

# On the construction of an UBA

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# General case

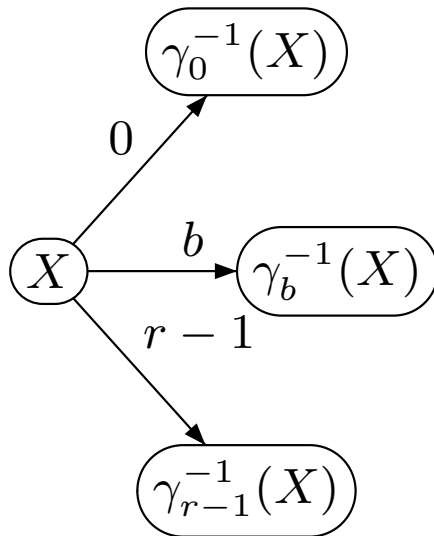
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Let  $b \in \{0, \dots, r - 1\}$  be a **digit** where  $r \geq 2$  is the basis of decomposition.

Let  $\gamma_b : \mathbb{N}^m \rightarrow \mathbb{N}^m$  be the affine function defined by:

$$\gamma_b(x_1, \dots, x_m) = (x_2, \dots, x_m, r \cdot x_1 + b)$$

An infinite graph is associated to any subset  $X \subseteq \mathbb{N}^m$ :



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**Case**  $m = 2, r = 2$ .

Let  $X = \{(x_1, x_2) \in \mathbb{N}^2; \alpha x_1 + \beta x_2 = \gamma\}$ , we have:

$$(x'_1, x'_2) \in \gamma_b^{-1}(X) \iff \gamma_b(x'_1, x'_2) \in X$$

$$\iff (x'_2, 2x'_1 + b) \in X$$

$$\iff \alpha x'_2 + \beta(2x'_1 + b) = \gamma$$

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$$\boxed{\alpha x_1 + \beta x_2 = \gamma} \xrightarrow{b} \boxed{2\beta x_1 + \alpha x_2 = \gamma - b\beta}$$

# Case $2x_1 - x_2 = 0$

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$$2x_1 - x_2 = 0$$

# Case $2x_1 - x_2 = 0$

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$$\alpha x_1 + \beta x_2 = \gamma \xrightarrow{b} 2\beta x_1 + \alpha x_2 = \gamma - b\beta$$

$$\begin{array}{c} -2x_1 + 2x_2 = 1 \\ \uparrow 1 \\ 2x_1 - x_2 = 0 \end{array}$$

# Case $2x_1 - x_2 = 0$

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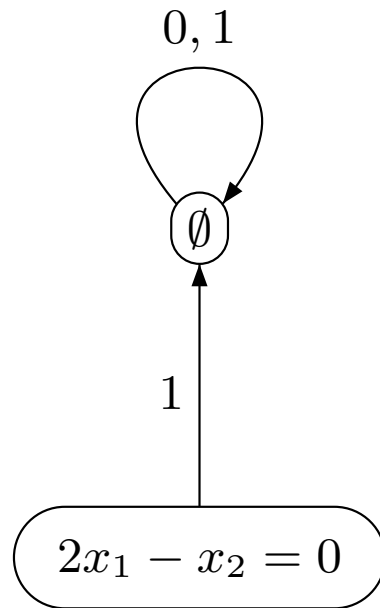
$$\alpha x_1 + \beta x_2 = \gamma \xrightarrow{b} 2\beta x_1 + \alpha x_2 = \gamma - b\beta$$

$$\begin{array}{c} -2x_1 + 2x_2 = 1 \\ \iff \\ (x_1, x_2) \in \emptyset \\ \uparrow 1 \\ 2x_1 - x_2 = 0 \end{array}$$

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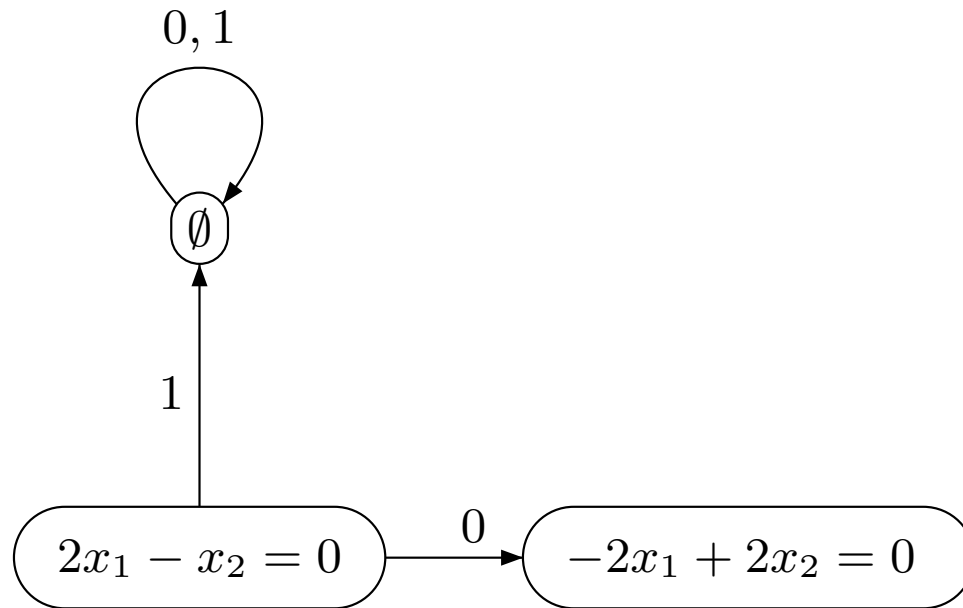




