Label-Guided Graph Exploration by a Finite Automaton

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Graph exploration

Goal

A mobile entity has to traverse every edge of an unknown anonymous graph.

Motivations (1)

Exploration by mobile agents

- Physical robot: exploration of environments unreachable by humans
- Software agent: network maintenance

Equivalence between logic and automata (Engelfriet, Hoogeboom)

Through characterization of string, tree or graph languages

- Automata with nested pebbles
- First-order logic with transitive closure

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Motivations (2)

USTCON (undirected st-connectivity)

- $G = \{V, E\}$ an undirected graph
- $s, t \in V$ two vertices of G

Are s and t in the same connected component of G?

- L = class of problems solvable by deterministic log-space computations
- SL ($\supseteq$ L) = class of problems solvable by symmetric non-deterministic log-space computations

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Reingold, STOC 2005
Undirected ST-Connectivity in Log-Space

USTCON $\in$ L $\Rightarrow$ SL=L

Unknown, anonymous

Unknown

- Unknown topology
- Unknown size (no upper bound)

Anonymous

- No node labeling
- Local edge labeling

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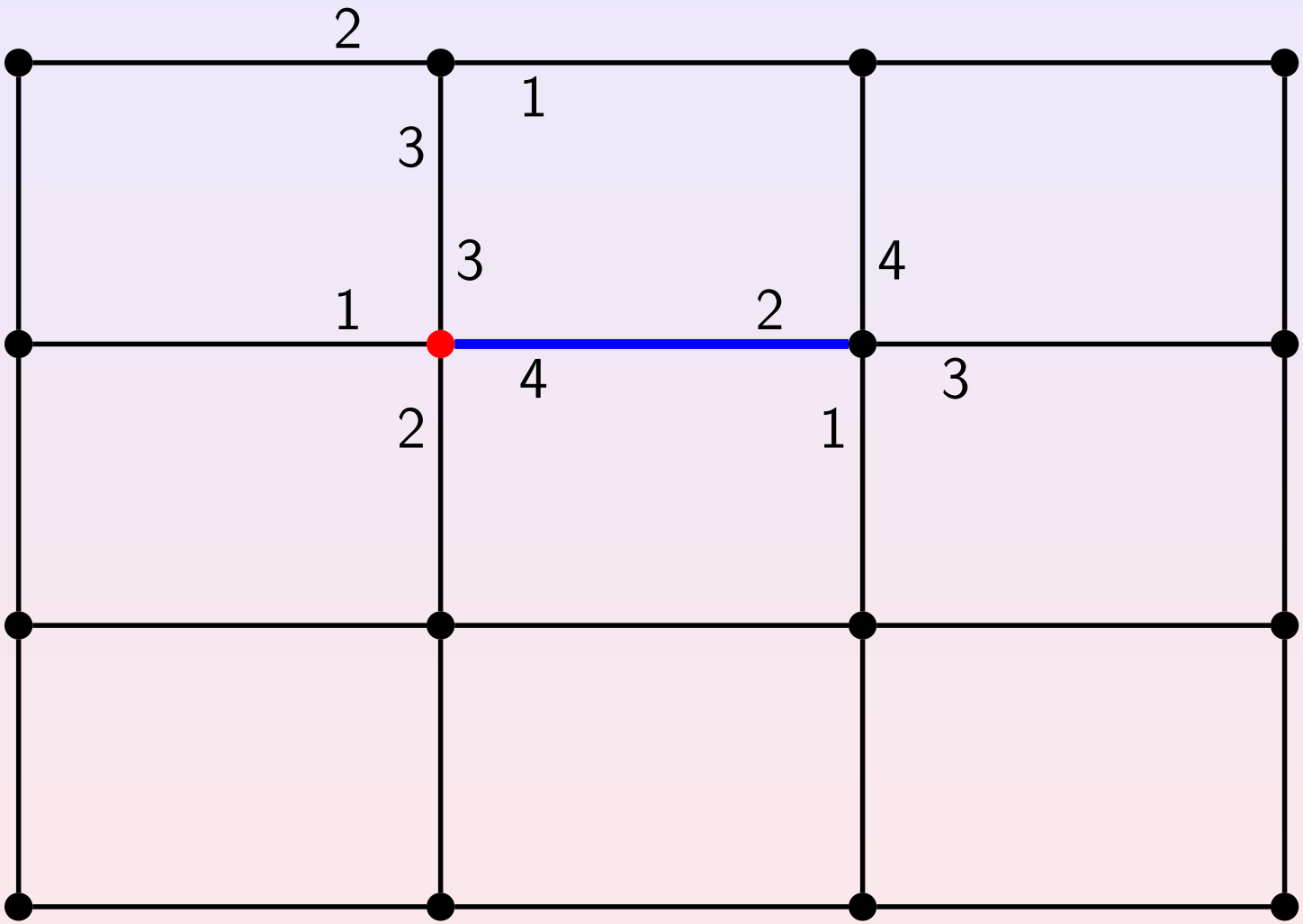
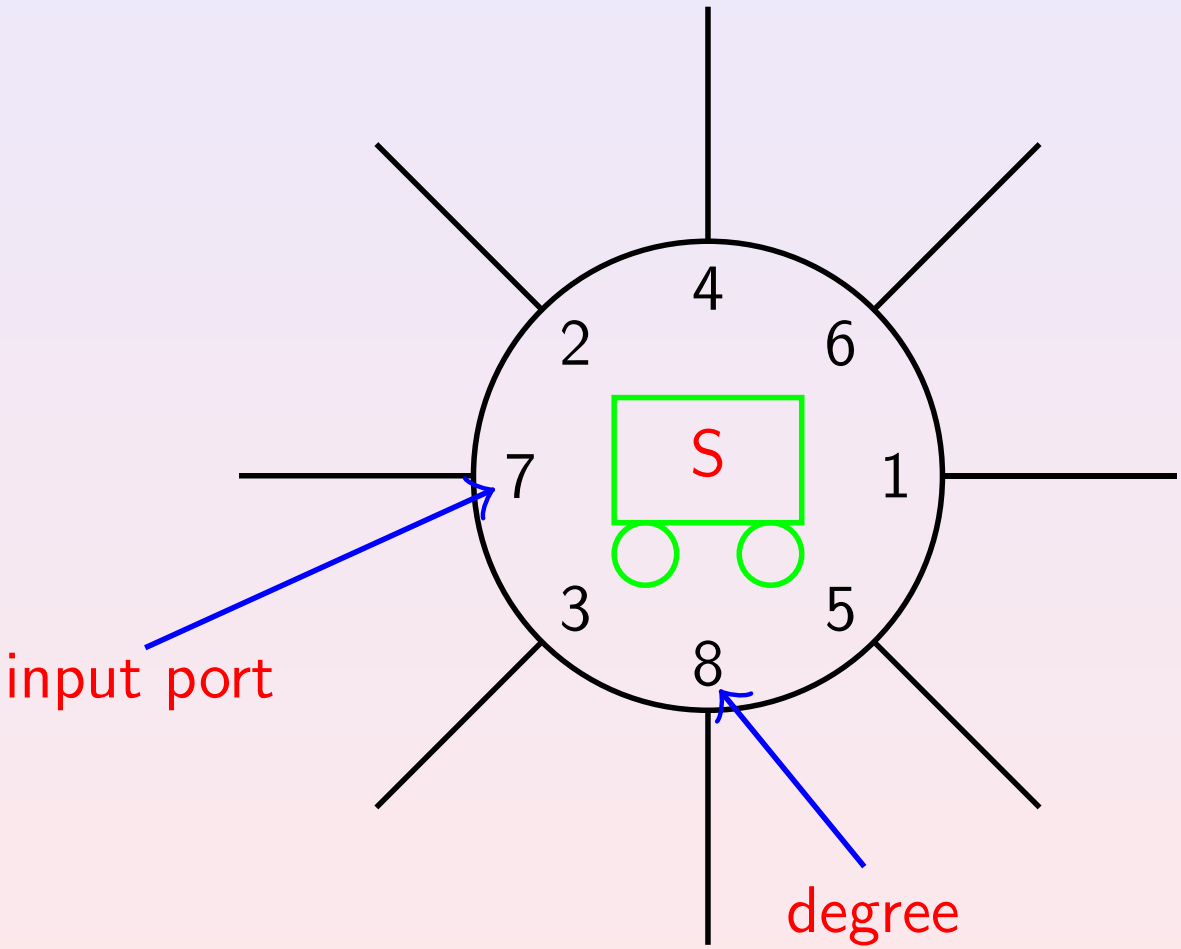
Example of an anonymous graph



Mealy automaton (1)



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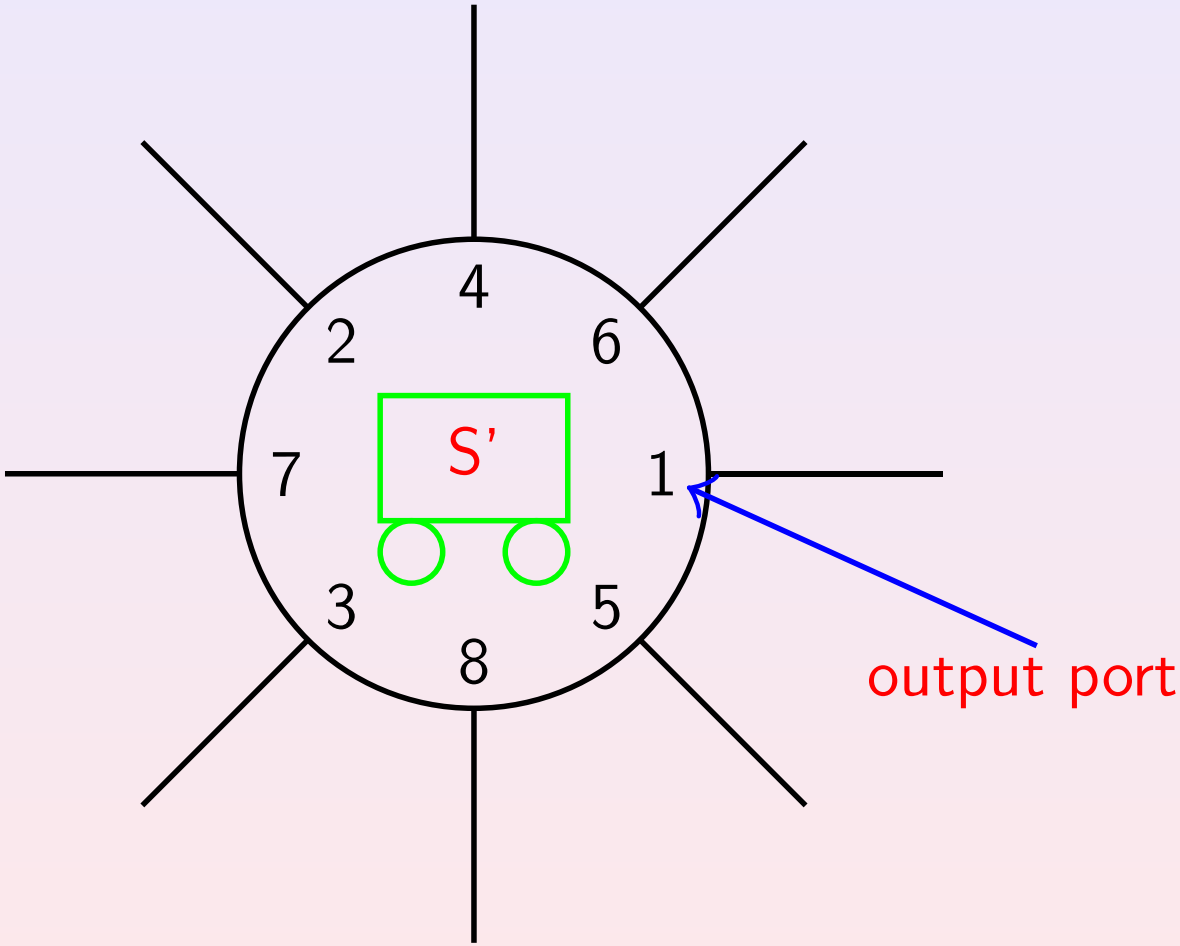


Mealy automaton (2)

