

Label-Guided Graph Exploration by a Finite Automaton

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DYNAMO, Les Ménuires
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Graph exploration

Goal

A mobile entity has to **traverse** every edge of an **unknown anonymous** graph.

Motivations (1)

Exploration by mobile agents

- **Physical robot**: exploration of environments unreachable by humans
- **Software agent**: network maintenance

Equivalence between logic and automata
(Engelfriet, Hoogeboom)

Through characterization of string, tree or graph languages

- Automata with nested pebbles
- First-order logic with transitive closure

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Motivations (2)

USTCON (undirected st-connectivity)

- $G = \{V, E\}$ an undirected graph
- $s, t \in V$ two vertices of G

Are s and t in the same connected component of G ?

- L = class of problems solvable by deterministic log-space computations
- $SL (\supseteq L)$ = class of problems solvable by symmetric non-deterministic log-space computations

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Reingold, STOC 2005

Undirected ST-Connectivity in Log-Space

$USTCON \in L \Rightarrow SL=L$

Unknown, anonymous

Unknown

- Unknown topology
- Unknown size (no upper bound)

Anonymous

- No node labeling
- Local edge labeling

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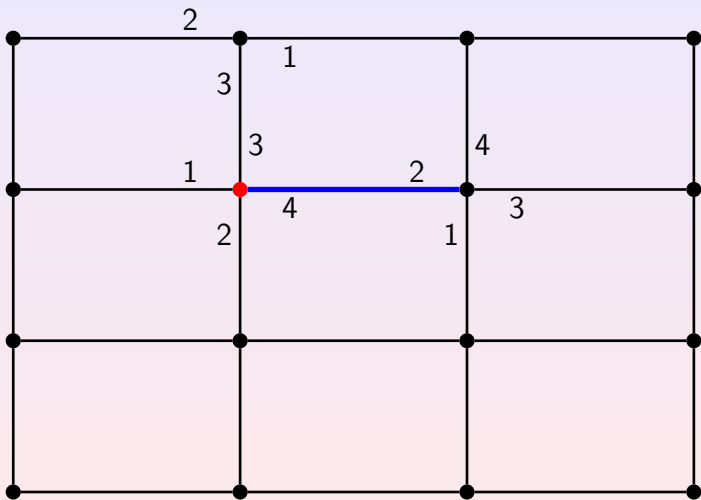
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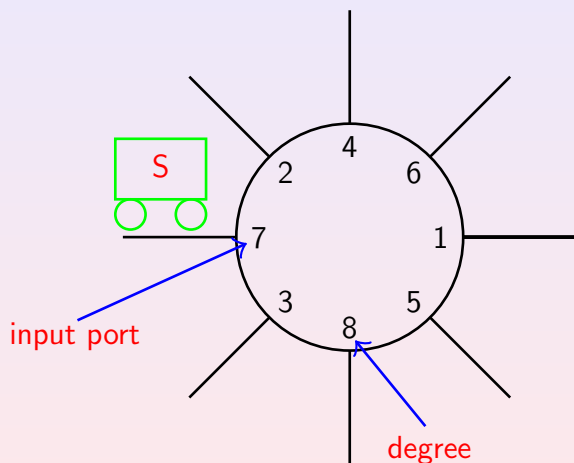
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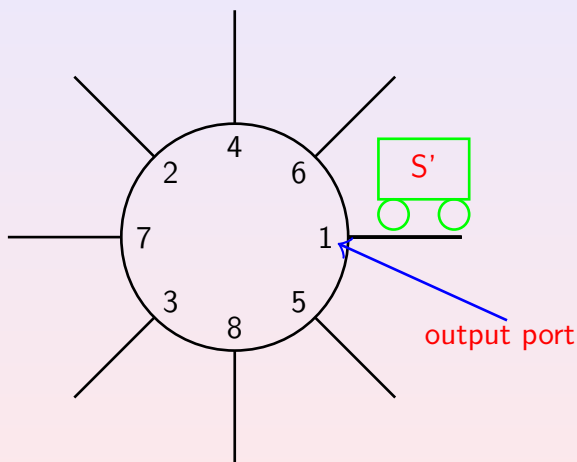
Example of an anonymous graph