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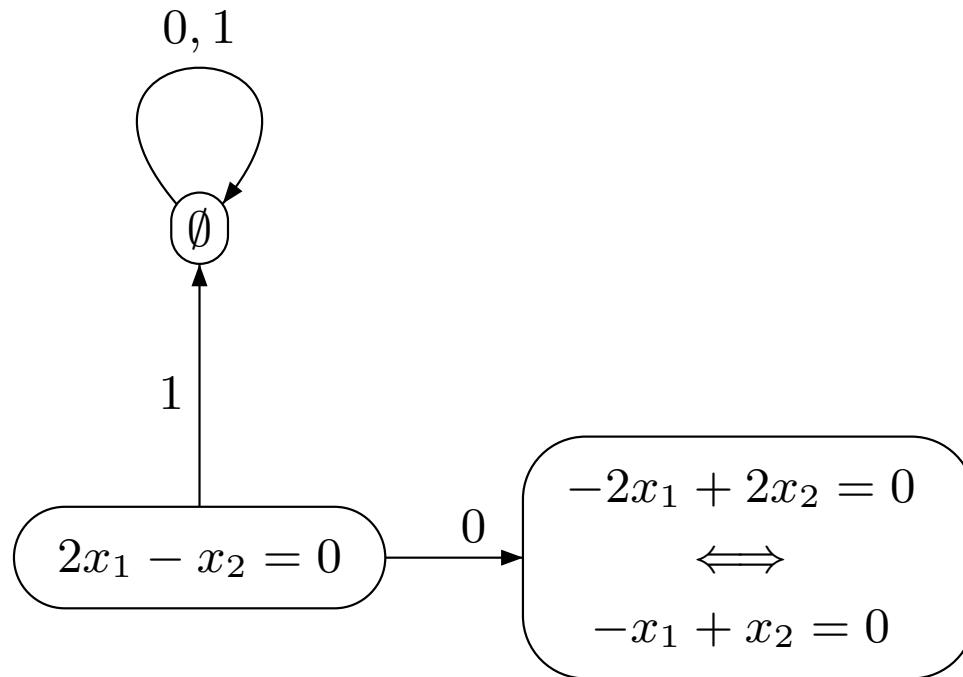
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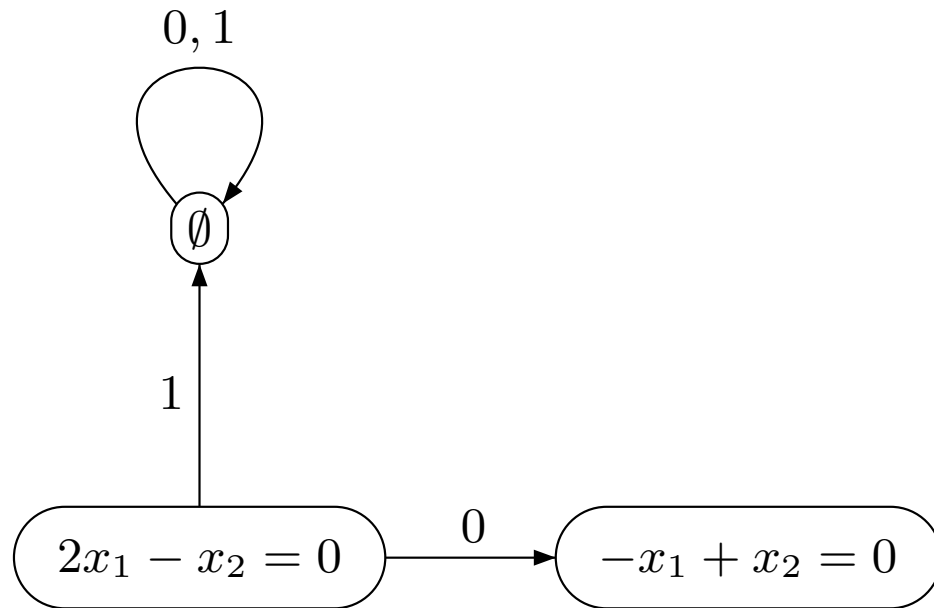
