Initial knowledge and impossibility results in distributed computing

David Ilcinkas

CNRS & Université de Bordeaux (LaBRI)

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Graph algorithms

Centralized case

- 1 process/algorithm
- Complete knowledge of the instance/graph

Distributed case

- n processes (one per vertex) cooperating in a graph
- Partial (initial) knowledge of the instance/graph

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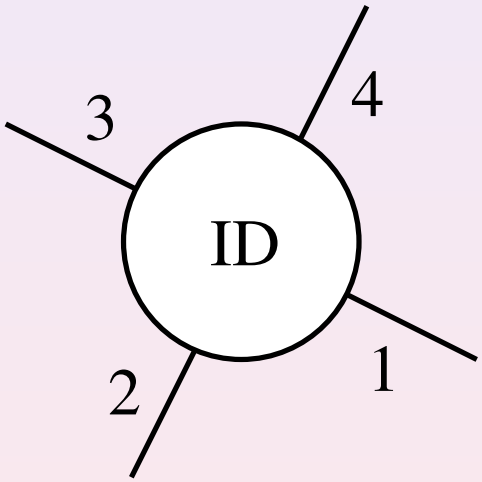
Types of knowledge

Local knowledge



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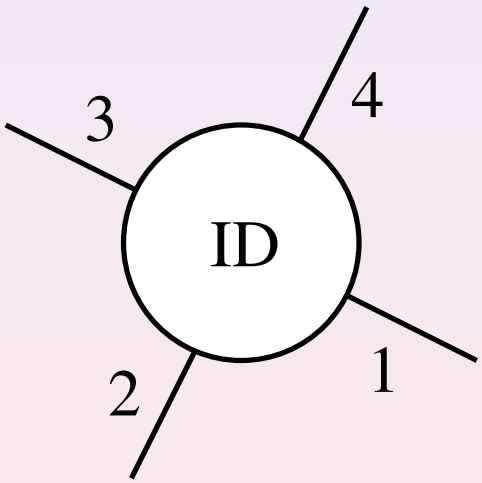

Global knowledge

- number n of vertices
- diameter
- maximum degree
- genus, girth, treewidth, etc.

Knowledge and performance

Observation

The existence/quality of the algorithmic solutions often depends on the availability of global knowledge about the network.

- [Lynch, PODC 1989]: A hundred impossibility proofs for distributed computing.
- [Fich, Ruppert, Distributed Computing, 2003]: Hundreds of impossibility results for distributed computing.

Some examples

Synchronous deterministic broadcast in n-node radio networks of diameter D

- Time $\Omega(n \log D)$ if no information
  [Clementi, Monti, Silvestri, TCS 2003]
- Time $O(D + \log^2 n)$ if complete knowledge
  [Kowalski, Pelc, Distributed Computing 2006]

Wakeup in arbitrary networks

[Awerbuch, Goldreich, Peleg, Vainish, J. of ACM, 1990]
Initial knowledge of the topology within radius ρ

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On the need of a priori knowledge

[A. Korman, J.-S. Sereni and L.Viennot. Toward more Localized Local Algorithms: Removing Assumptions concerning Global Knowledge. PODC'2011]

Result

A rather general method to transform an algorithm so that it does not need initial global knowledge anymore.

- the transformed algorithm has, up to a small constant, the same time complexity
- the method applies to a wide family of both deterministic and randomized algorithms; in particular most of those for MIS, Maximal Matching, coloring...

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Case study: graph exploration with termination

Graph exploration...

A mobile entity (robot/agent) must visit at least once each vertex of an anonymous graph.

...with termination

The mobile entity must stop after finite time, i.e. detect termination.