Input

- S : current state
- i : input port number
- d : node's degree

Output

- S' : new state
- j : output port number

Transition function

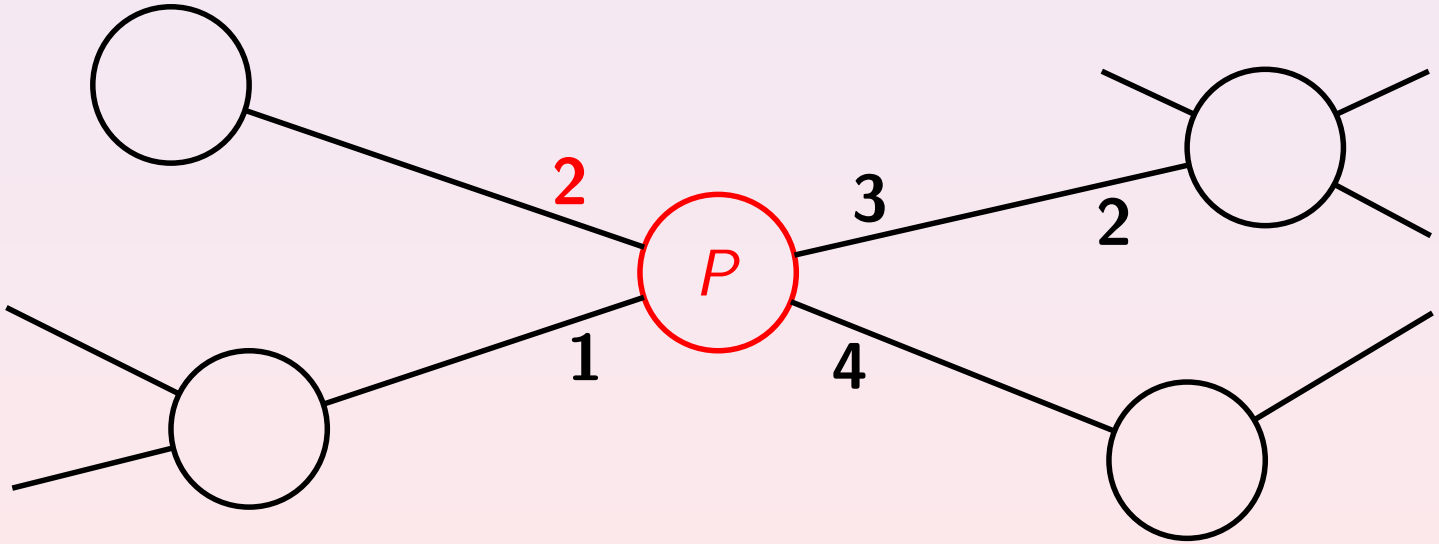
- $f(S, i, d) = (S', j)$

Exploration of the neighborhood

Star

The star can be explored by a 2-state automaton

- PlusOne P : $f(i, d, P) = (i + 1 \mod d, B)$
- Backtrack B : $f(i, d, B) = (i, P)$



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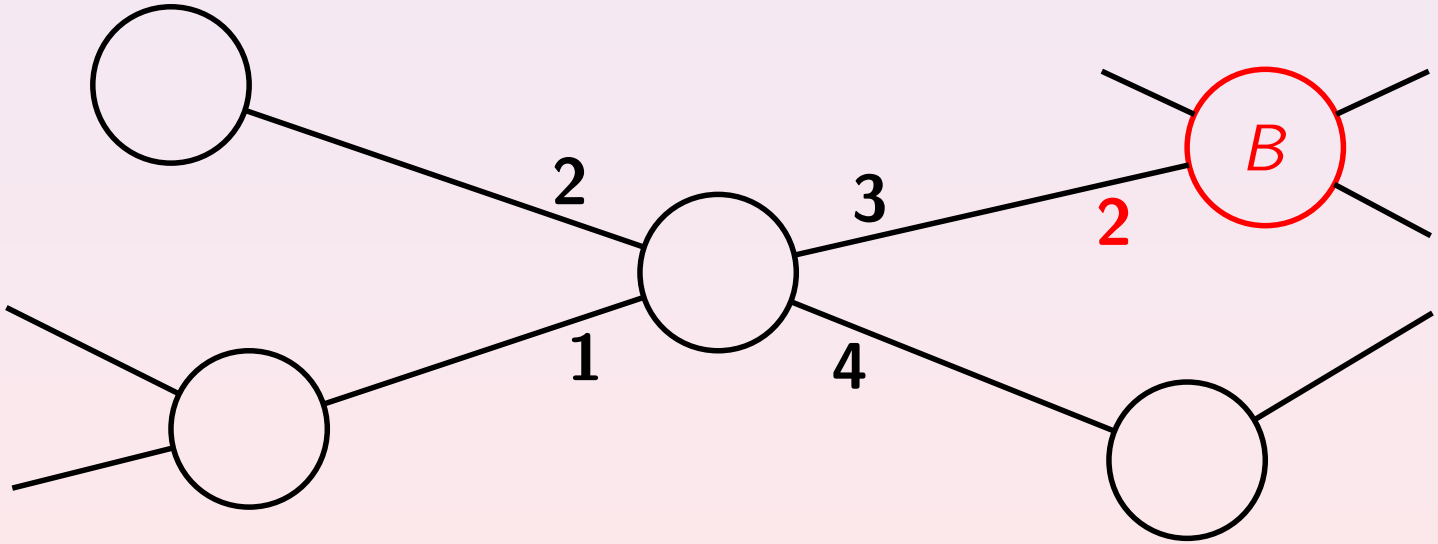

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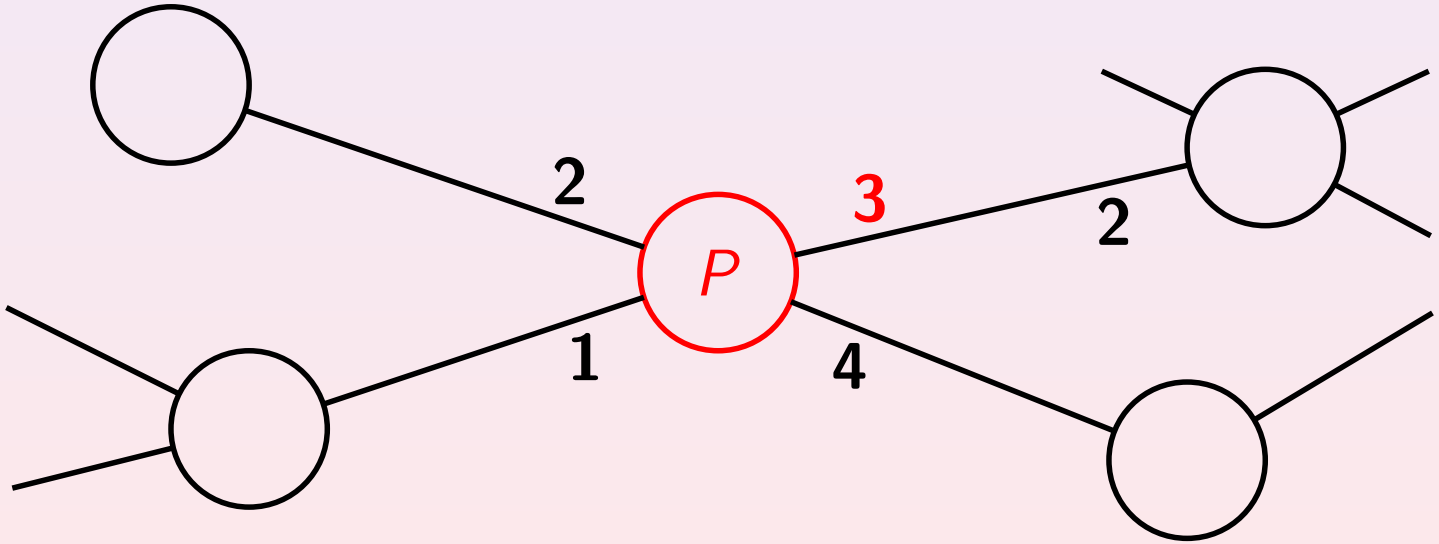

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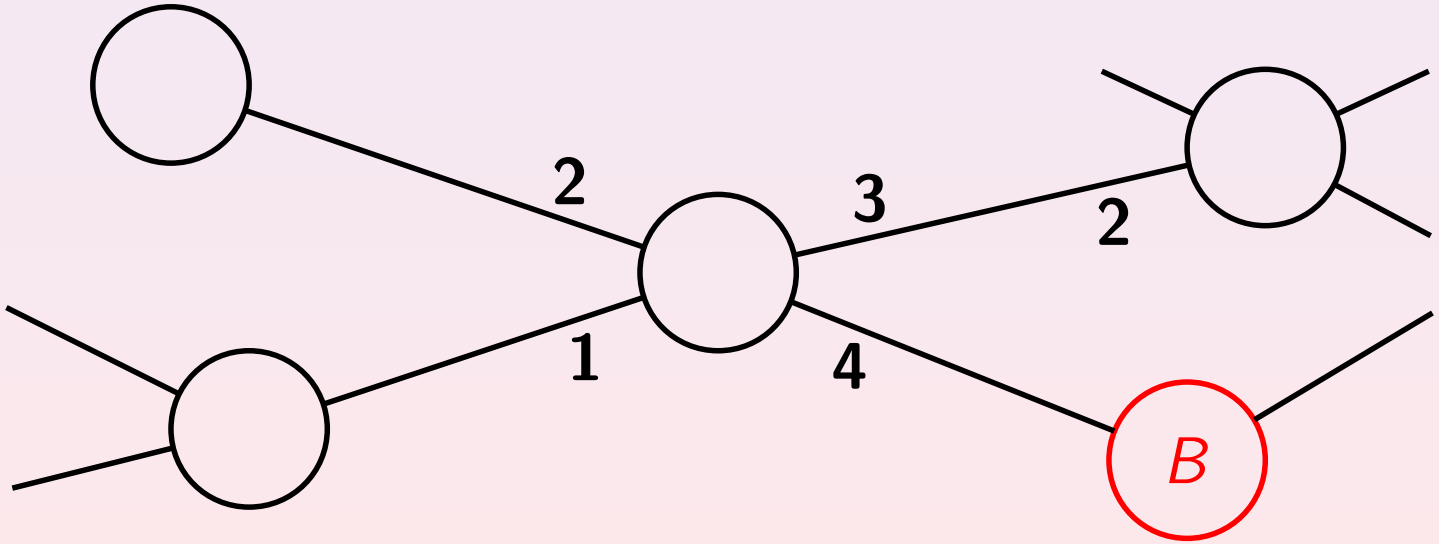

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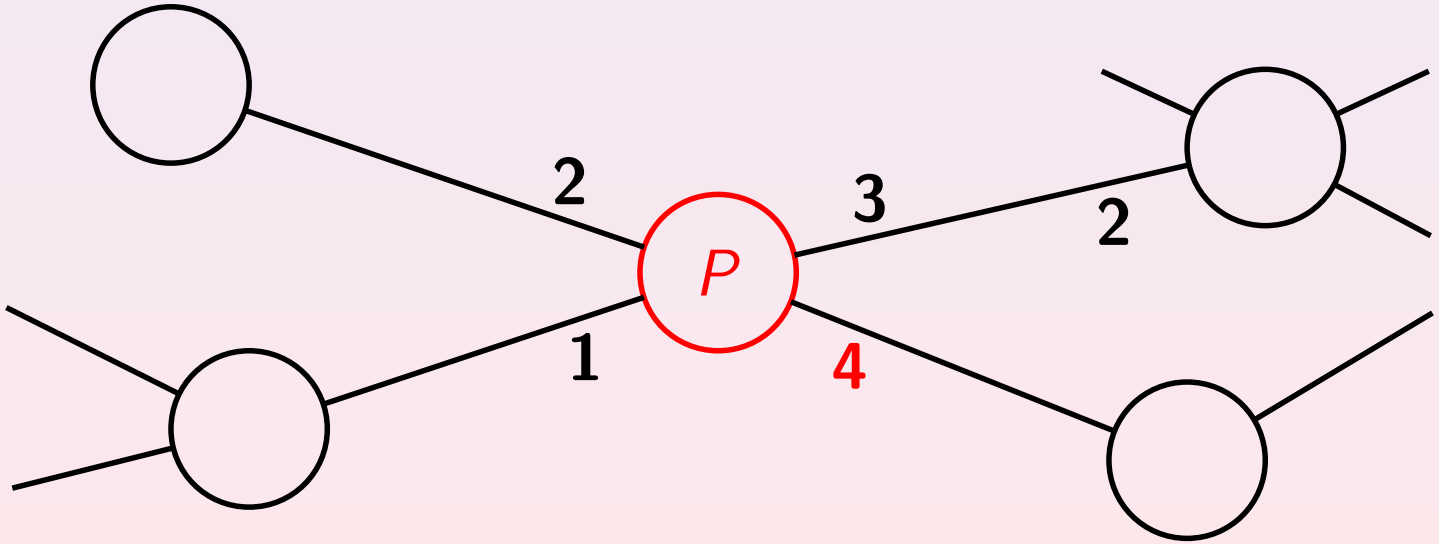

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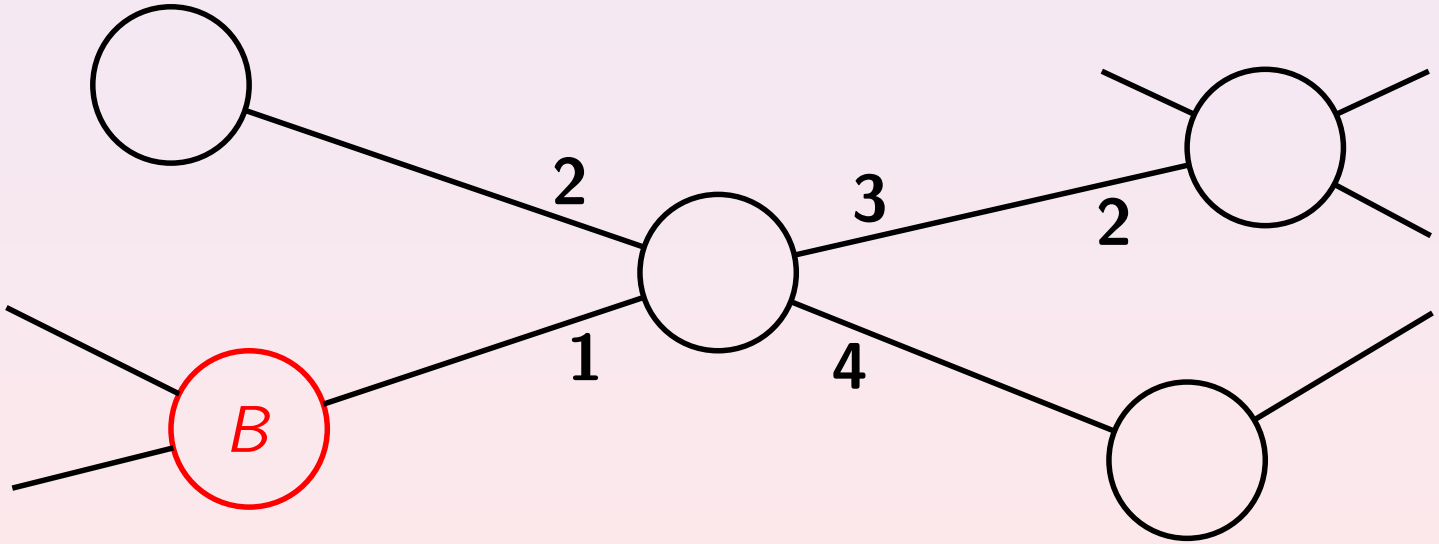