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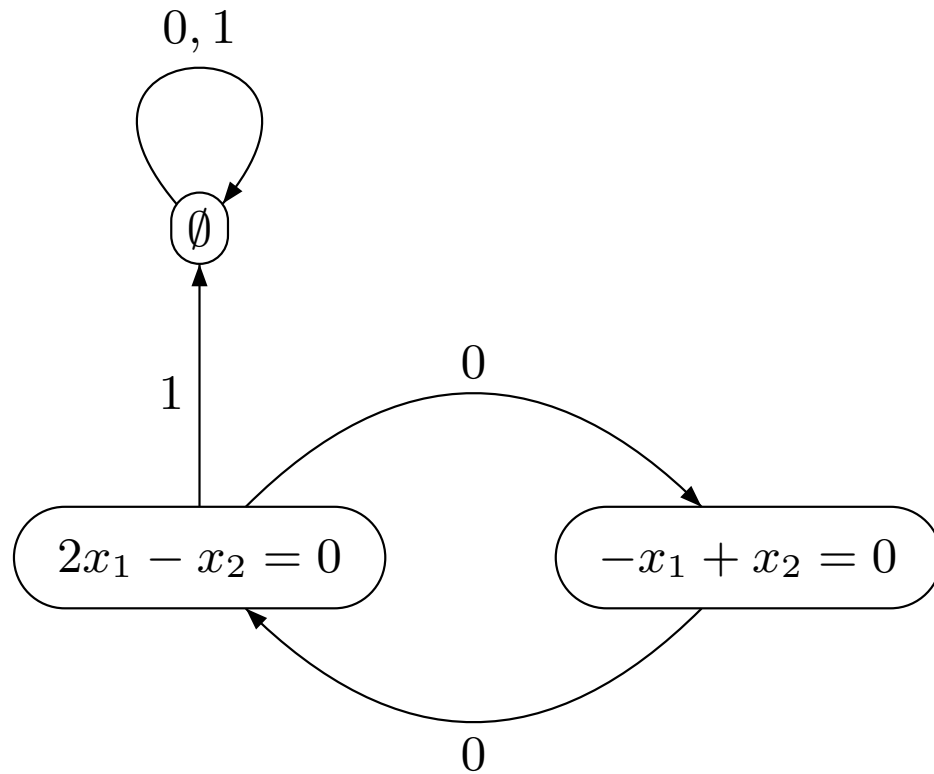
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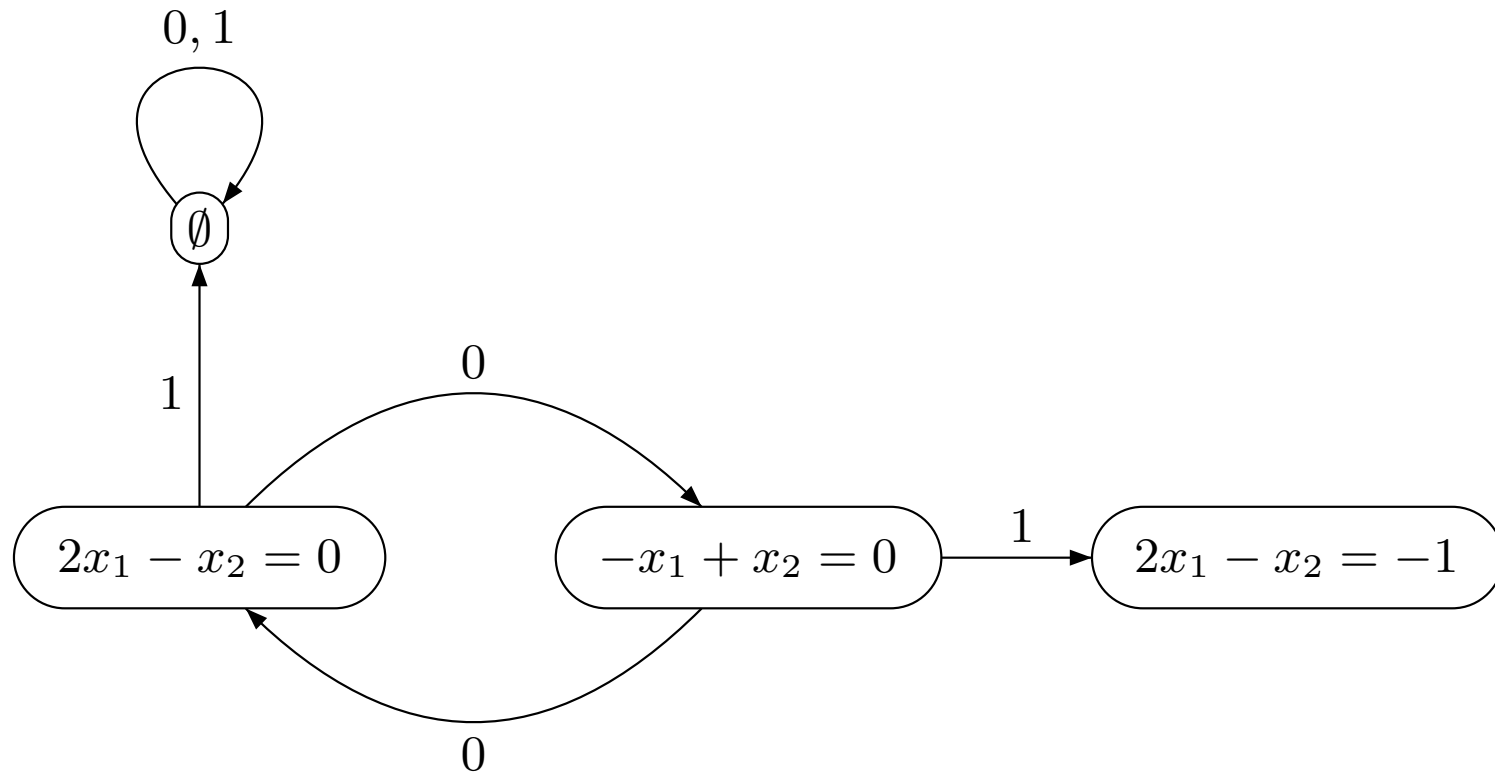
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