Anonymous :

- vertices with no IDs
- local labeling of the edges (port numbers)

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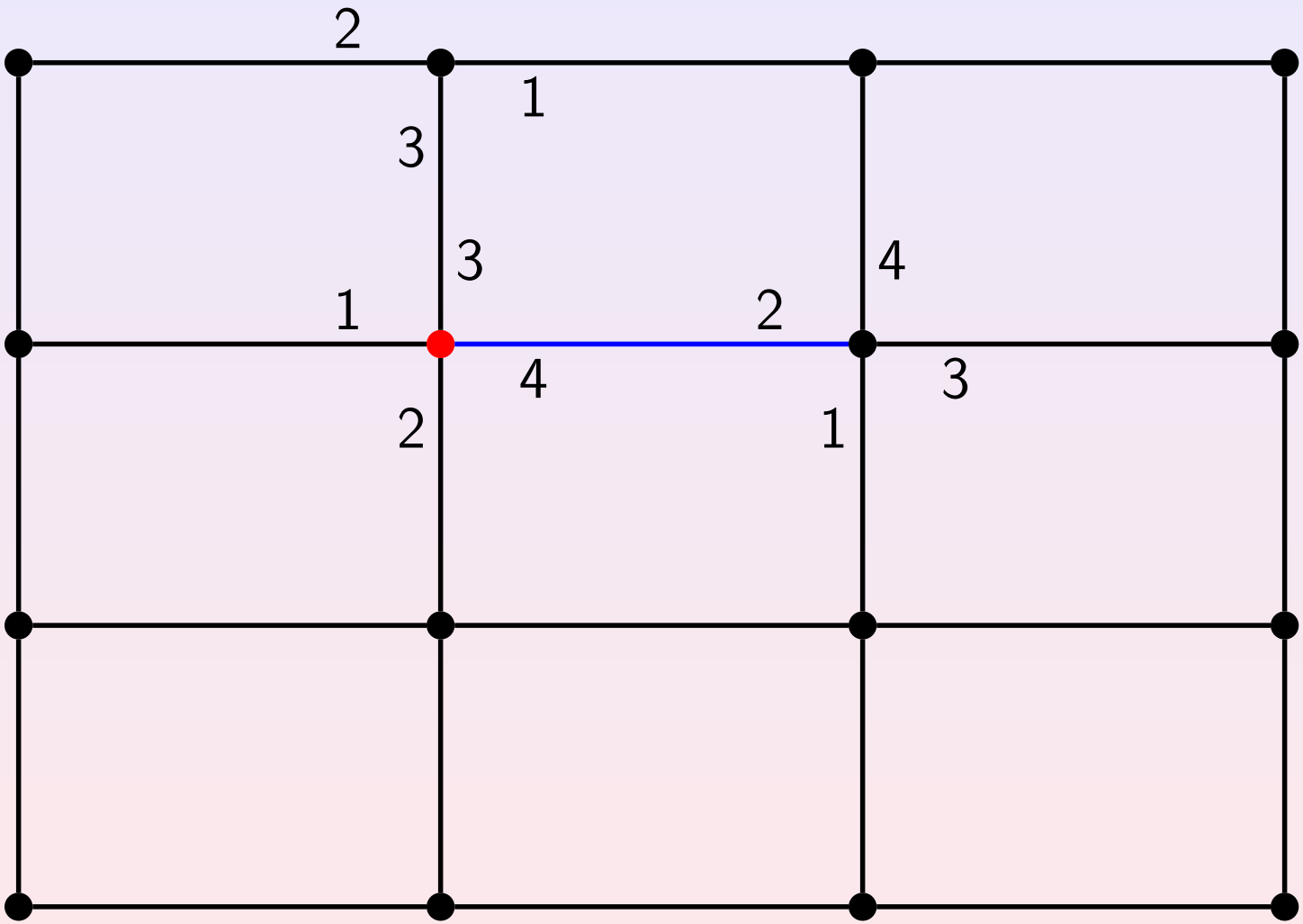
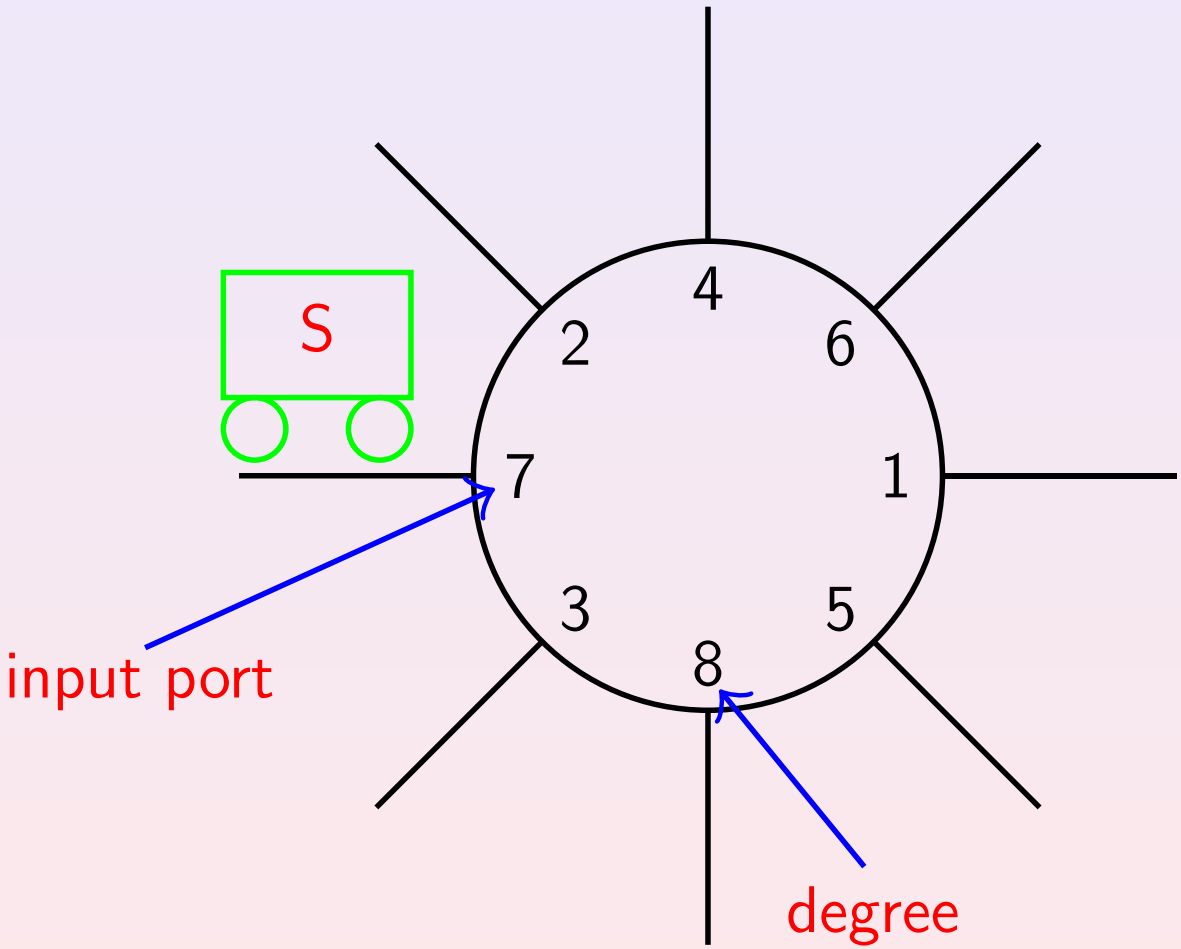
Example of an anonymous graph