Mealy automaton (1)



Mealy automaton (1)



Mealy automaton (2)

Input

- S : current state
- i : input port number
- d : node's degree

Output

- S' : new state
- j : output port number

Transition function

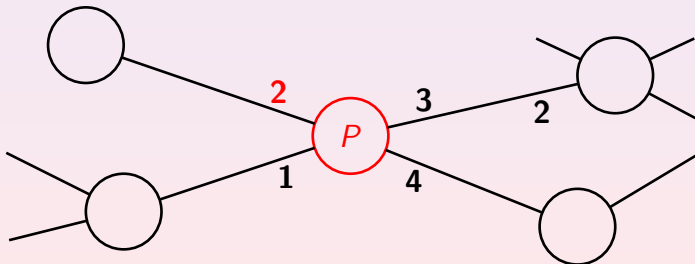
- $f(S, i, d) = (S', j)$

Exploration of the neighborhood

Star

The star can be explored by a 2-state automaton

- PlusOne $P : f(i, d, P) = (i + 1 \bmod d, B)$
- Backtrack $B : f(i, d, B) = (i, P)$

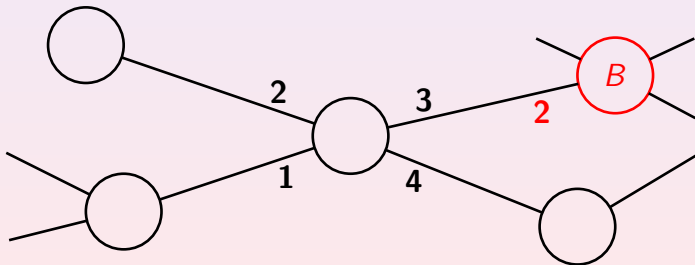


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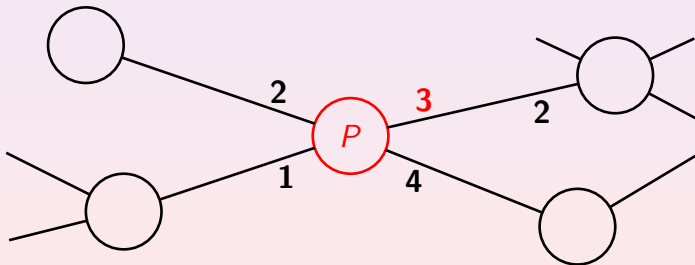


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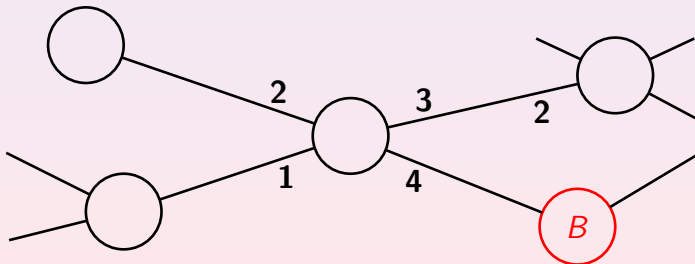


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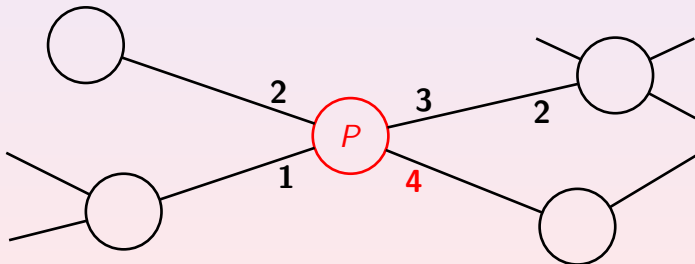