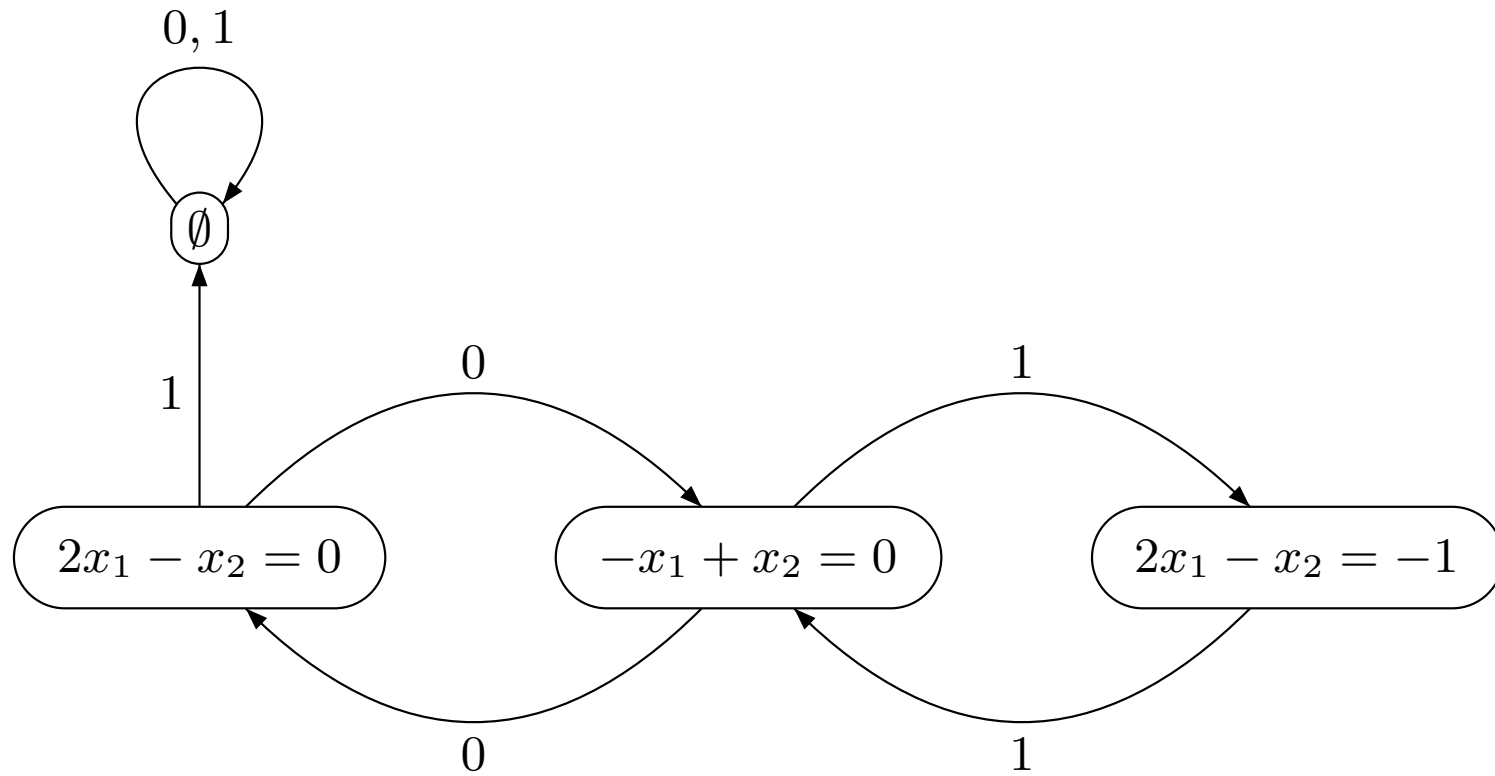
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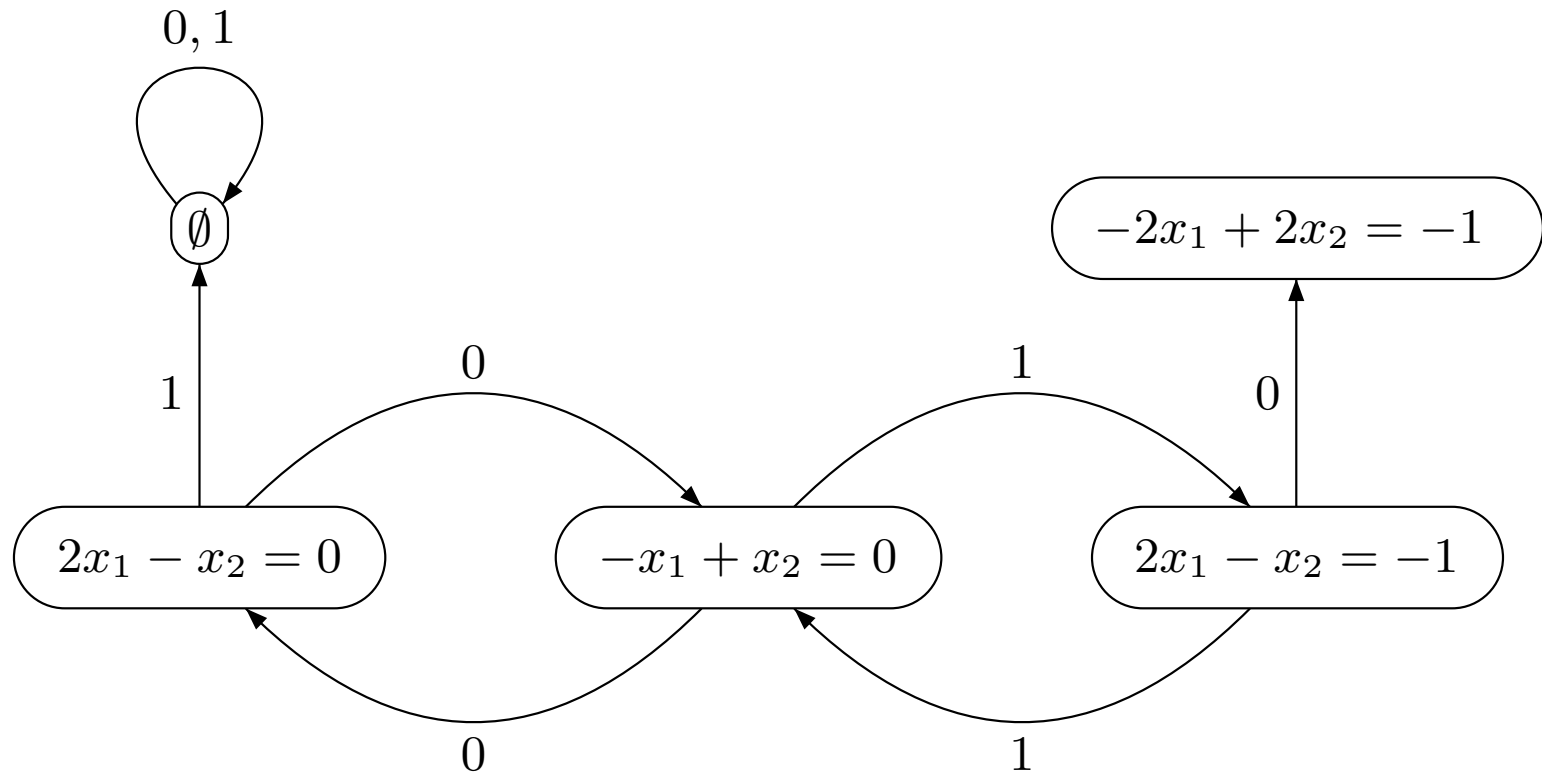
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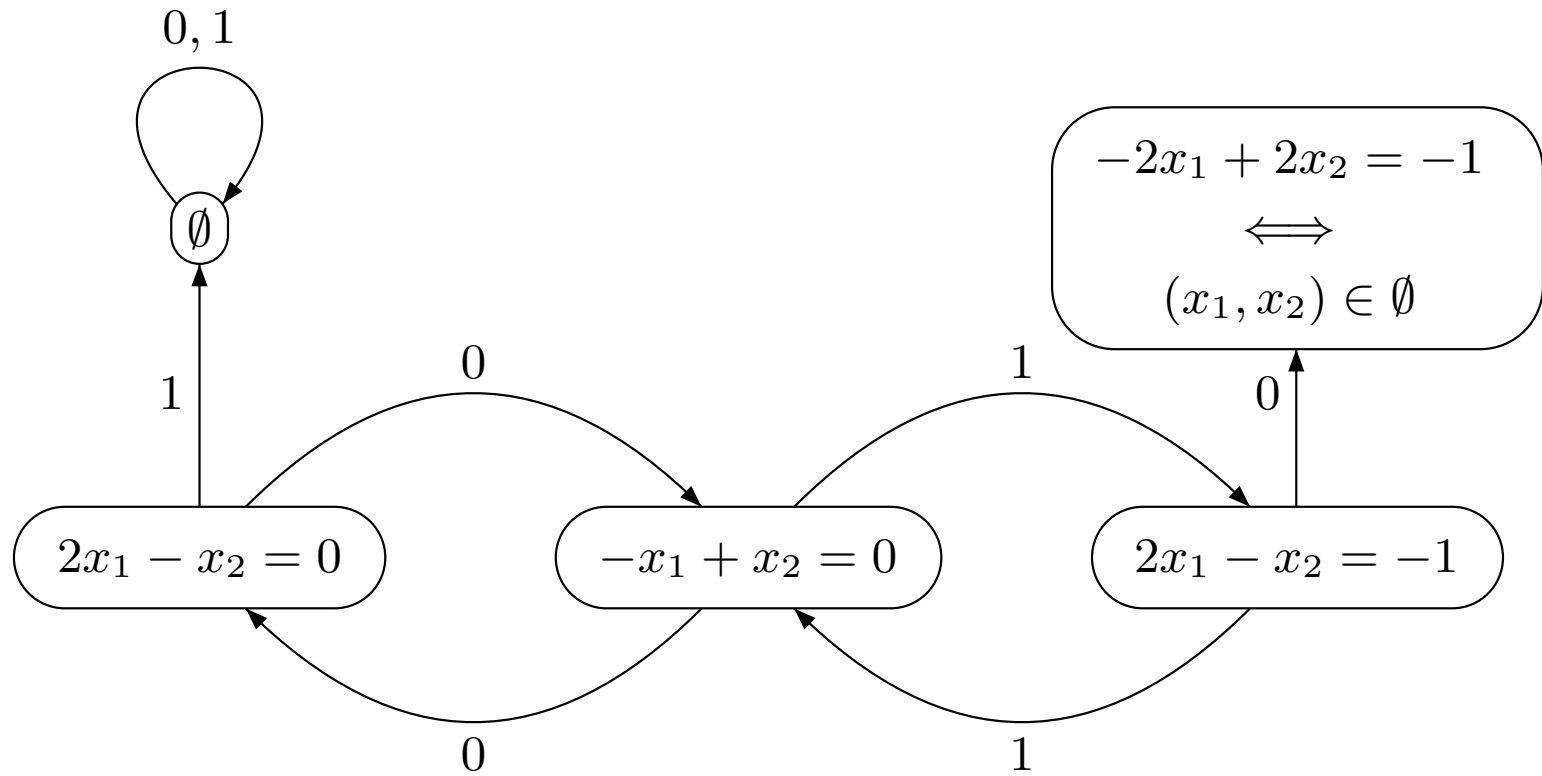
