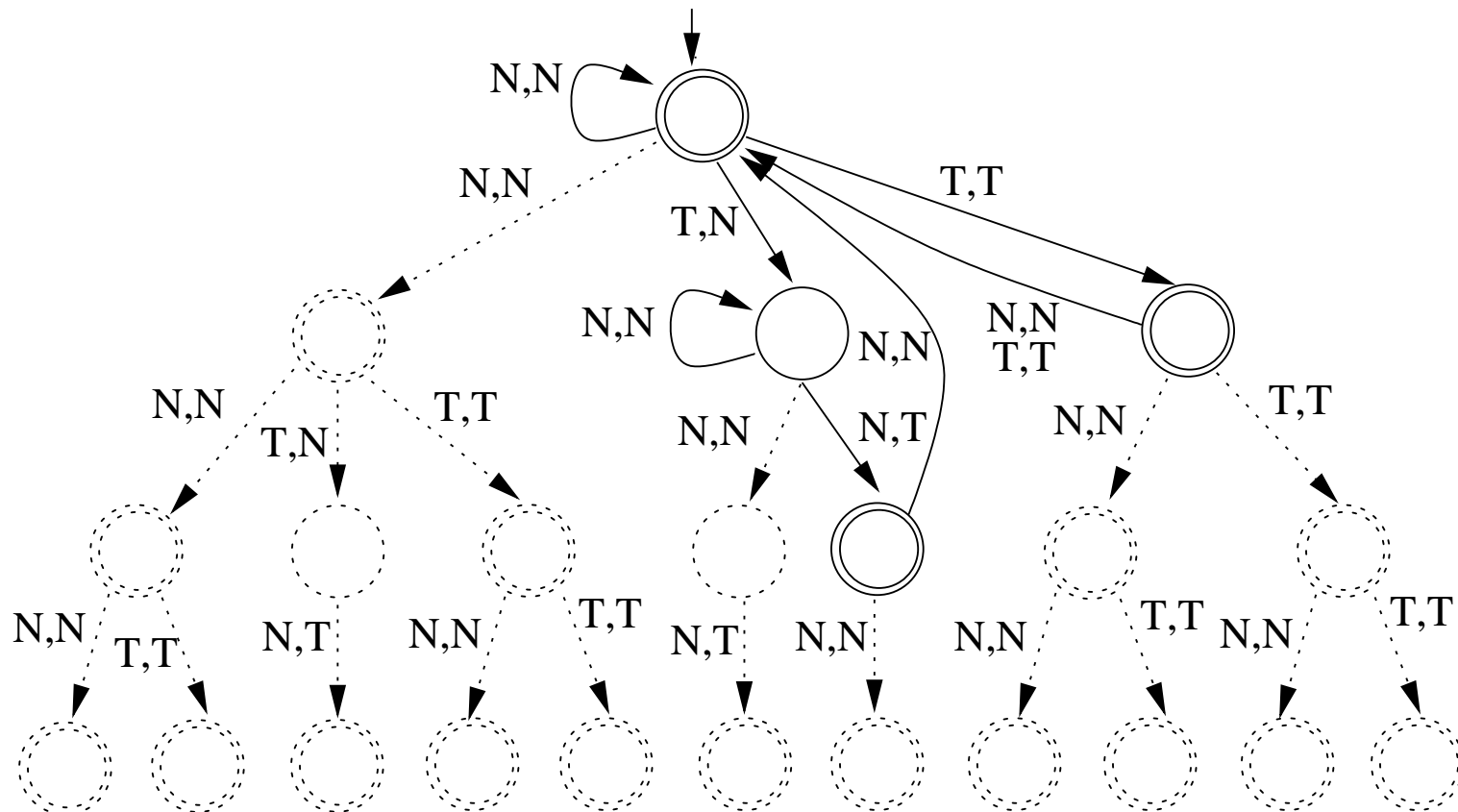
## Example

Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3



## Example

Inference of an overapproximation of  $\tau^*$  from a complete sample of size 3



## A general model-checking algorithm

Problem 1:  $\tau^*(Init) \cap Bad = \emptyset ?$

input: a length-preserving transducer  $\tau$ ,  
 a regular set of initial configurations  $Init$   
 and a regular set of bad configurations  $Bad$

$i := 1$ ; /\*  $i$  can be initialised differently too. \*/

repeat

$C := \tau^*(Init^{\leq i})$ ;

$\overline{C} := \Sigma^{\leq i} \setminus C$ ;

if  $Bad \cap C \neq \emptyset$  then output: property violated;

$A := inference(C, \overline{C})$ ;

$i := i + 1$ ;

until  $\tau(L(A)) \subseteq L(A)$  and  $Init \subseteq L(A)$  and  $L(A) \cap Bad = \emptyset$ ;

output: property satisfied

As inference algorithm one can use for example Trakhtenbrot-Barzdin

## Results

- The model-checking algorithm always terminates if  $\tau^*(Init)$  is regular.
- Model-checking problem 1 is decidable if  $\tau^*(Init)$  is regular.
- This already follows from [Pachl 87]. It is sufficient to enumerate all regular languages and check them for invariance. Here we provide a clever enumeration.
- The algorithm can be easily used for  $\tau^*$  as well.
- Model-checking problem 2 is decidable if  $\tau^*$  is regular.

## Experiments

Prototype implementation in Prolog using FSA Library

Experiment	$T$ [sec]	G[%]	Experiment	$T$ [sec]	G[%]
Bakery	0.03	50	Dijkstra	1.16	92
Bakery comm. liv.	0.36	90	PDS	0.04	63
Bakery counters 3P	8.69	70	Petri net/Read. Wr.	323	90
Bakery counters 4P	143	92	Faulty PN/Rd. Wr.	1.48	54
Bakery 5P unary	229	45	Szymanski	0.76	94
ABP	0.03	50	Rev. Lists	1.64	90
Burns	0.77	98	Rev. Lists/Transd.	40.5	69

## Conclusion and Perspectives

- General algorithm for Regular model-checking
- Termination guaranteed for regular reachability sets
- Try other inference algorithms
- Use dedicate algorithms for generating reachable configurations of bounded length.