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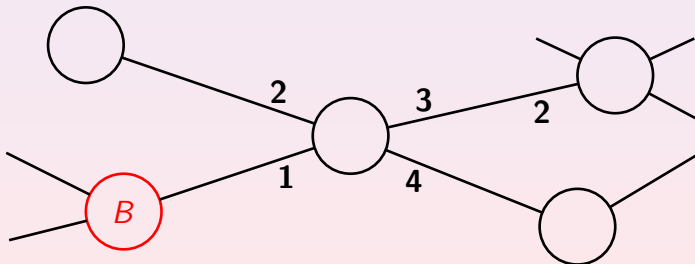


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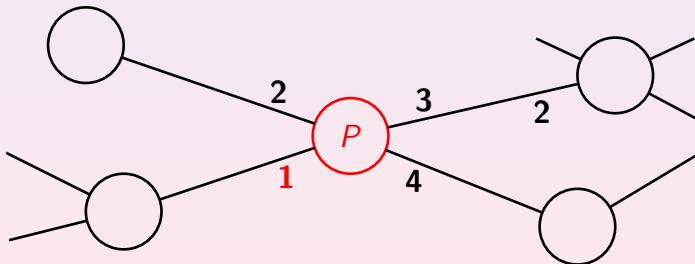


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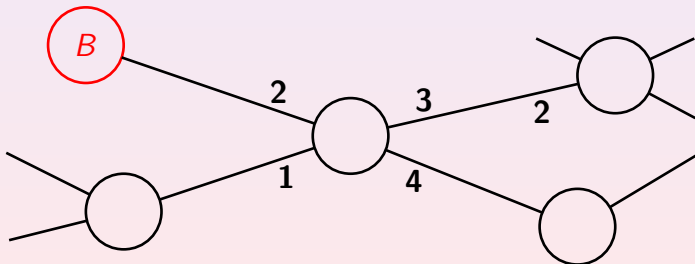


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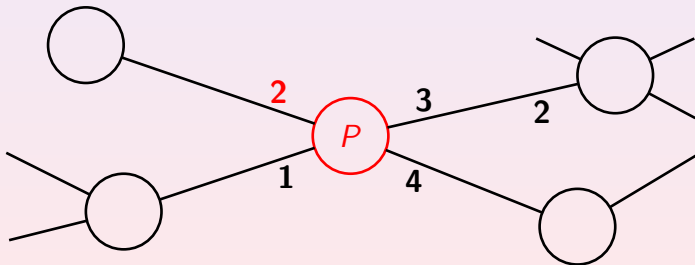


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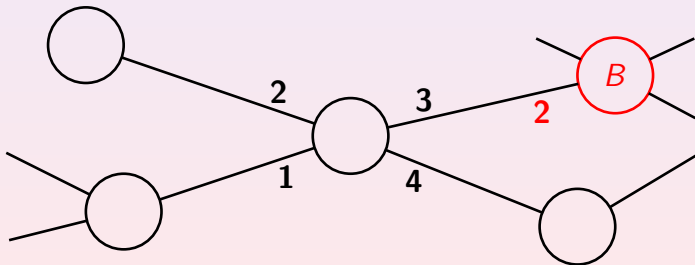


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Outline

- 1 Introduction
- 2 Related work
 - Impossibility results
 - Exploration of trees
- 3 Our model and results
- 4 Coloring algorithms
- 5 Conclusion

Impossibility results (1)

Budach, Math. Nachrichten, 1978
Automata and Labyrinths

No finite automaton can explore all graphs.

A pebble is a node-marker that can be dropped at and removed from nodes.

— Seminar talk at Berkeley, 1967
Maze threading automata

No finite automaton with a finite number of pebbles can explore all graphs.

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Impossibility results (2)

Rollik, Acta Informatica, 1980
Automaten in planaren Graphen

No finite team of finite cooperative automata can explore all (cubic planar) graphs.

A JAG (Jumping Automaton for Graphs) is a team of finite automata that cooperate constantly. Moreover an automaton can jump to a vertex occupied by another automaton.