Agent's model



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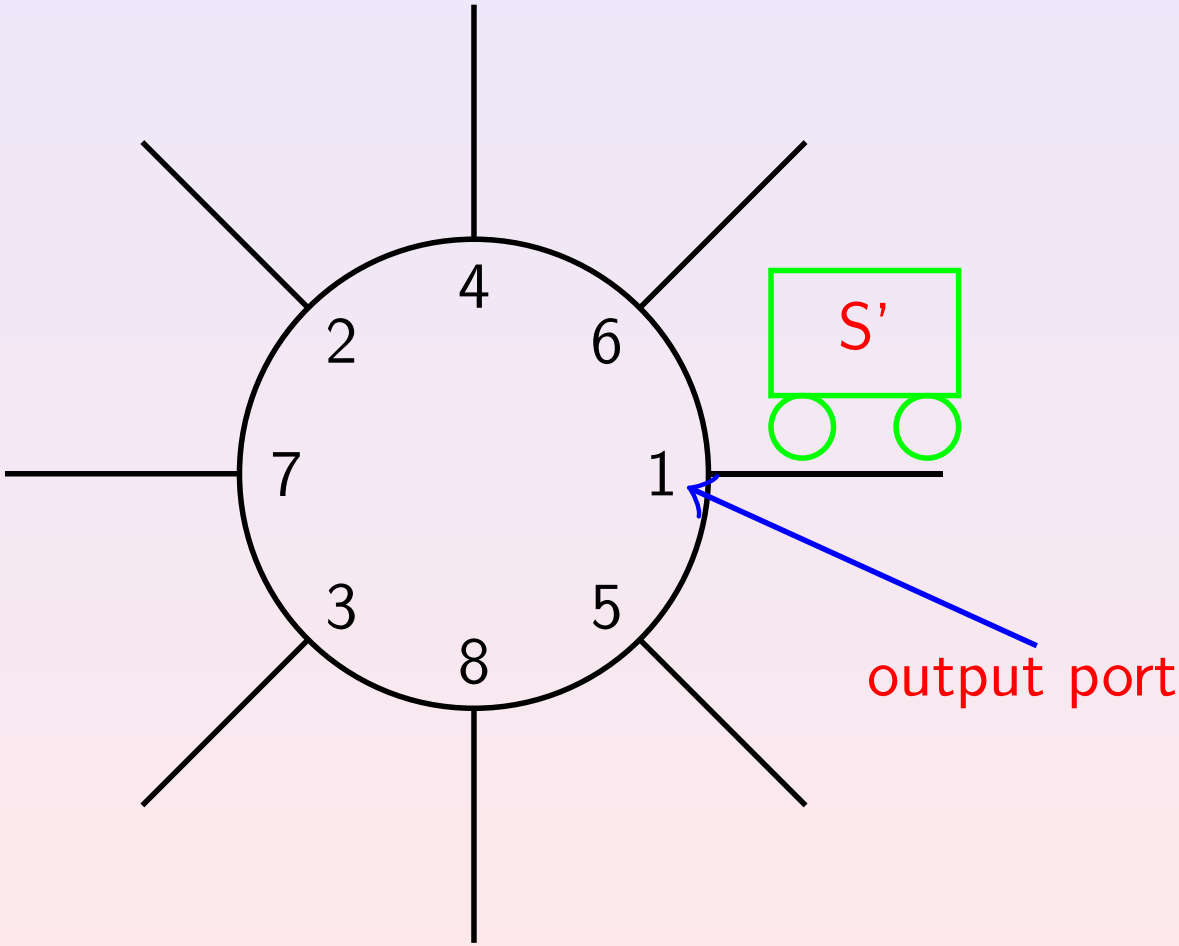

Positive result

Theorem

There exists an agent able to solve the graph exploration with termination problem if it knows an upper bound $\hat{n}$ on the number n of nodes of the to-be-explored graph.

Proof

Explore all paths of length $\hat{n}$ coming out of the starting vertex, and then stop.

Corollary

The knowledge of an upper bound on n is sufficient to solve graph exploration with termination.

Is this knowledge necessary?

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- Assume there exists an agent exploring without the knowledge of an upper bound on n.
- Agent on a triangle: termination at time k.
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Are you convinced?
Is it a satisfactory proof? No: parity of n.

Attempt to formalize

A priori knowledge / advice

- The agent has to explore graph G.
- At the beginning of the algorithm's execution, the agent is provided an advice (a priori knowledge) from an oracle $\mathcal{O}$ as a binary string $\mathcal{O}(G)$.

$\mathcal{O}$ induces a partition $(F_0, F_1, \cdots)$ of the graph family $\mathcal{G}$: $F_i = \{G \in \mathcal{G} | \mathcal{O}(G) = i\}$

To know an upper bound on n

$$\forall i \; \exists \hat{n}_i \; \forall G \in F_i \;\; n(G) \leq \hat{n}_i$$

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Negative result (bis)

Theorem

An agent without the knowledge of an upper bound on n cannot explore all cycles.

Reminder: to know upper bound: $\forall i\ \exists \hat{n}_i\ \forall G \in F_i \quad n(G) \leq \hat{n}_i$

Proof

- Assume there exists an agent exploring without the knowledge of an upper bound on n.
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End of story?