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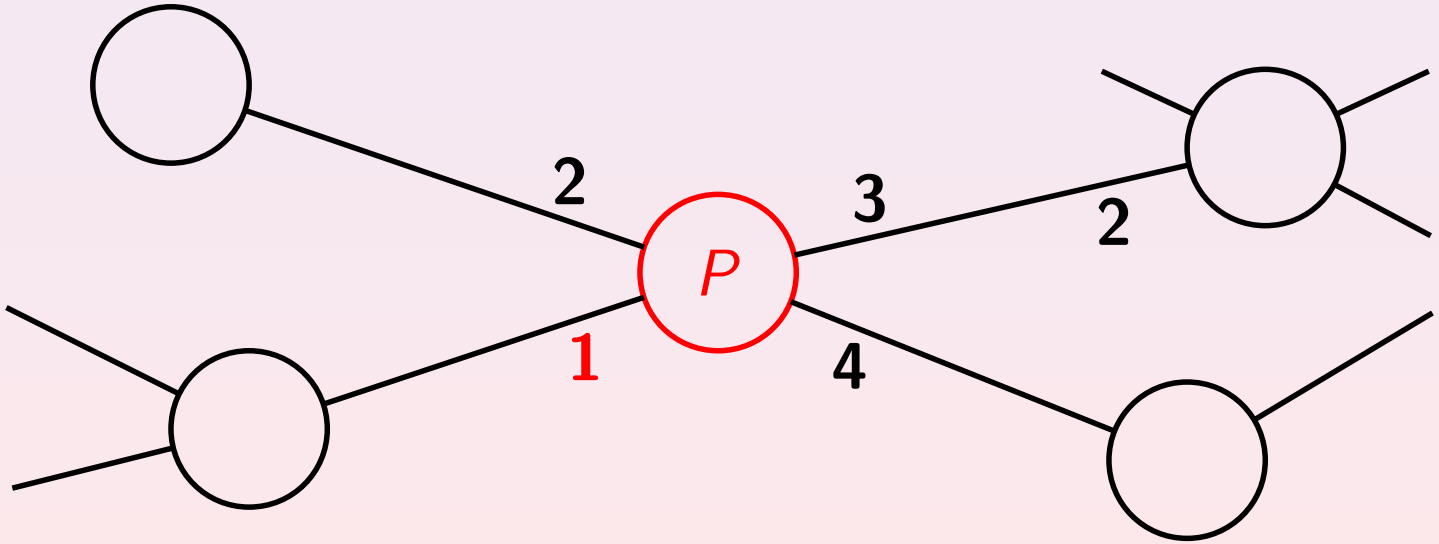

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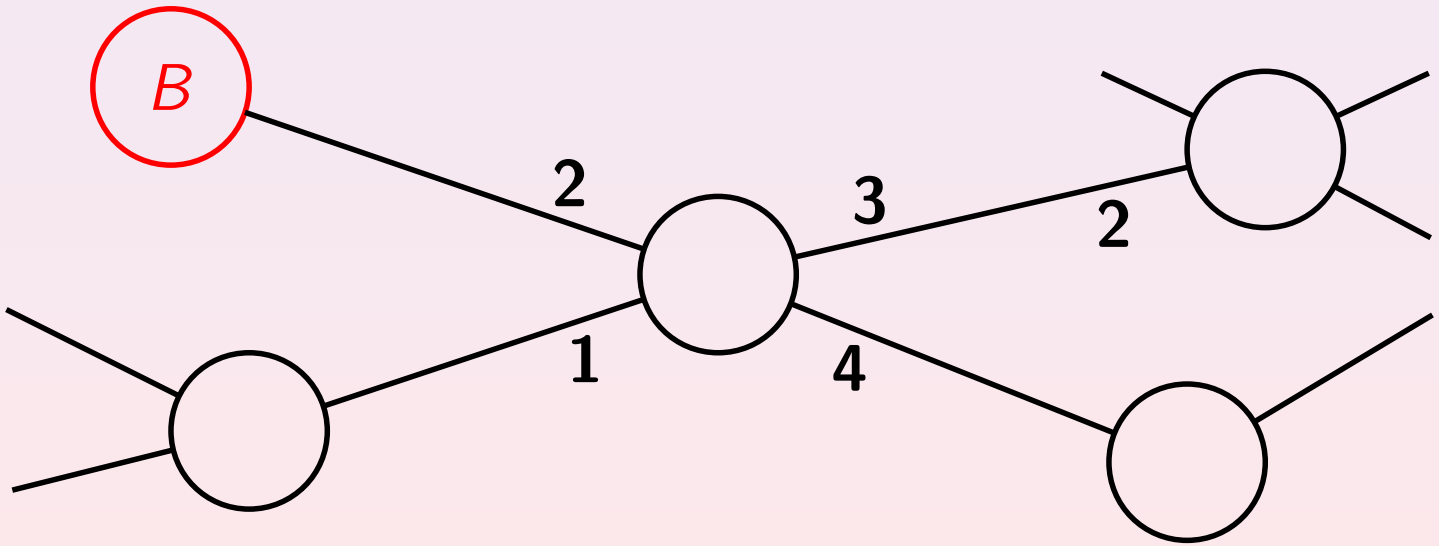

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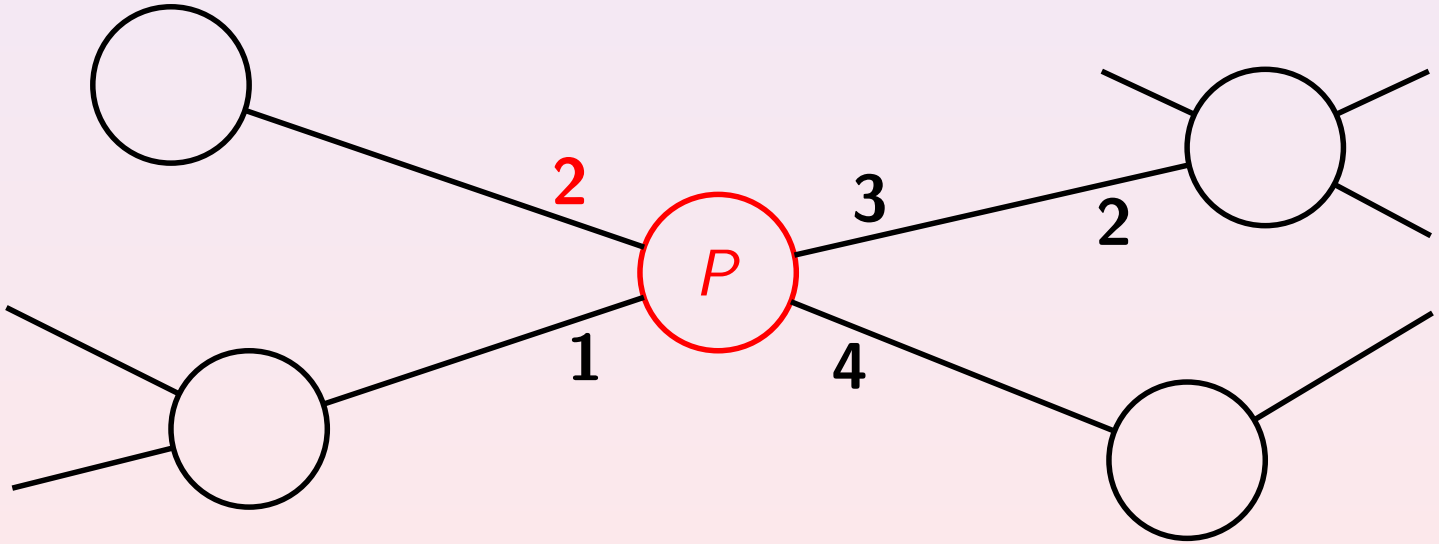

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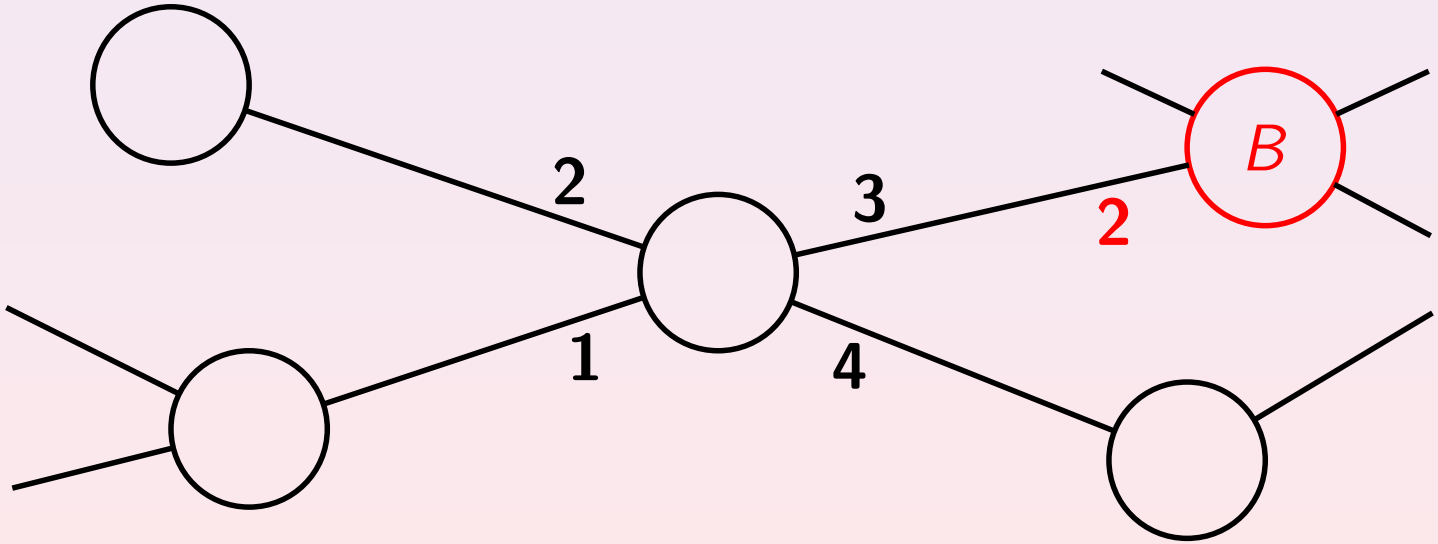

Outline

Impossibility results (1)

Budach, Math. Nachrichten, 1978
Automata and Labyrinths

No finite automaton can explore all graphs.

A pebble is a node-marker that can be dropped at and removed from nodes.

Rabin, Seminar talk at Berkeley, 1967
Maze threading automata

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Automaten in planaren Graphen

No finite team of finite cooperative automata can explore all (cubic planar) graphs.

A JAG (Jumping Automaton for Graphs) is a team of finite automata that cooperate constantly. Moreover an automaton can jump to a vertex occupied by another automaton.

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Universality for trees

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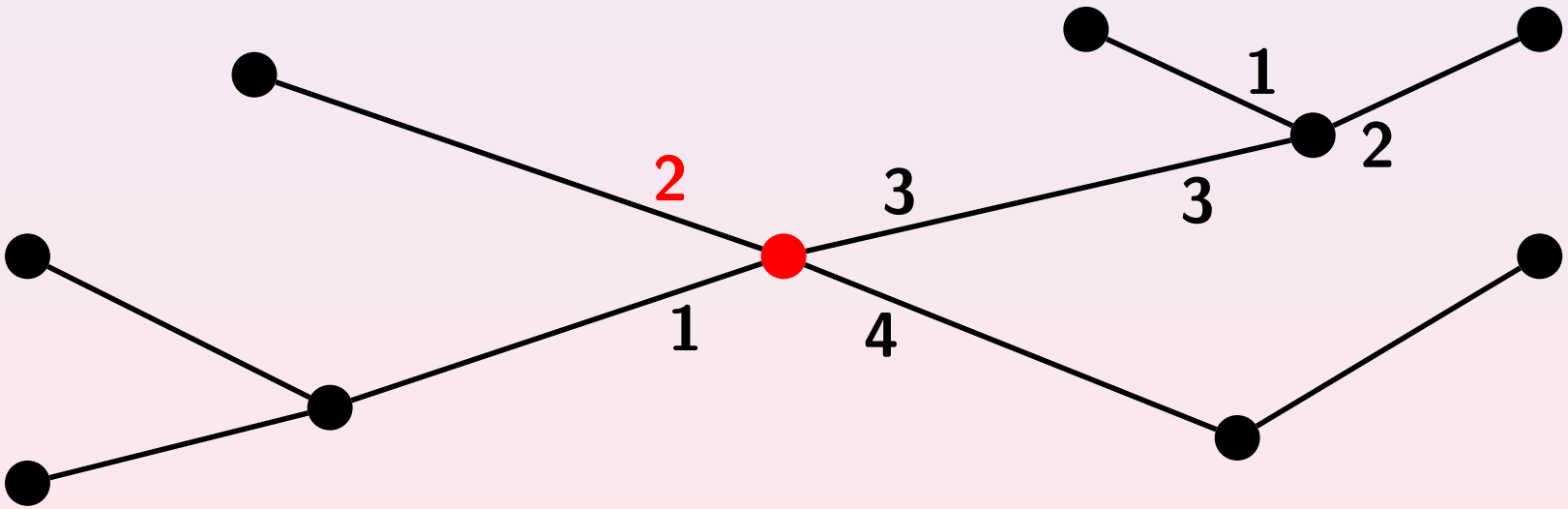
Tree exploration with little memory

An oblivious automaton (one single state) can explore all trees.