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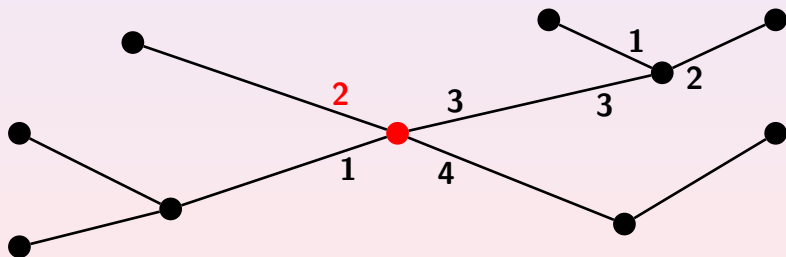
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Universality for trees

Diks, Fraigniaud, Kranakis, Pelc, SODA 2002

Tree exploration with little memory

An oblivious automaton (one single state) can explore all trees.

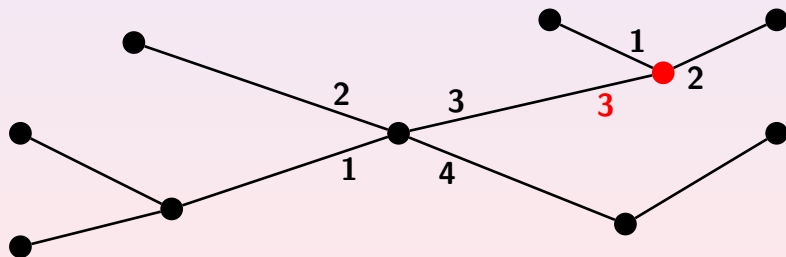


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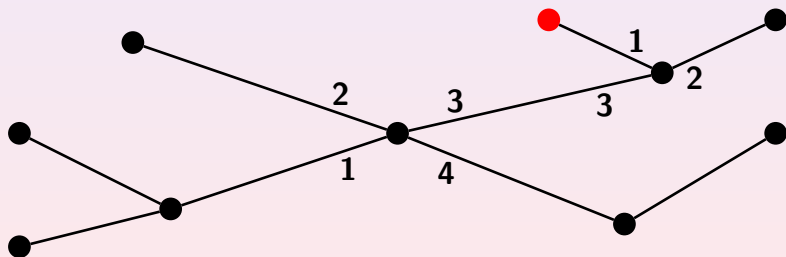


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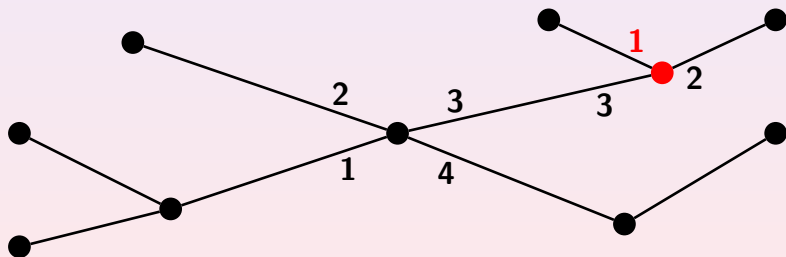


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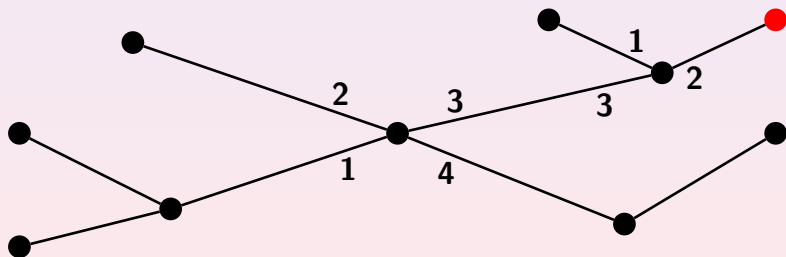