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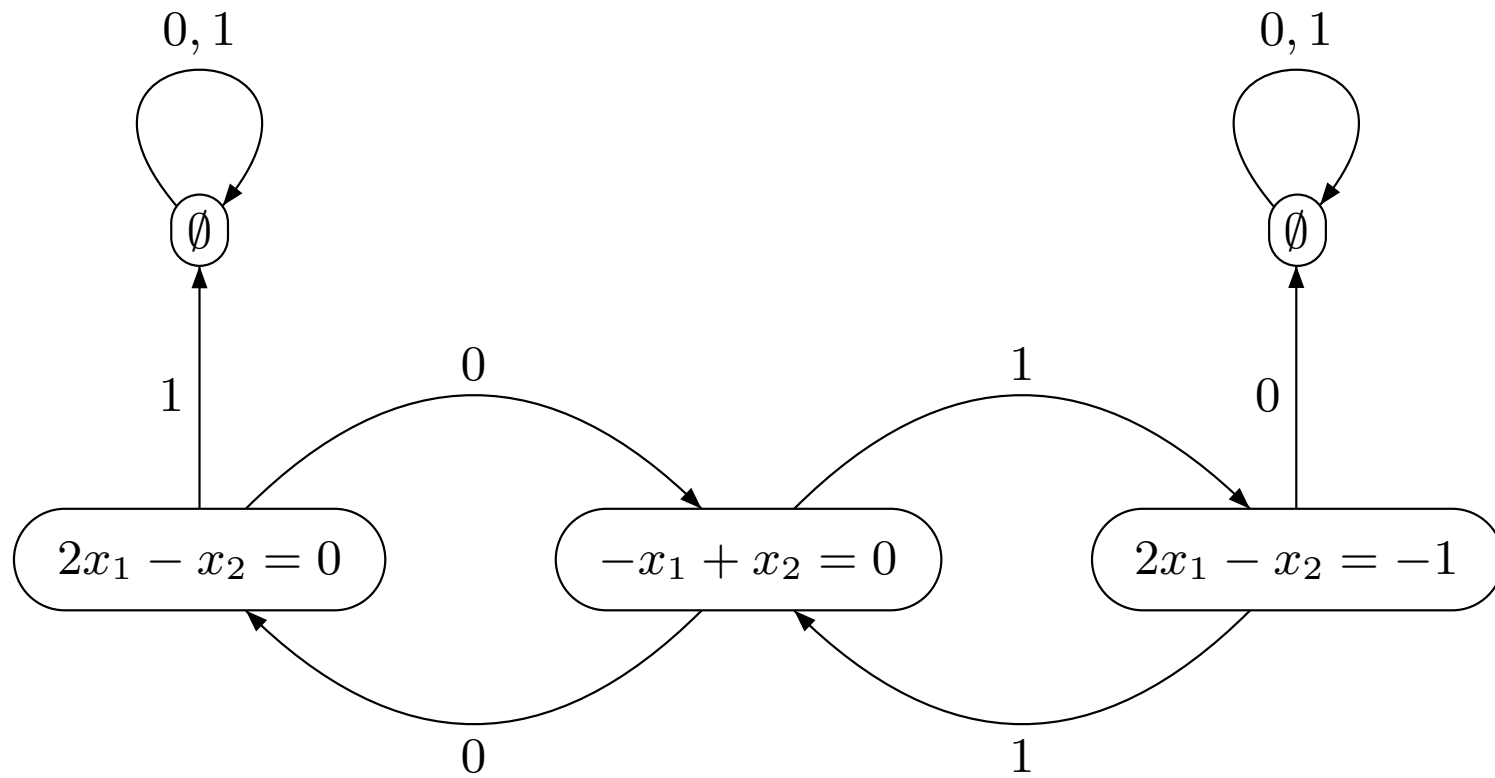
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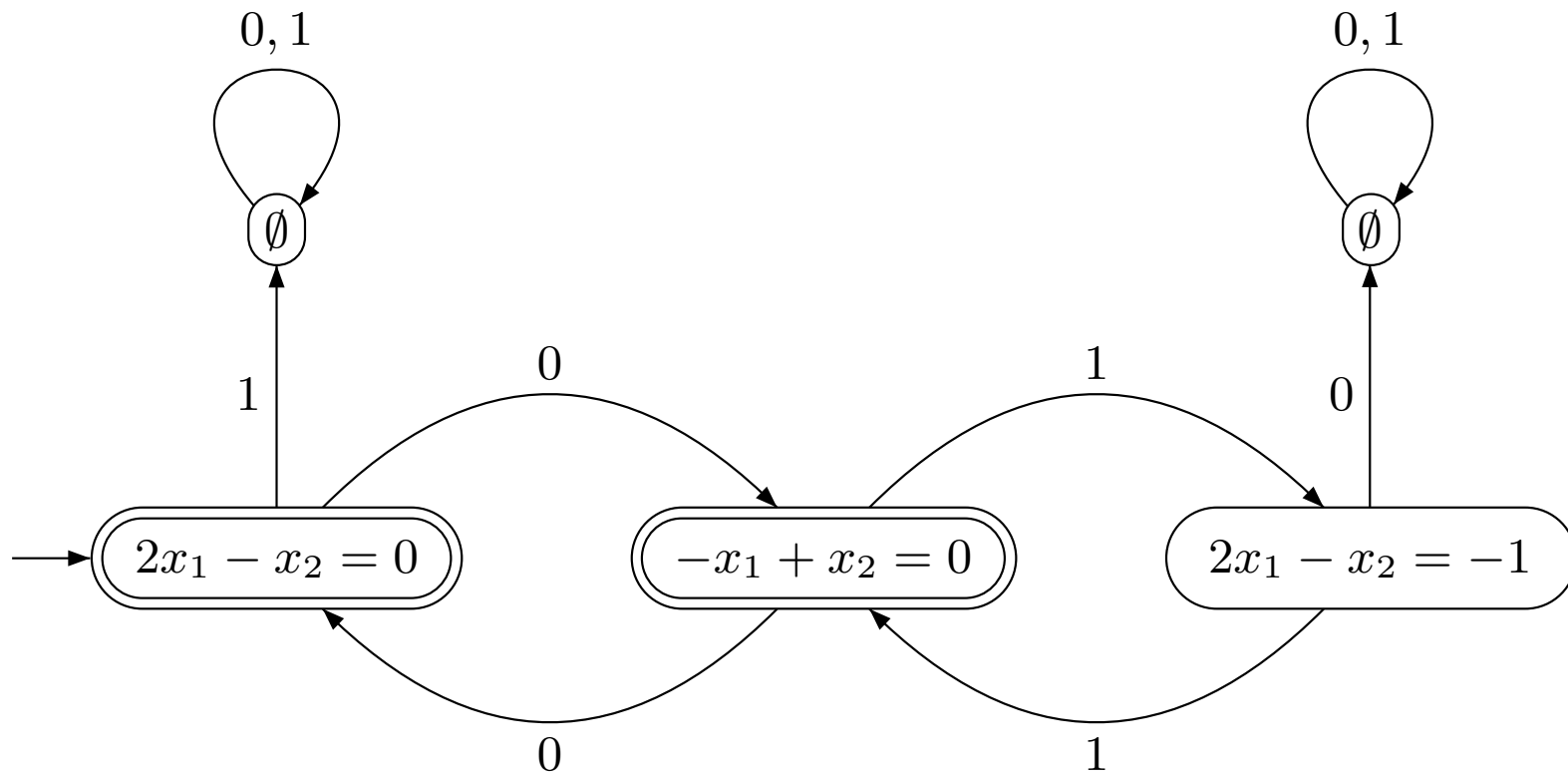