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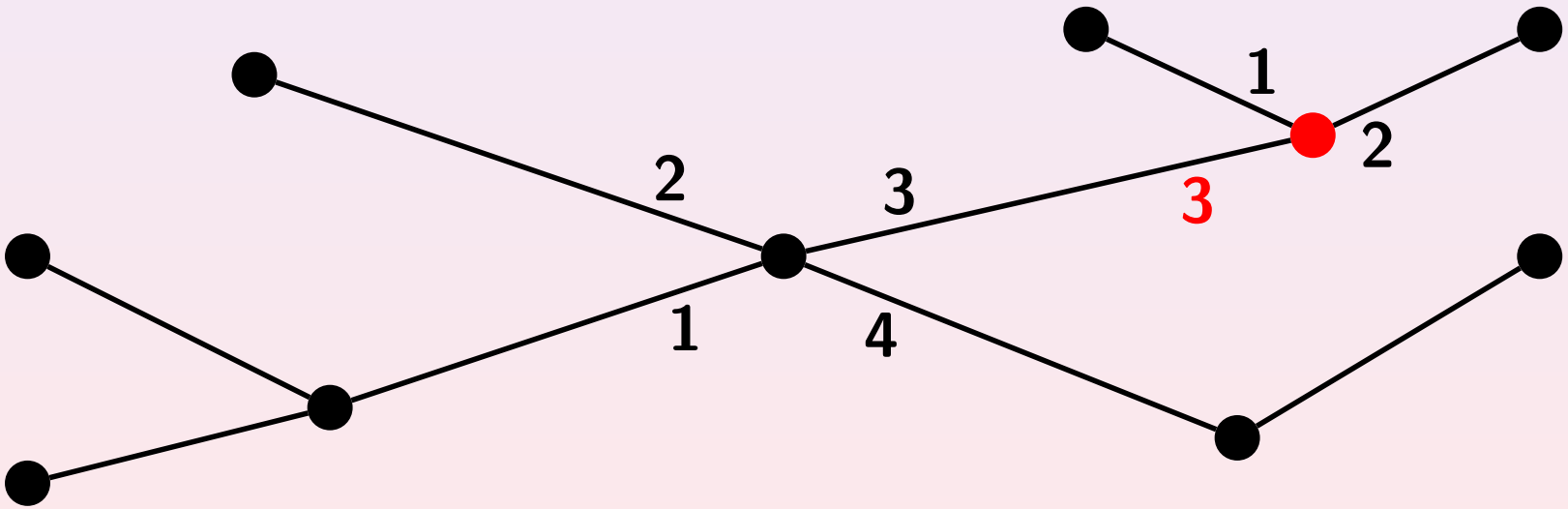

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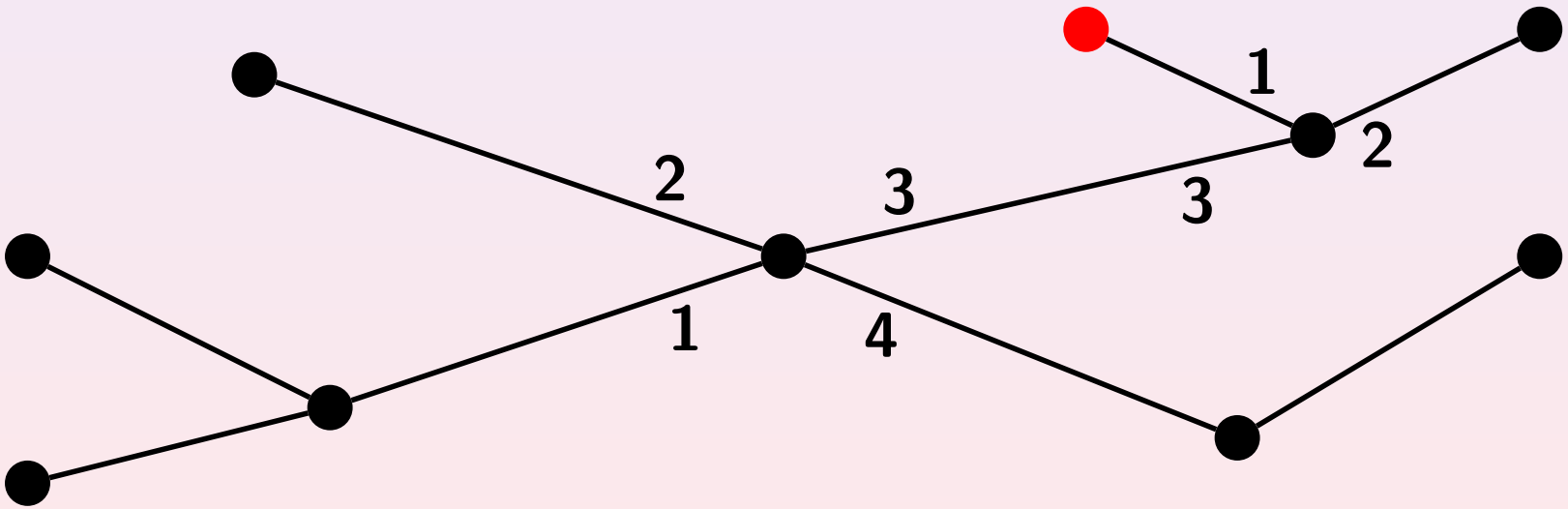

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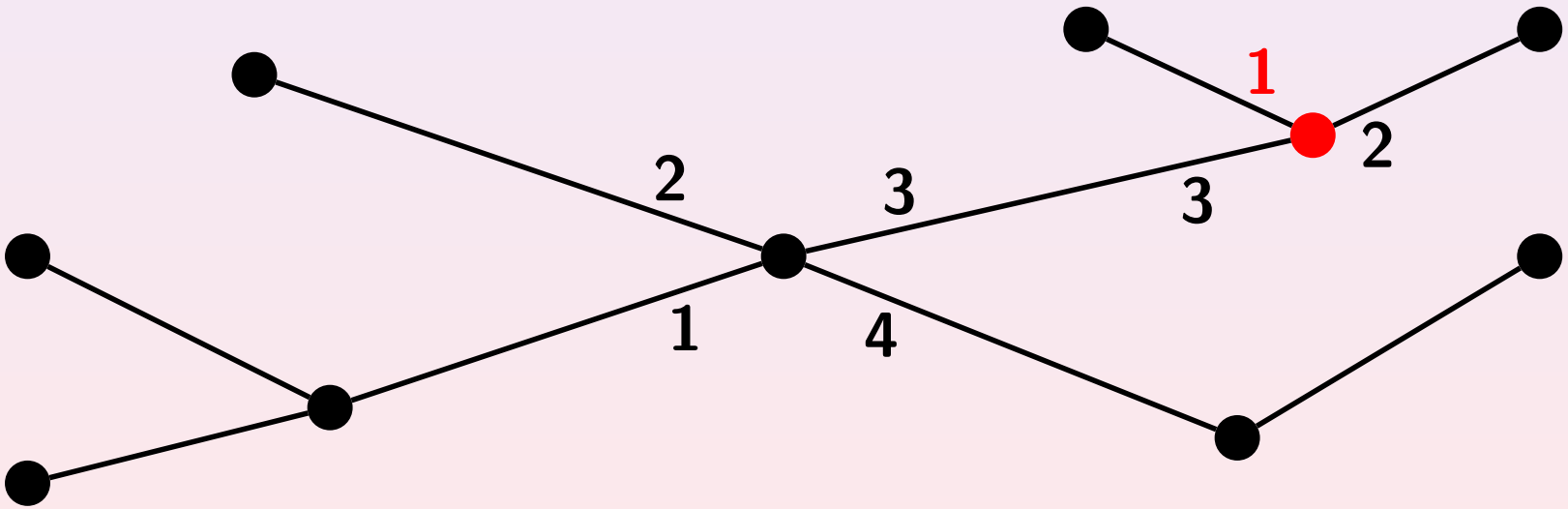

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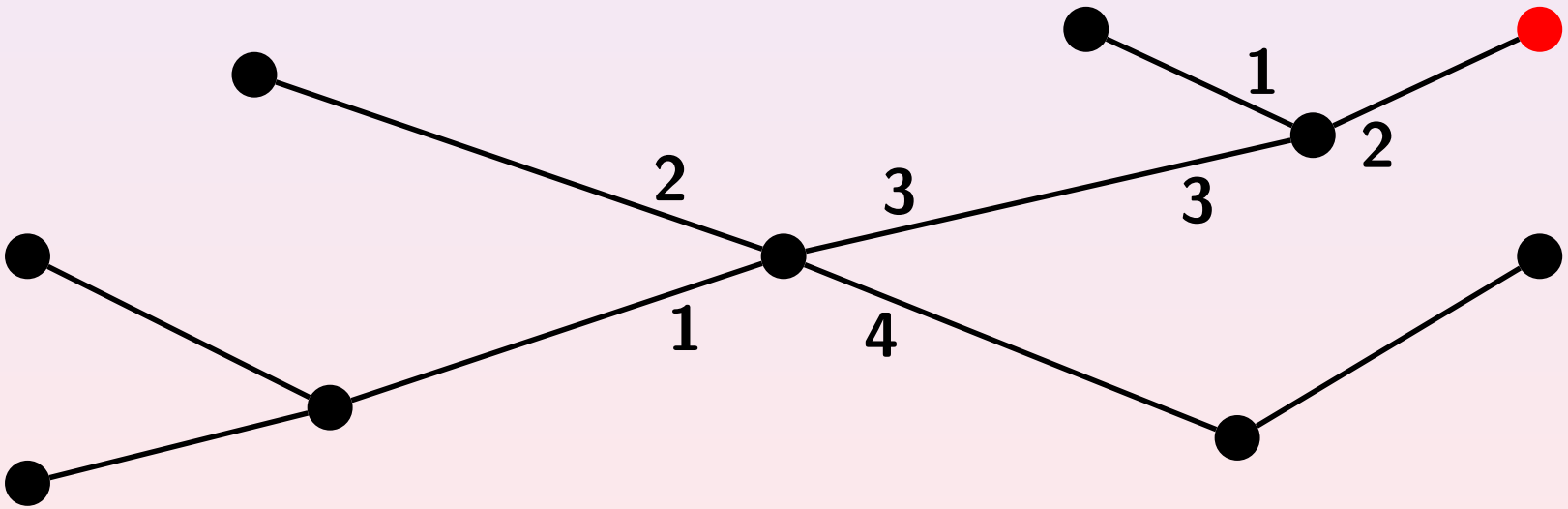

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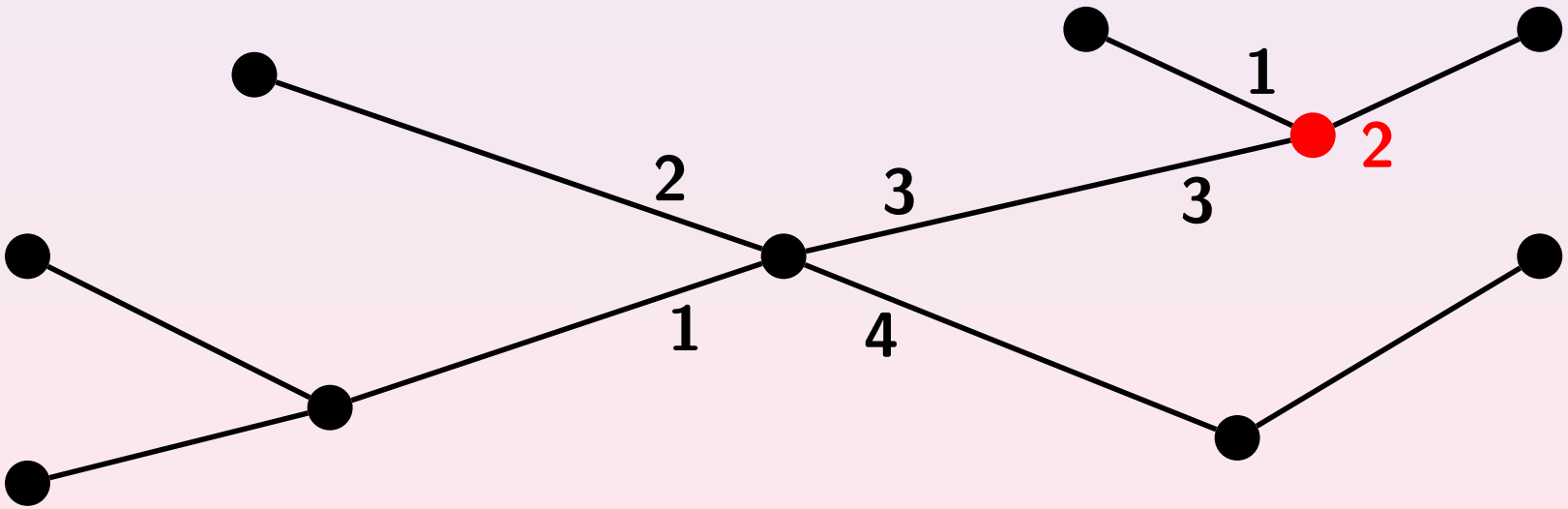