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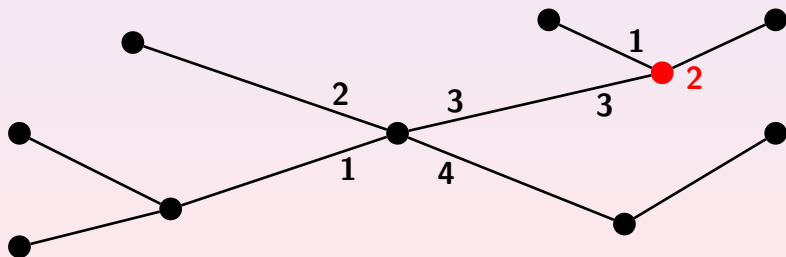


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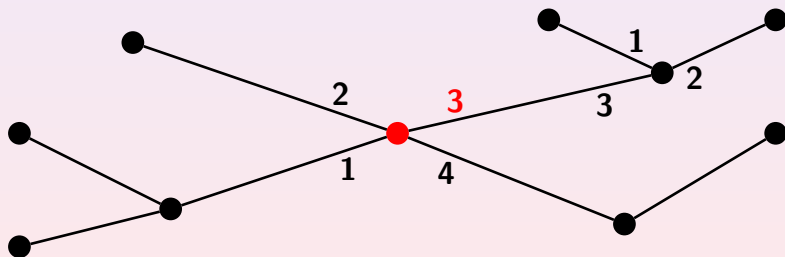


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Outline

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- 2 Related work
- 3 Our model and results**
 - Our model
 - Results
- 4 Coloring algorithms
- 5 Conclusion

Our model

Model

- An oracle colors (labels) the graph to help the automaton.
- The finite automaton can read the color of the node as an input of its transition function.

Goal

Use the smallest possible number of colors.

Our results

Theorem 1: Three colors

There exist a **finite automaton** and an algorithm coloring in **three colors** such that the automaton can explore **all graphs**.

Theorem 2: Two colors

There exist an automaton of $O(\log \Delta)$ memory bits and an algorithm coloring in only two colors such that the automaton can explore all graphs of maximum degree Δ .

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- 4 Coloring algorithms**
 - Spanning tree
 - Three colors
 - Two colors
- 5 Conclusion

Spanning tree

Basic idea

spanning tree: the label tells which edges are in the tree

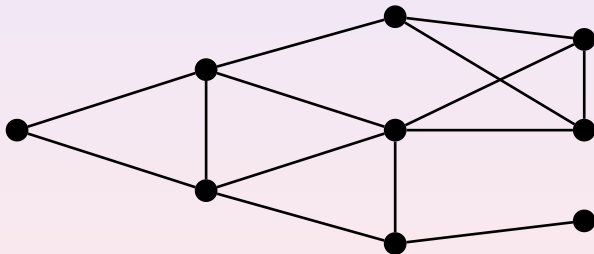


Use the tree exploration algorithm

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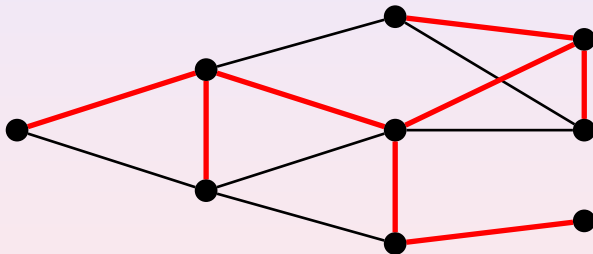


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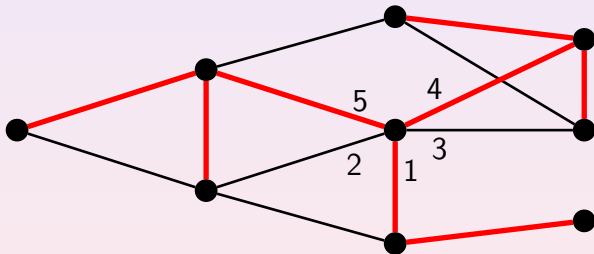