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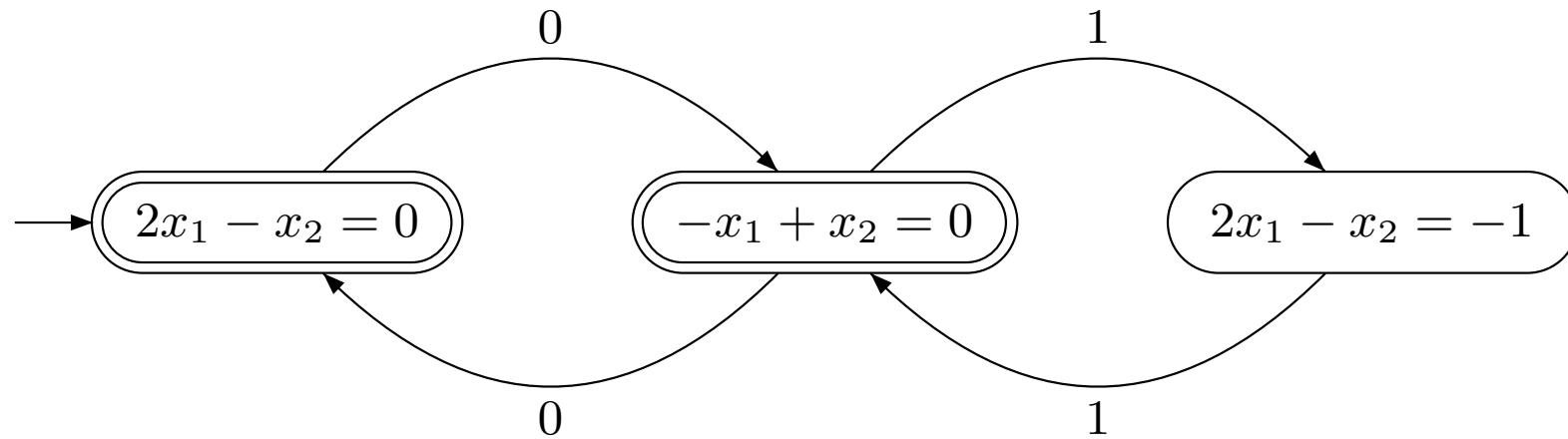
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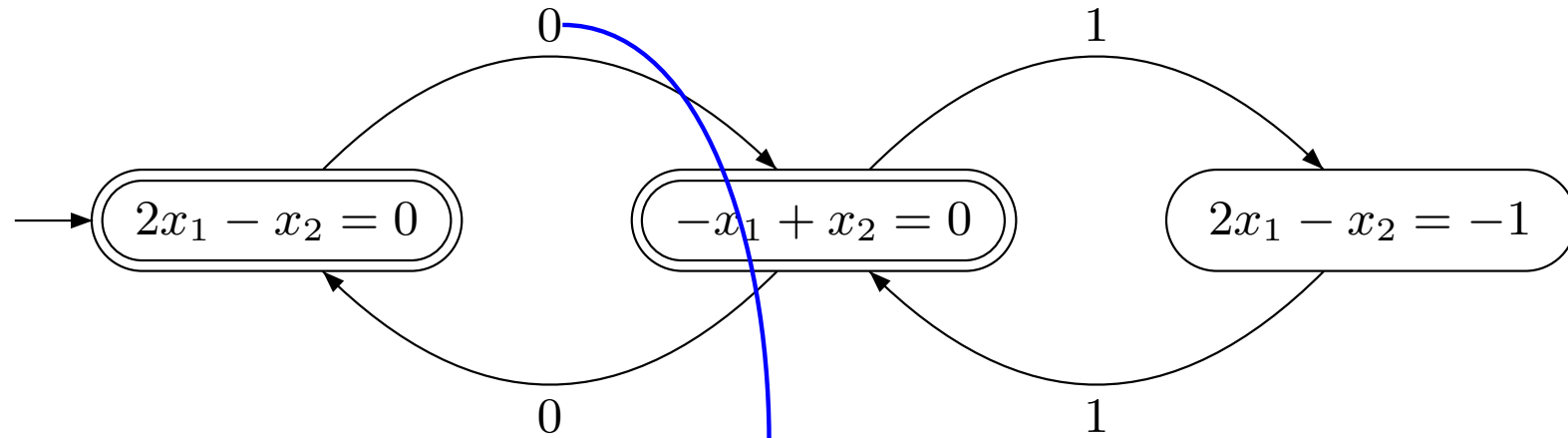
# Case $2x_1 - x_2 = 0$