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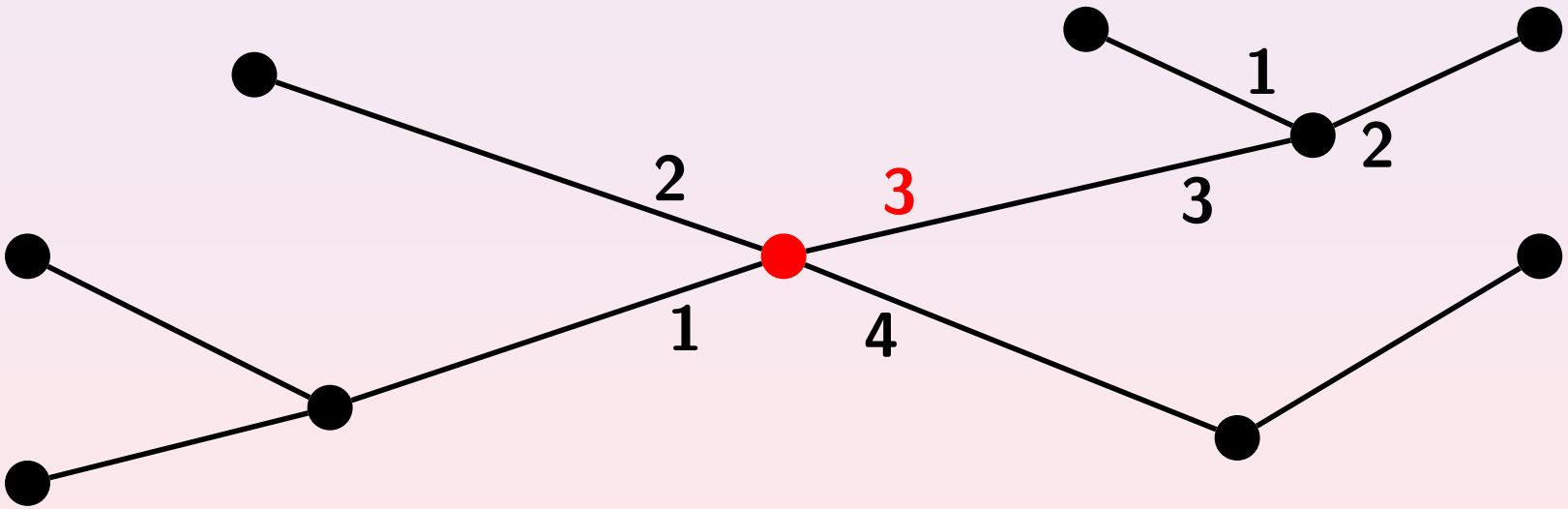

Whiteboards

A whiteboard is a local storage where an automaton can write, read and erase information.

Fraigniaud, WSS 2001
On a Space-optimal Distributed Traversal Algorithm

An oblivious automaton (one single state) can explore all graphs of maximum degree Δ using only $O(log\,\Delta)$ memory bits per node.

Algorithm: When visiting a node for the ith times, take the edge $i \mod d$ where d is the degree of the node.

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Outline

Our model

Model

- An oracle colors (labels) the graph to help the automaton.
- The finite automaton can read the color of the node as an input of its transition function.

Goal

Use the smallest possible number of colors.

Our results

Theorem 1: Three colors

There exist a finite automaton and an algorithm coloring in three colors such that the automaton can explore all graphs.

Theorem 2: Two colors

There exist a automaton of $O(log\Delta)$ memory bits and an algorithm coloring in only two colors such that the automaton can explore all graphs of maximum degree Δ.

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Basic idea

spanning tree: the label tells which edges are in the tree

Use the tree exploration algorithm

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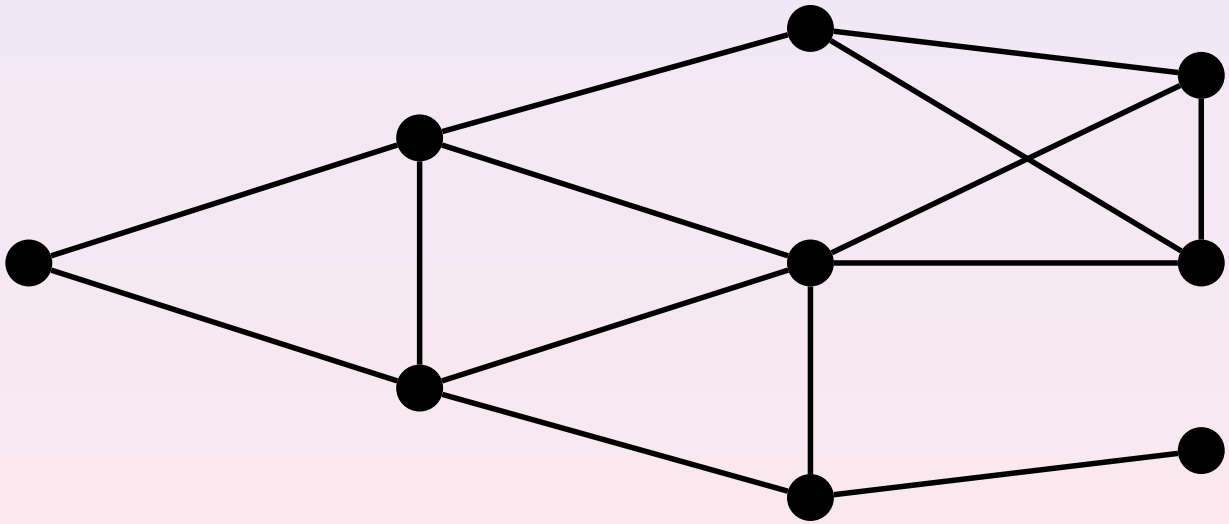

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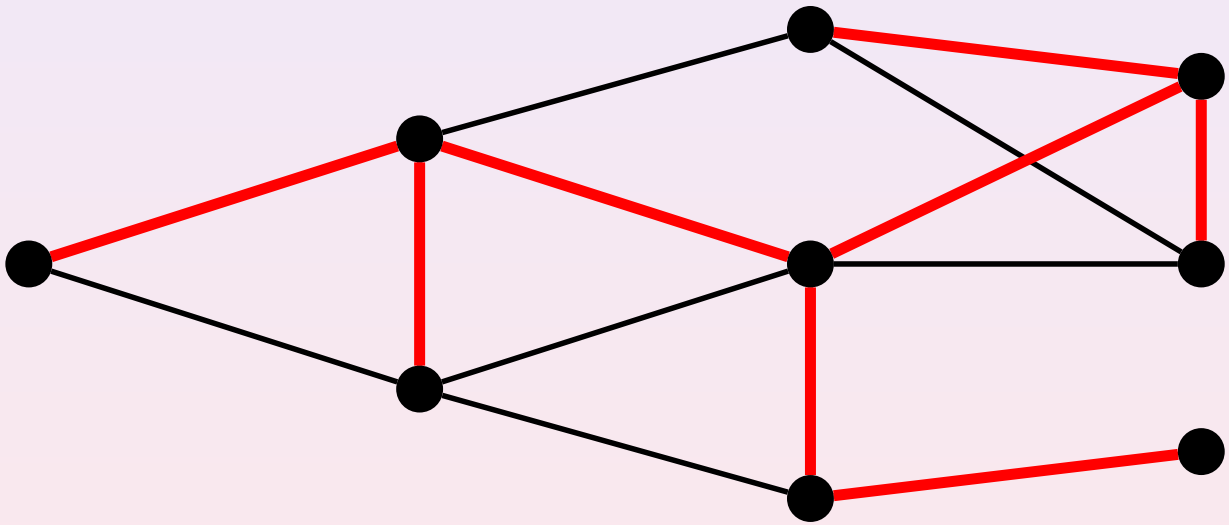

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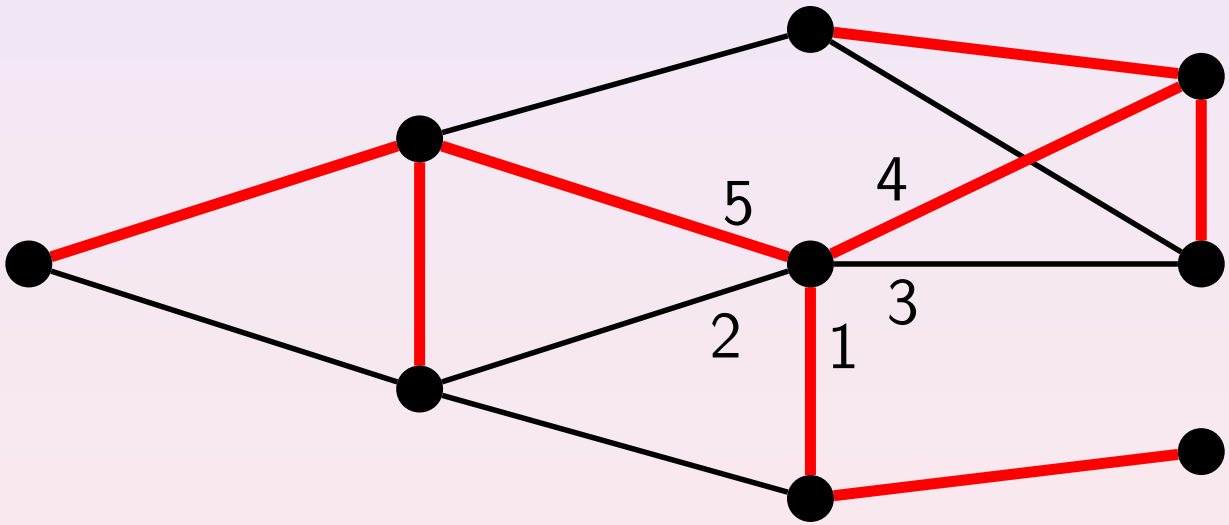

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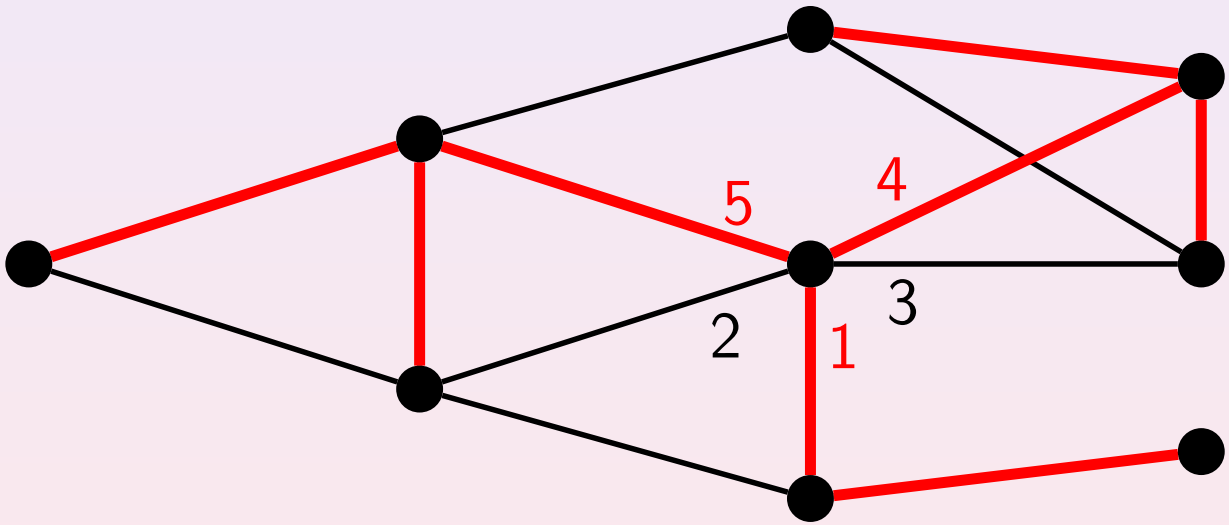