Set of all cycles $\subset$ set of all graphs

Corollary

The knowledge of an upper bound on n is also necessary to explore all graphs.

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WRONG!
It is not monotonic...
$\mathcal{O}(G) = 0$ if G is a path and $\mathcal{O}(G) = n(G)$ otherwise

Comparison between oracles

Idea: concentrate on the information that the agent cannot compute alone

Definition: $\mathcal{O} \succeq \mathcal{O}'$

There exists a mobile agent algorithm $\mathcal{A}$ such that $\mathcal{O} + \mathcal{A}$ can simulate the output of $\mathcal{O}'$.

[Chandra, Toueg, JACM 1996], [Godard, Métivier, ToCS 2003]

Theorem (new try)

For every $\mathcal{O}$ permitting to solve the exploration in all graphs, there exists $\mathcal{O}'$ giving an upper bound, such that $\mathcal{O} \succeq \mathcal{O}'$.

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$\mathcal{O}$ and the corresponding exploration algorithm $\mathcal{A}_{\mathcal{O}}$ give an upper bound on the size of G: the stopping time of $\mathcal{A}_{\mathcal{O}}$.

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Corollary

The equivalence class of every oracle giving an upper bound on n is minimal for the problem of exploring all graphs.

"Problem" 1: Can several minimal classes exist?

"Problem" 2

The equivalence class of every oracle giving an upper bound on n is minimal for the problem of exploring all paths.

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Back to the starting point

Theorem

The empty/neutral oracle is strictly weaker than every exploration oracle for the family of all graphs.

Proof

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Can't we say anything more?

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Every oracle inducing a finite number of parts F_i is strictly weaker than every exploration oracle for the family of all graphs.

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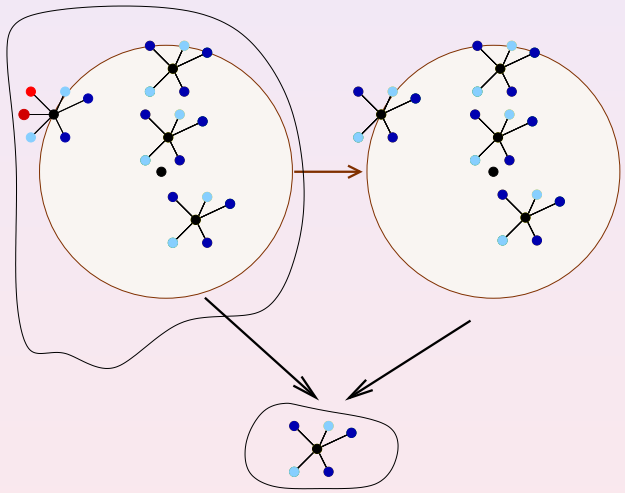
Precise characterisation

Definition

The graph H is a quasi-covering of radius r of the graph G if there exists a vertex u of H such that the ball in H centered at u and of radius r is indistinguishable from G.



See [J. Chalopin, E. Godard and Y. Métivier. Local Terminations and Distributed Computability in Anonymous Networks. Distributed Computing 2008]