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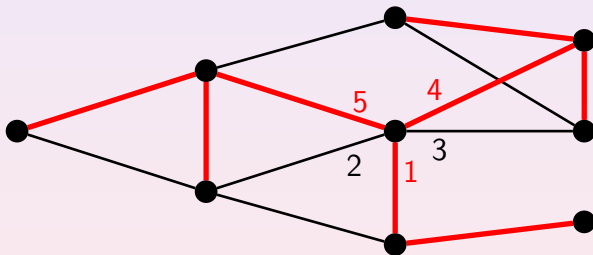


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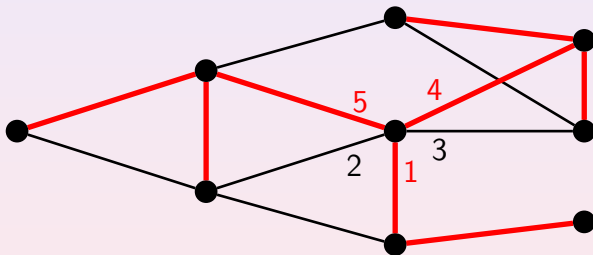


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Use the tree exploration algorithm

Rooted tree

Enhanced labeling

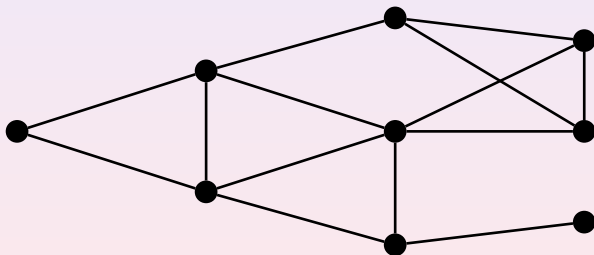
only indicate edge leading to the parent \rightarrow Δ colors



Rooted tree

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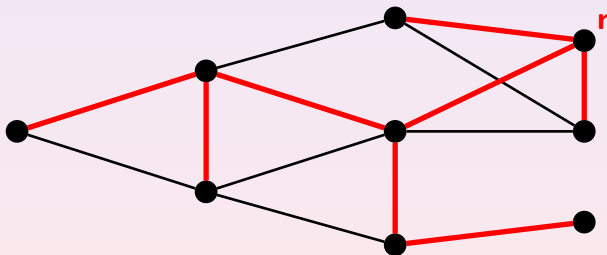
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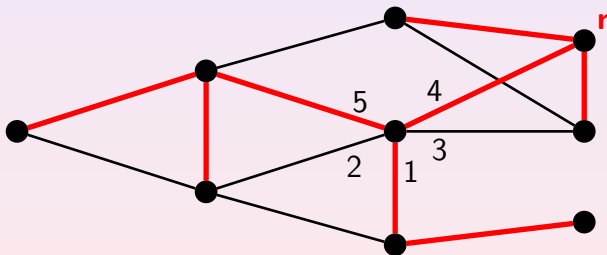
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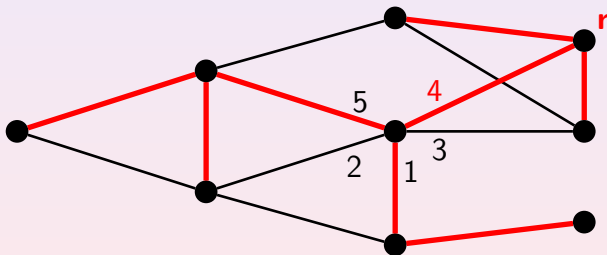
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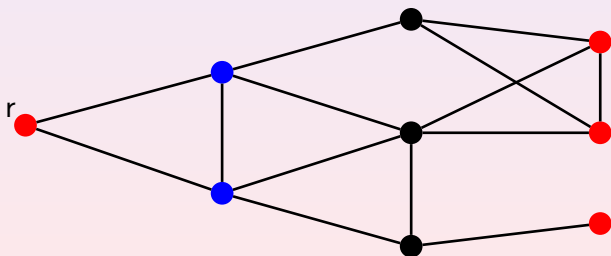
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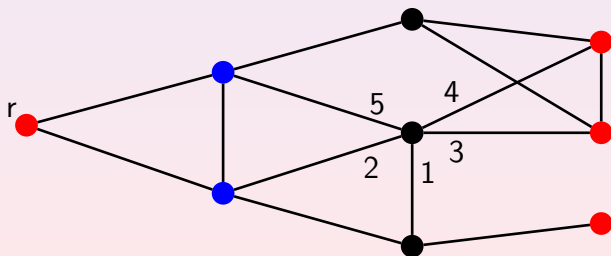
Three colors are enough