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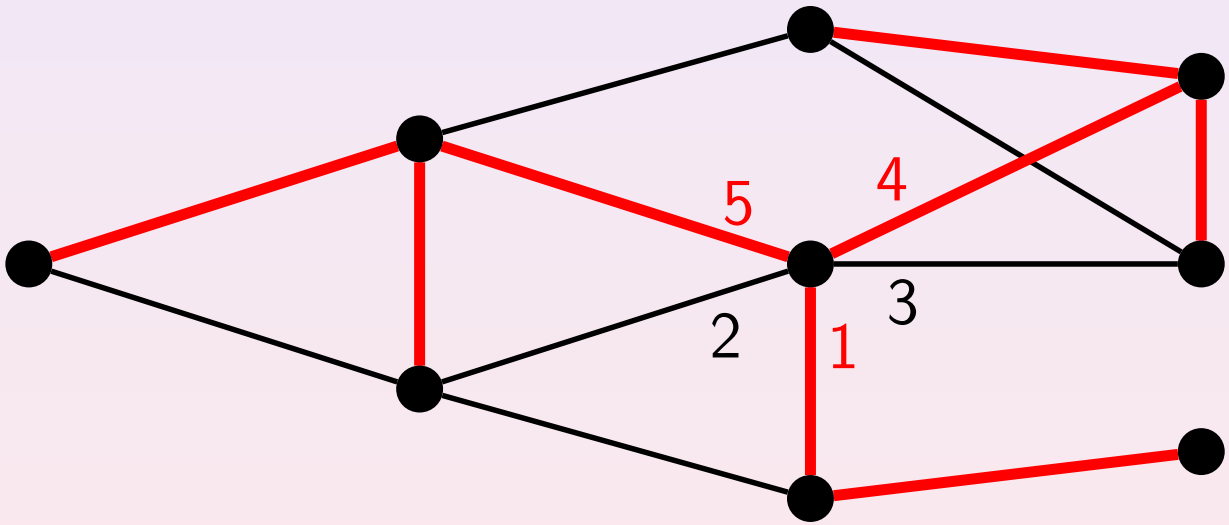

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Rooted tree

Enhanced labeling

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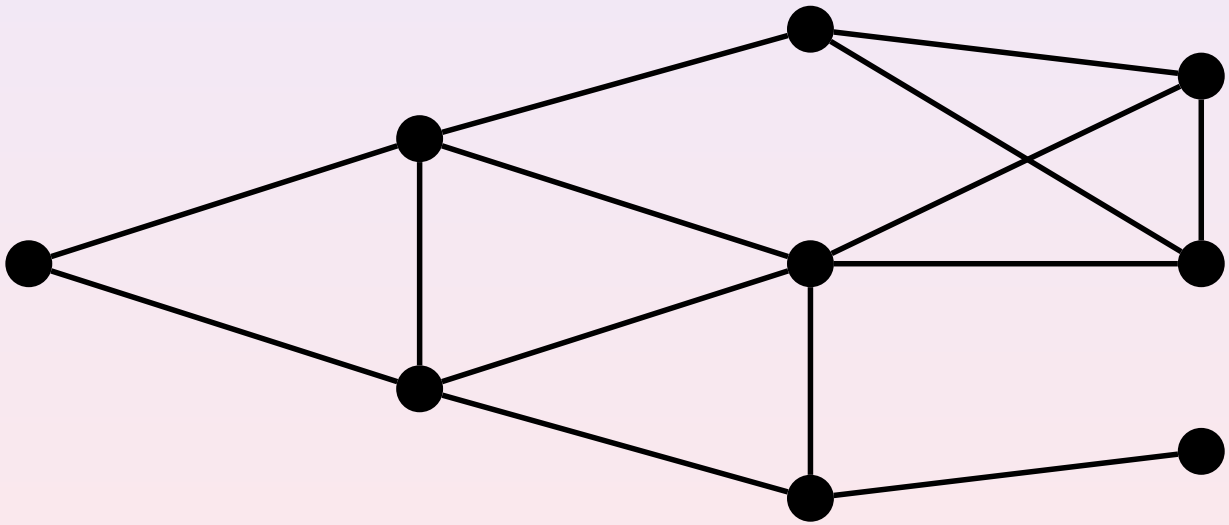

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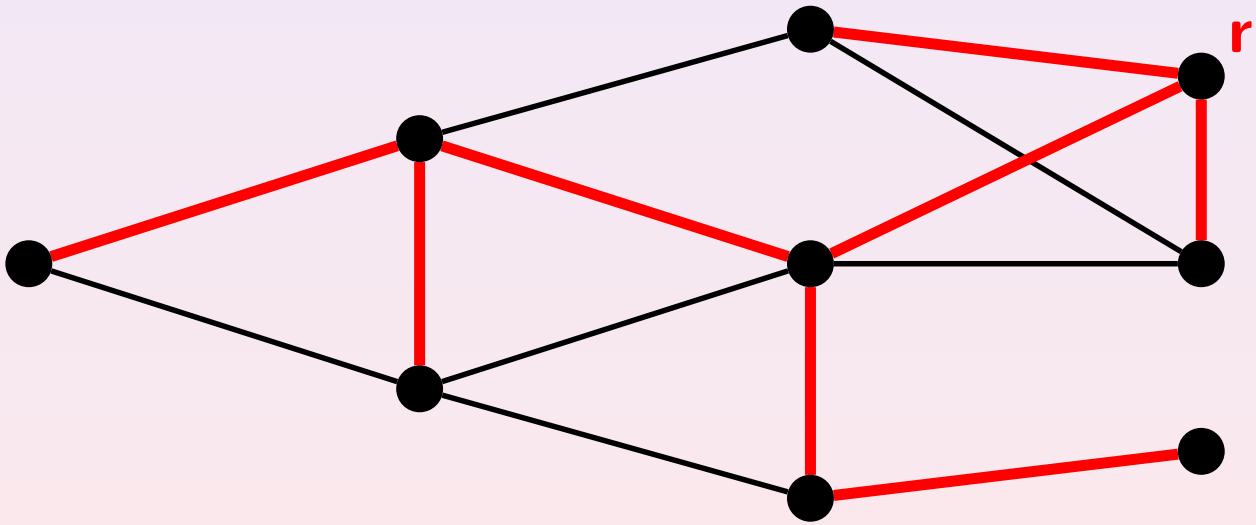

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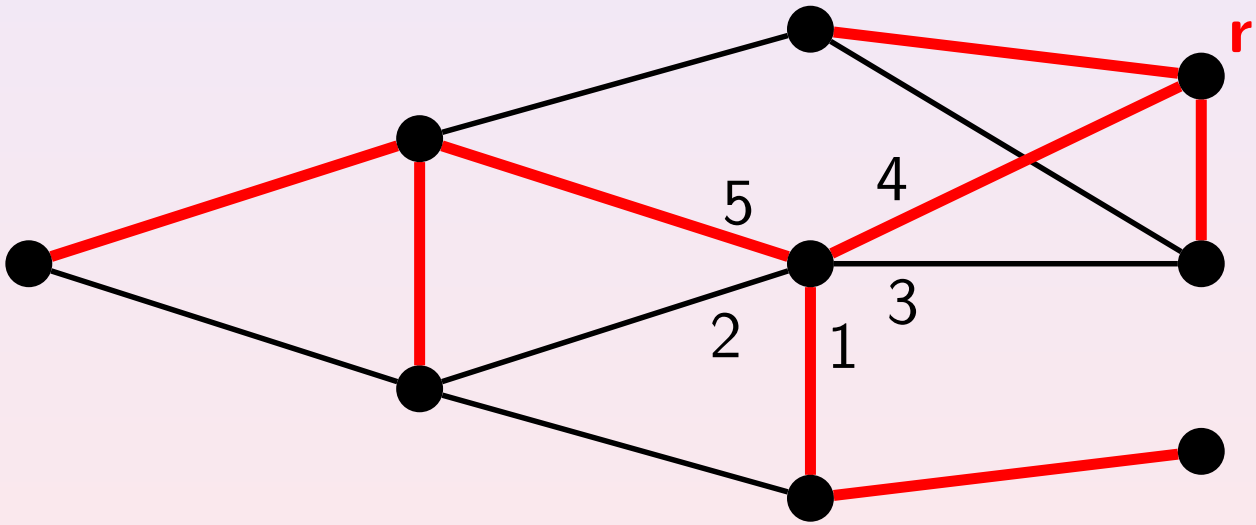

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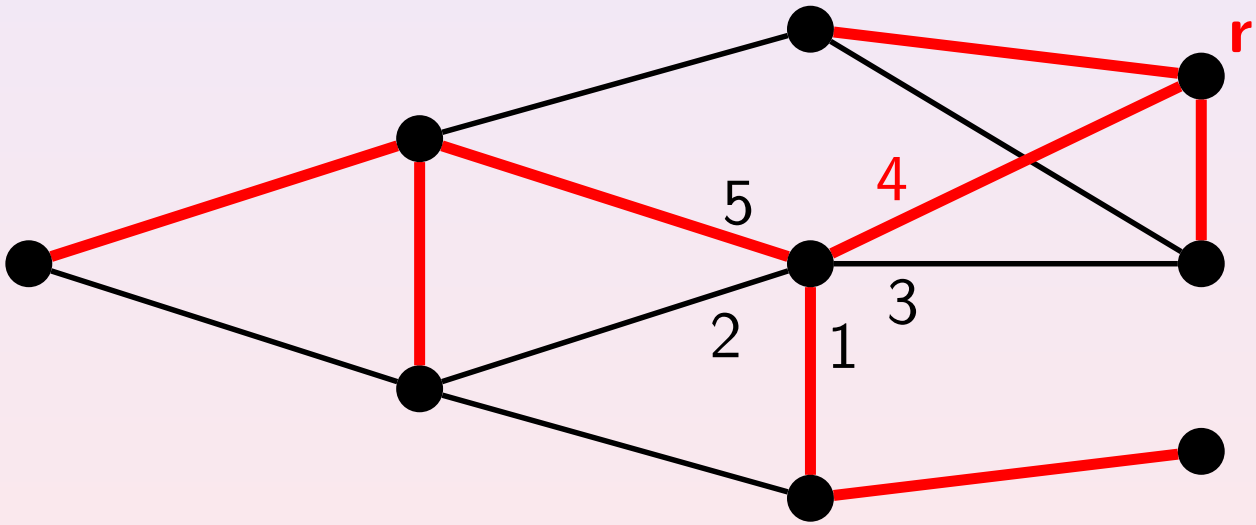
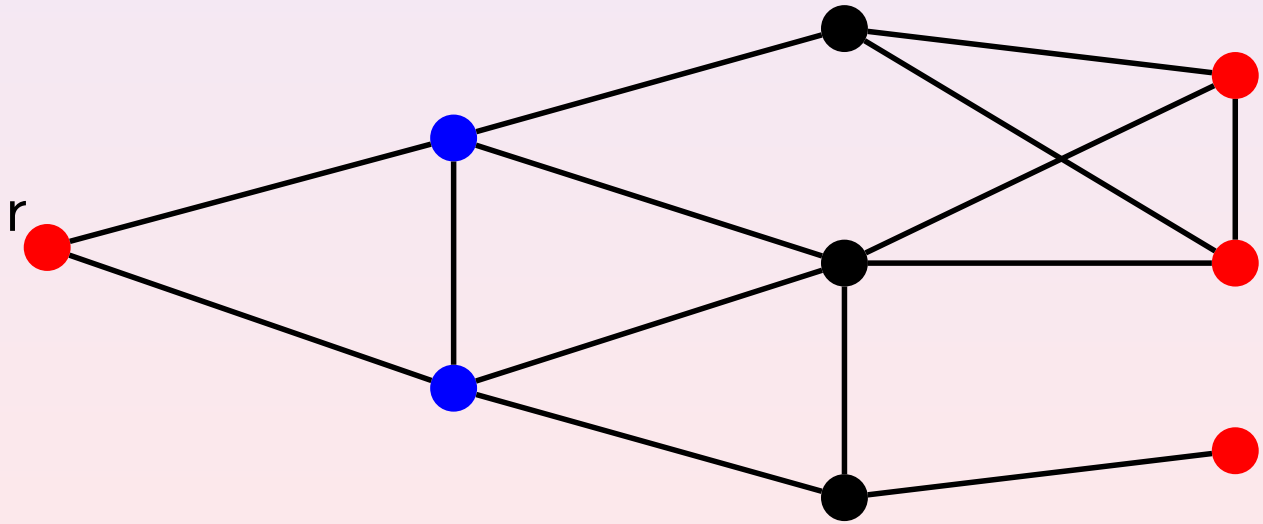


Three colors are enough

- choose arbitrarily a node as the root
- colors all nodes according to their distance d to the root
  - distance $d \cong 0[n]$ red
  - distance $d \cong 1[n]$ blue
  - distance $d \cong 2[n]$ black



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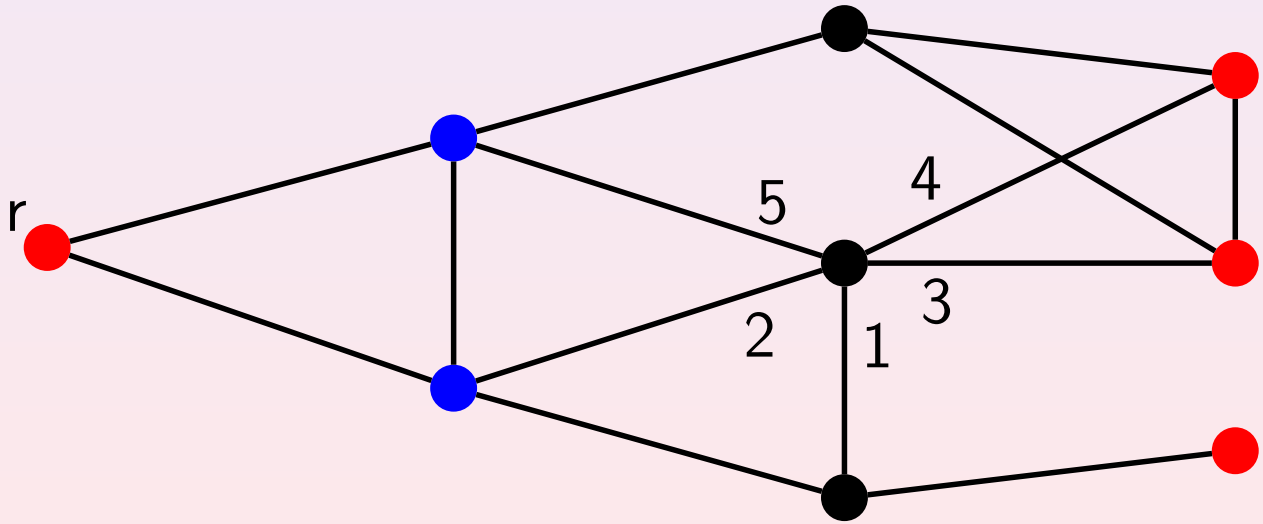