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$$\begin{array}{l} x_1 = \text{(binary)} = 0 \text{ (decimal)} \\ x_2 = \text{(binary)} = 0 \text{ (decimal)} \end{array}$$

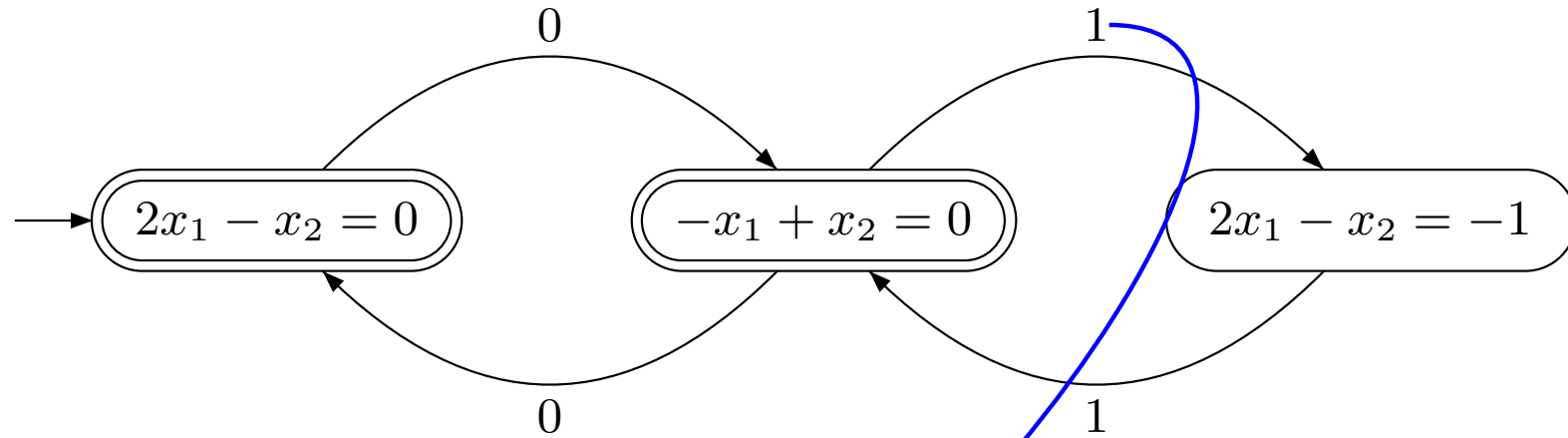
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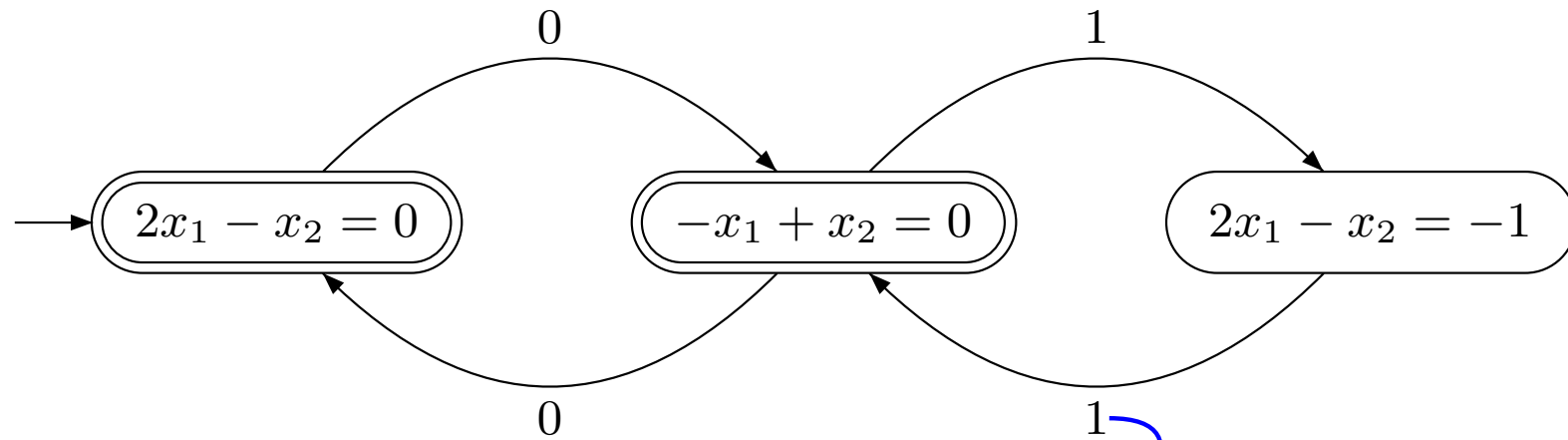
# Case $2x_1 - x_2 = 0$