- choose arbitrarily a node as the root
- color all nodes according to their distance d to the root
 - distance $d \cong 0[n]$ red
 - distance $d \cong 1[n]$ blue
 - distance $d \cong 2[n]$ black



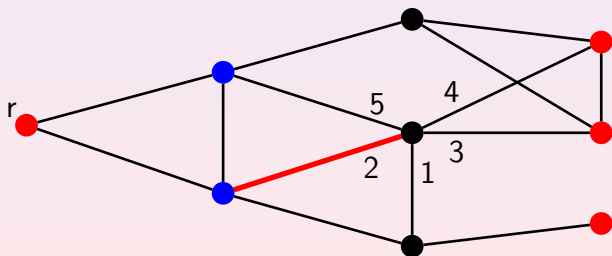
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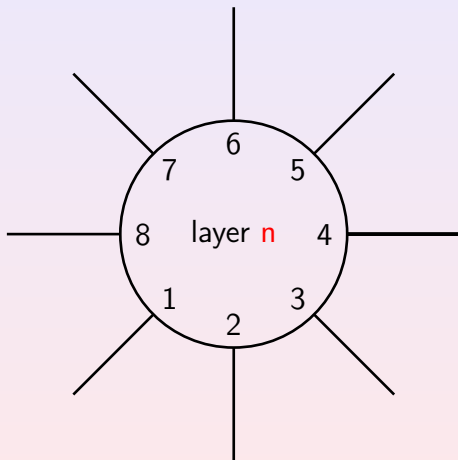


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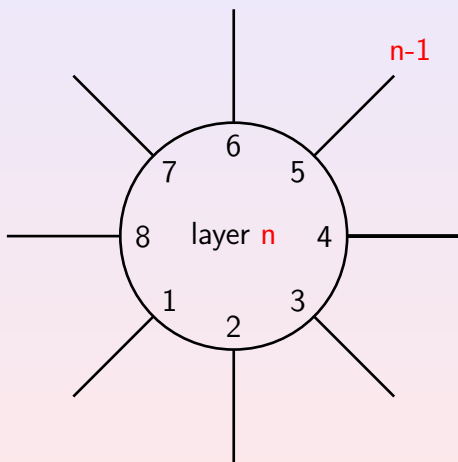
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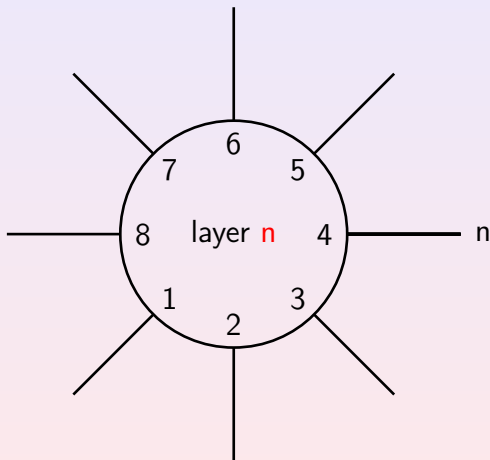
How to find the parent