Precise characterisation

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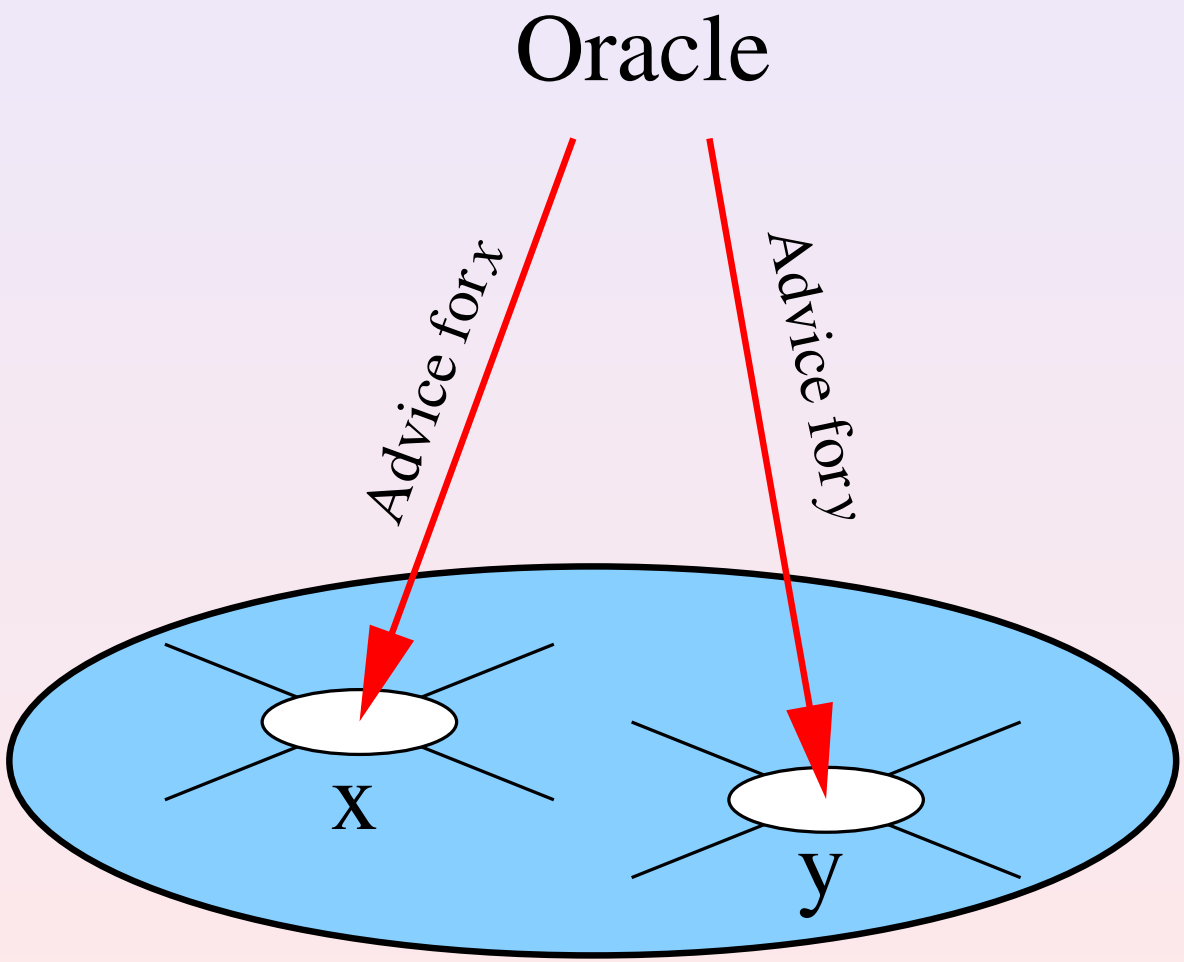
Characterisation

An oracle allows an agent to solve exploration in a family $\mathcal{F}$ if and only if there exists no part F_i of $\mathcal{F}$ containing quasi-coverings of unbounded radius of a common graph $G \in F_i$.

Global knowledge given by an oracle

General framework in distributed computing:



Distributed computing with advice

Design of two elements

the oracle $\mathcal{O}$

- takes the whole graph G as input
- returns advice $\mathcal{O}(G, x)$ to every node (or agent) x

the algorithm $\mathcal{A}$

- executed locally by every node (or agent)
- using the pieces of advice given by the oracle

Qualitative and quantitative approaches

Drawback of the qualitative approach

Difficult to compare

- Algorithm knowing n
- Algorithm knowing D
- Algorithm knowing the neighborhood

Quantitative approach

What is the minimum size of advice permitting to solve problem $\mathcal{P}$?

Quantitative questions about the required knowledge, regardless of what kind of knowledge is supplied.