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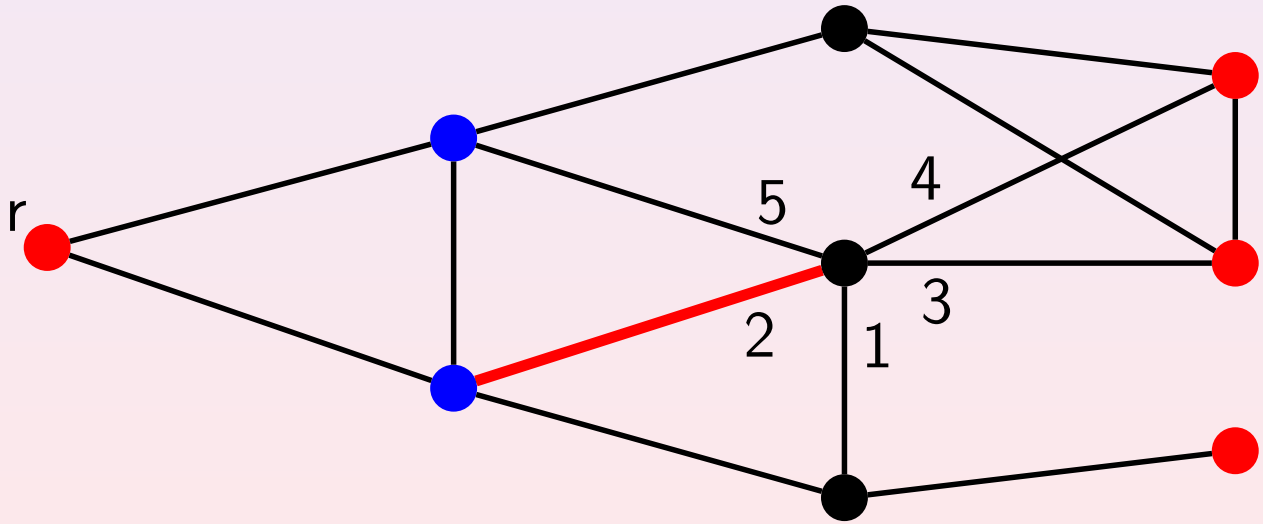
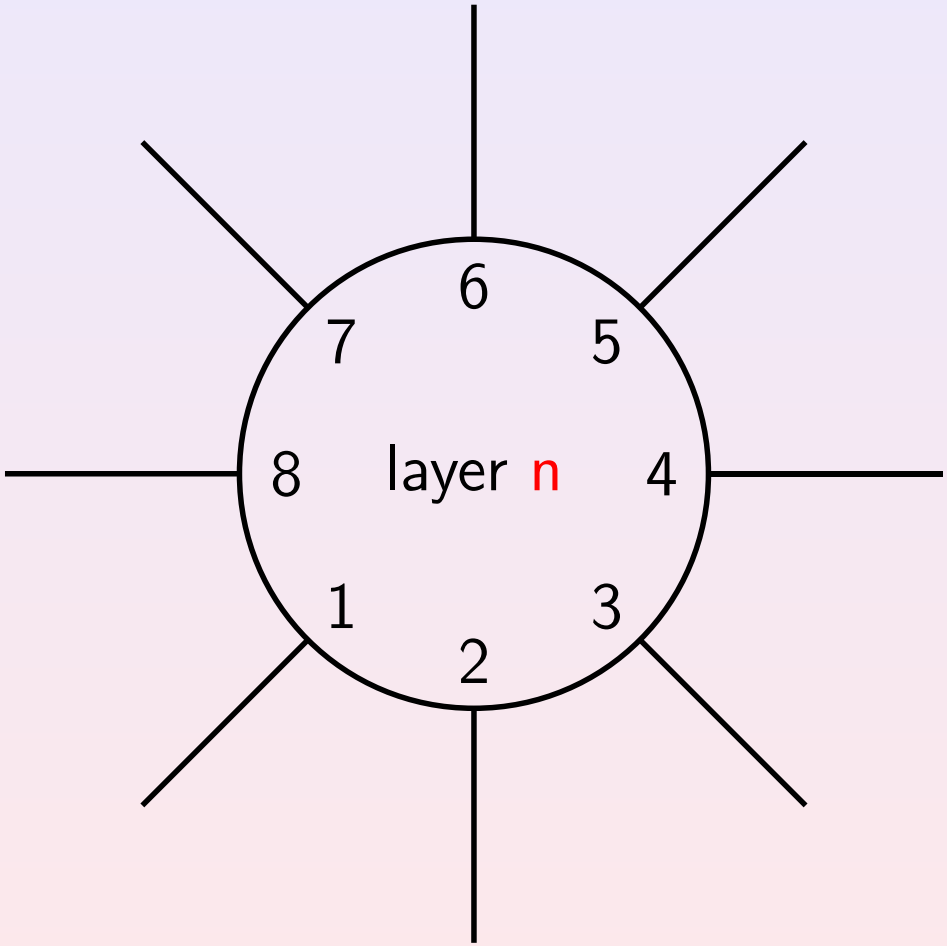
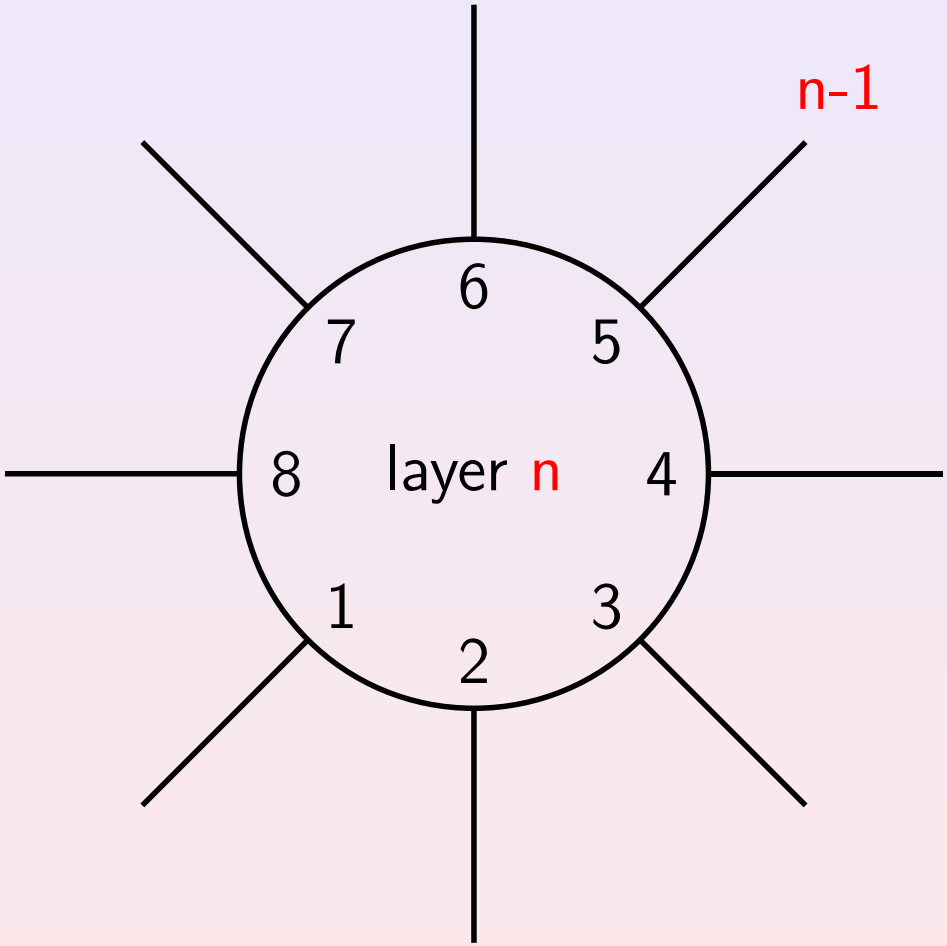


How to find the parent



How to find the parent



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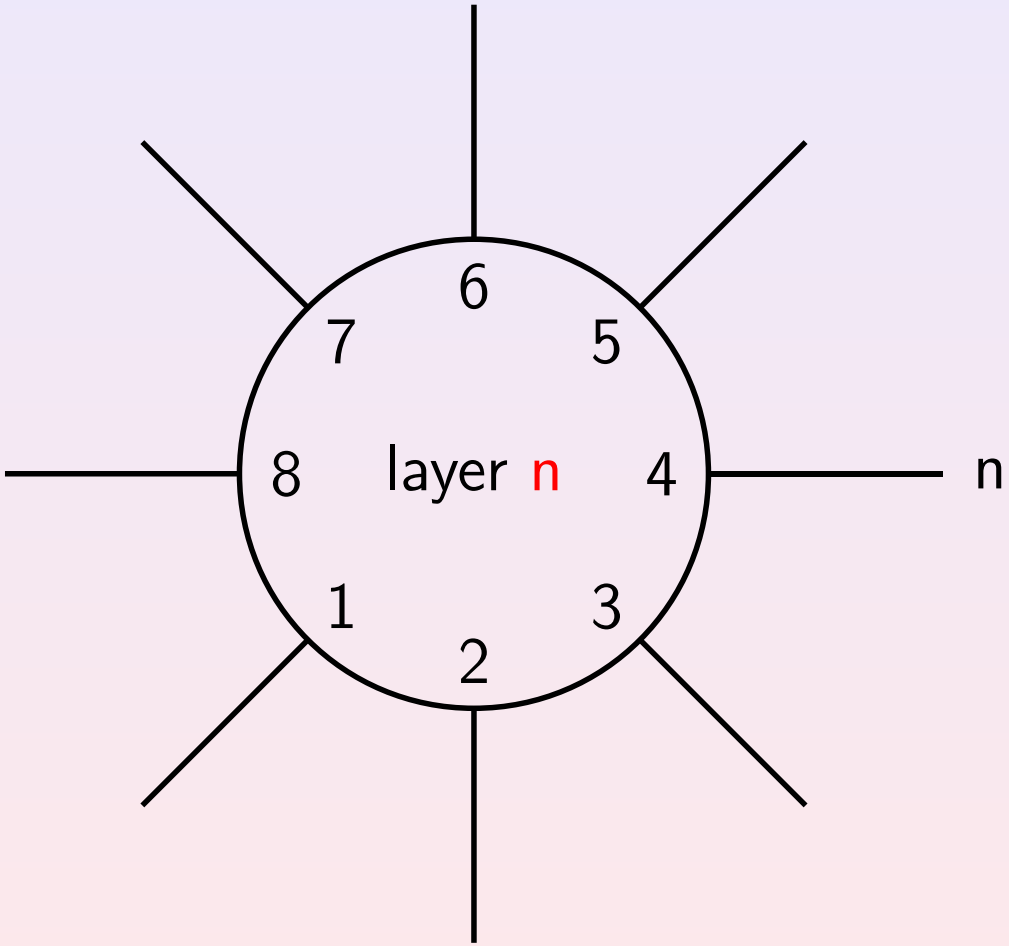

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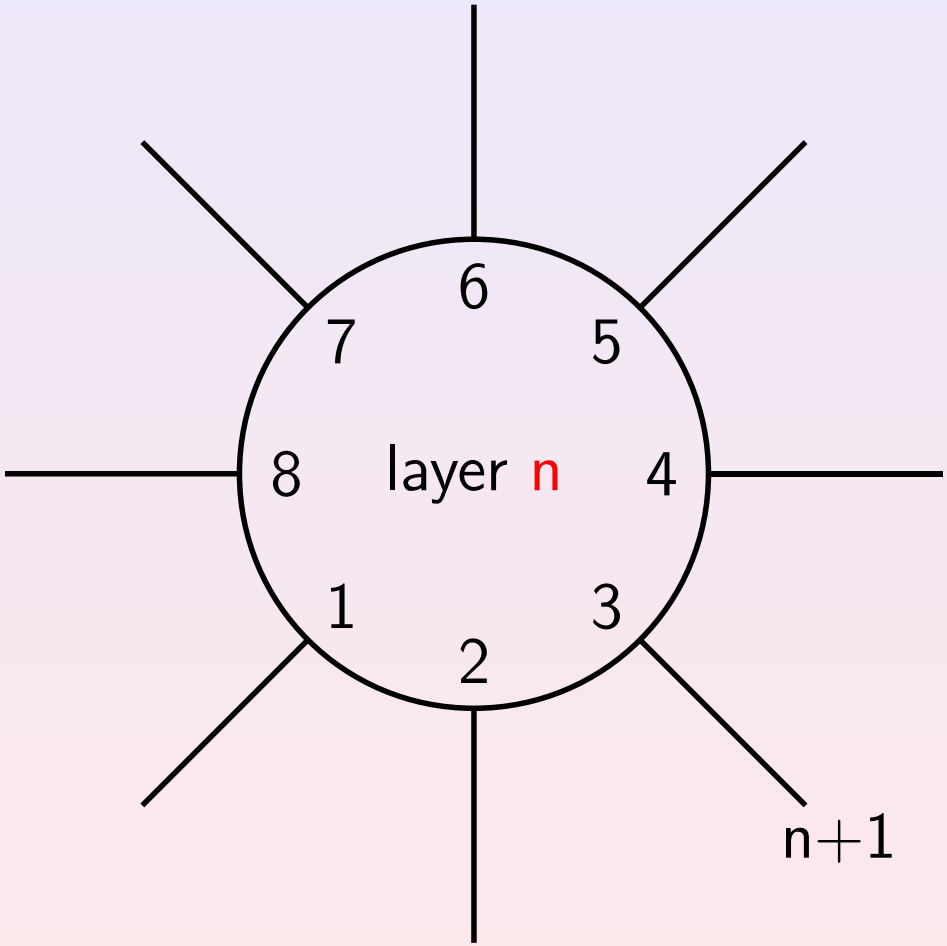