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$x_1$	$= 1$	(binary)	$= 1$	(decimal)
$x_2$	$= 0$	(binary)	$= 0$	(decimal)

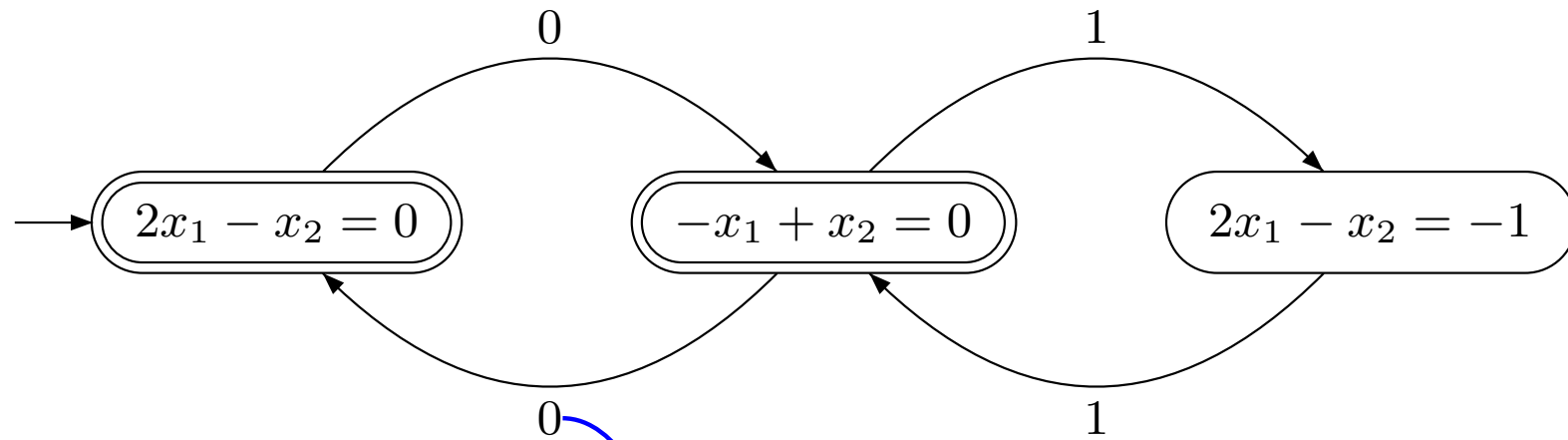
# Case $2x_1 - x_2 = 0$



$x_1$	$= 1$	(binary)	$= 1$	(decimal)
$x_2$	$= 01$	(binary)	$= 2$	(decimal)

A blue arrow points from the '1' in the second row to the '01' in the second row.

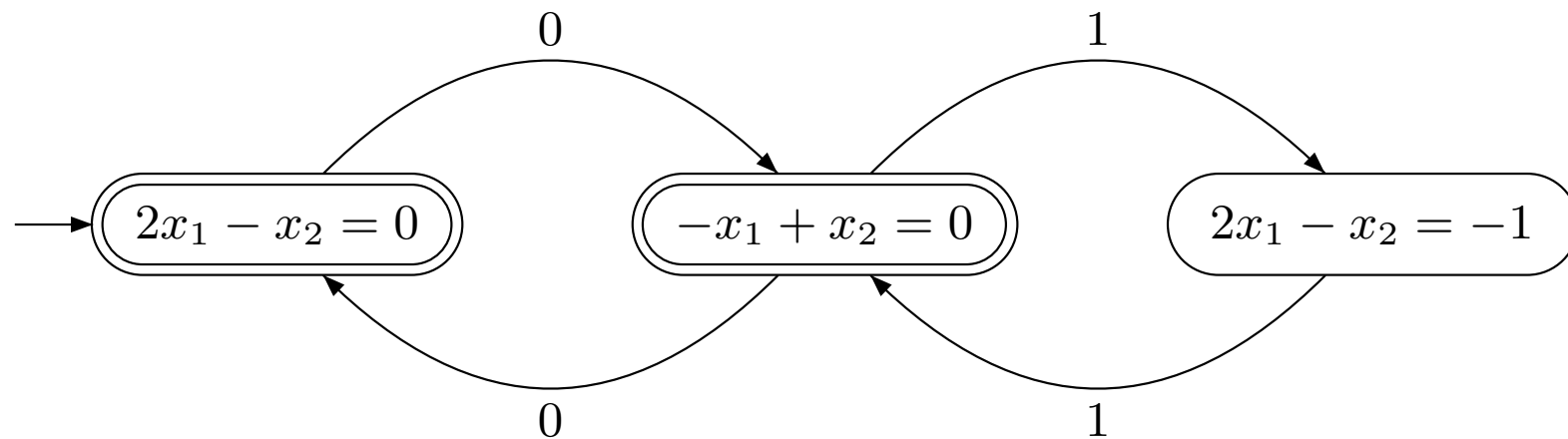
# Case $2x_1 - x_2 = 0$



$x_1 = 10$  (binary) = 1 (decimal)  
 $x_2 = 01$  (binary) = 2 (decimal)

# Case $2x_1 - x_2 = 0$

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$$x_1 = 10 \text{ (binary)} = 1 \text{ (decimal)}$$

$$x_2 = 01 \text{ (binary)} = 2 \text{ (decimal)}$$