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One possible application

Two fundamental tasks

Disseminating a message M from a source to all the nodes of a network

- Wakeup: a node cannot communicate before it has received the message M
- Broadcast: a node can communicate at any time

Problem

Achieving these tasks using a number of messages linear in n

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Results

[P. Fraigniaud, D. Ilcinkas, and A. Pelc. *Communication algorithms with advice*. JCSS 2010]

Wakeup

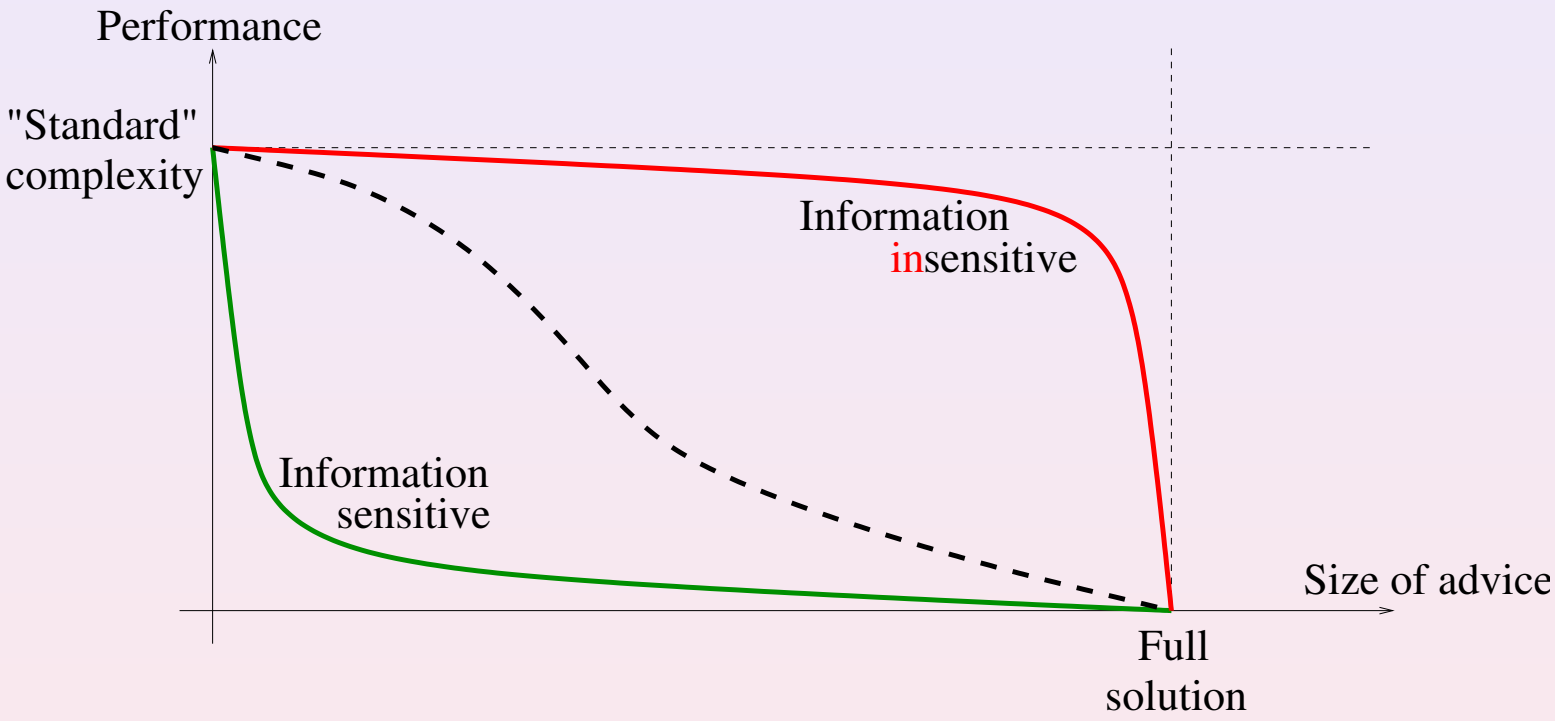
Minimum oracle size is $\Theta(n \log n)$ bits.

Broadcast

Minimum oracle size is $\Theta(n)$ bits.

Permits to clearly distinguish the two problems.

Another possible point of view



Two things to remember

- Proving that some initial knowledge is necessary must be done with extra care.
- The advice/oracle point of view can be insightful.

Thank you

for your attention