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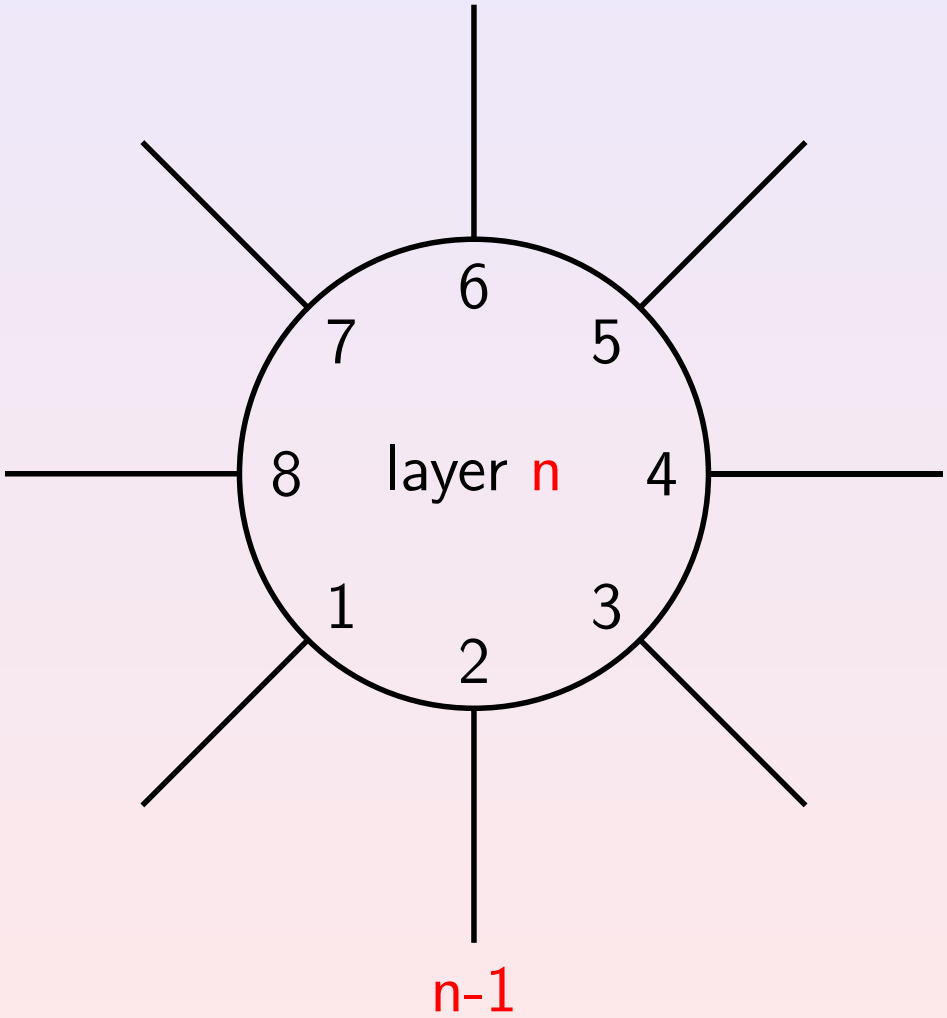

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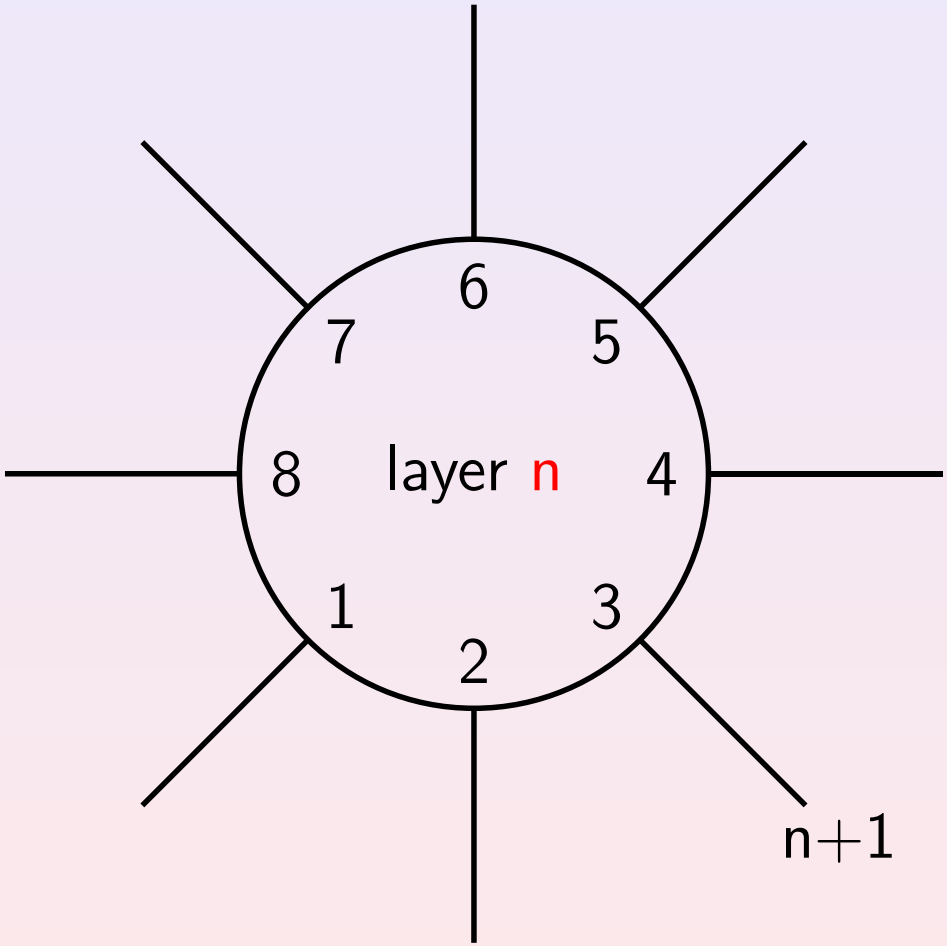

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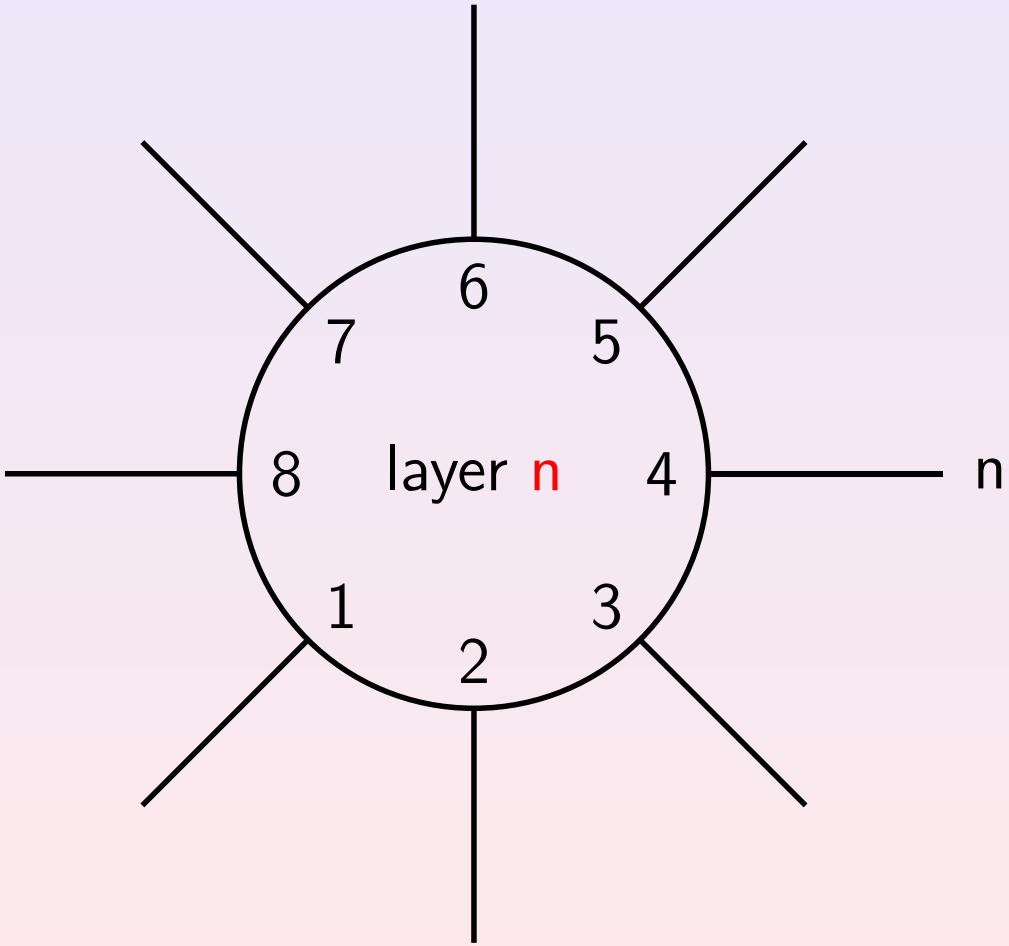

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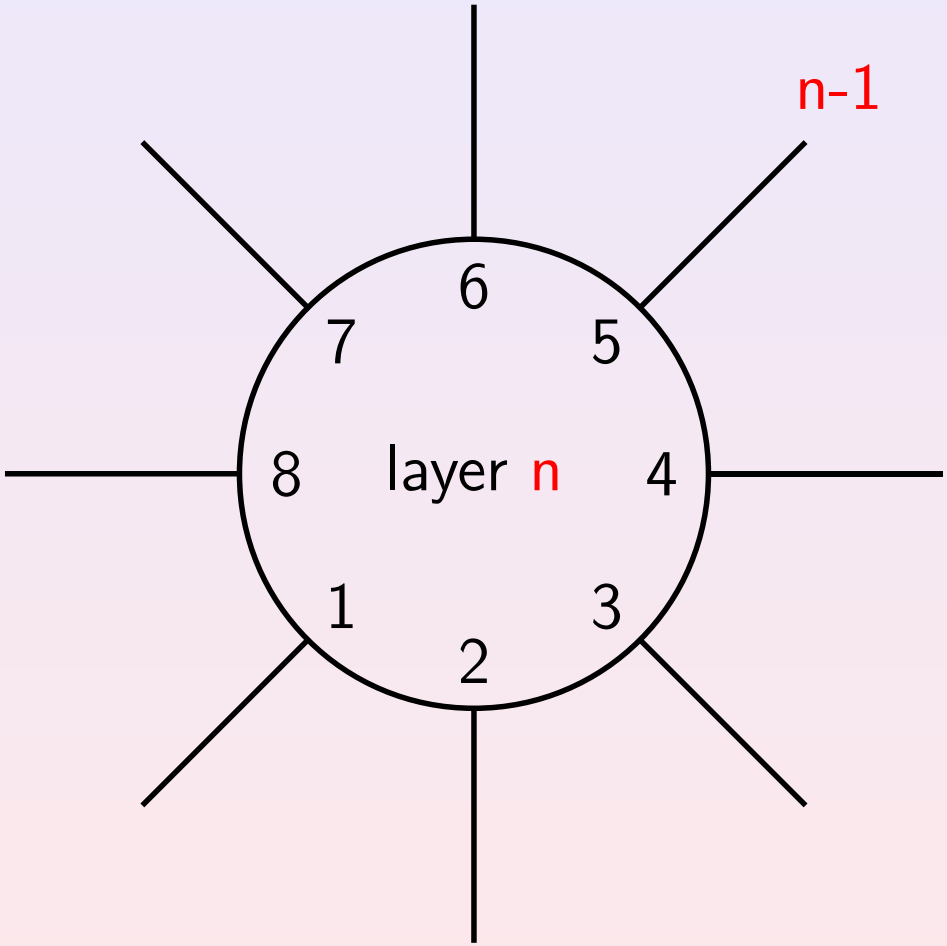

Only two colors?

- Layer 1: red
- Layer 2: blue
- Layer 3: red
- Layer 4: red
- Layer 5: red
- Layer 6: blue
- Layer 7: blue
- Layer 8: blue

Outline

Open problem

Conclusion

- Three colors are sufficient for arbitrary graphs.
- Two colors are necessary and sufficient for graphs of constant degree.

Open problem

Are two colors sufficient for arbitrary graphs?

Open problem

Conclusion

- Three colors are sufficient for arbitrary graphs.
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Open problem

Are two colors sufficient for arbitrary graphs?