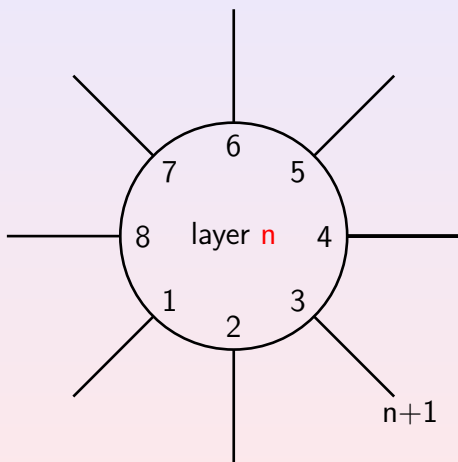
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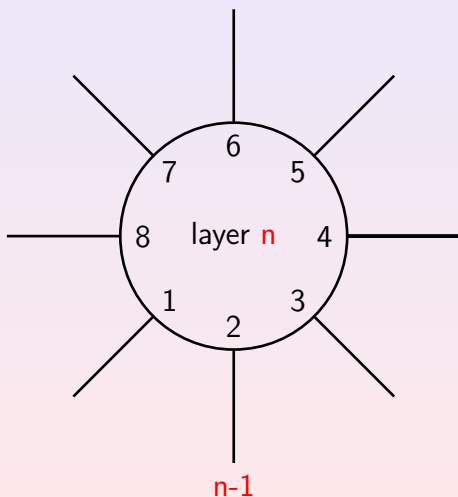
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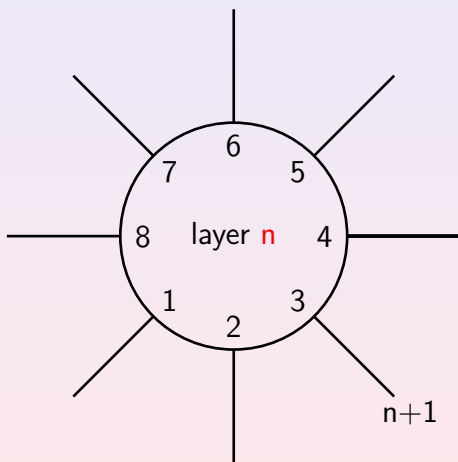
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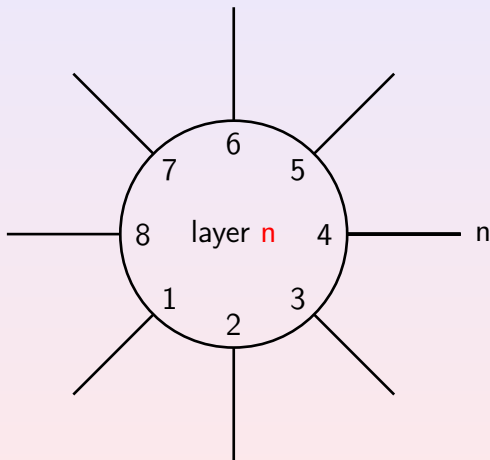
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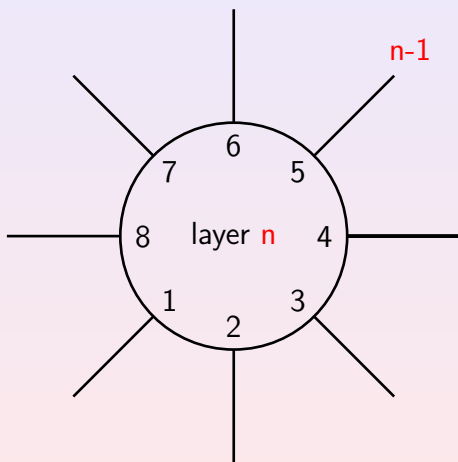
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How to find the parent



How to find the parent



Only two colors?

- Layer 1: red
- Layer 2: blue
- Layer 3: red
- Layer 4: red
- Layer 5: red
- Layer 6: blue
- Layer 7: blue
- Layer 8: blue

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Open problem

Conclusion

- **Three colors** are sufficient for **arbitrary graphs**.
- **Two colors** are necessary and sufficient for graphs of **constant degree**.

Open problem

Are two colors sufficient for arbitrary graphs?

Open problem

Conclusion

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Open problem

Are **two colors** sufficient for **arbitrary graphs**?