

Graph Exploration By A Finite Automaton

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Graph exploration

- Goal
 - A mobile entity has to **traverse** every edge of an **unknown graph**

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 - A mobile entity has to **traverse** every edge of an **unknown graph**
- Motivation
 - exploration of environments unreachable by humans
 - network maintenance
 - map drawing

Unknown and Anonymous graphs

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- Graph of unknown size and of unknown topology

Unknown and Anonymous graphs

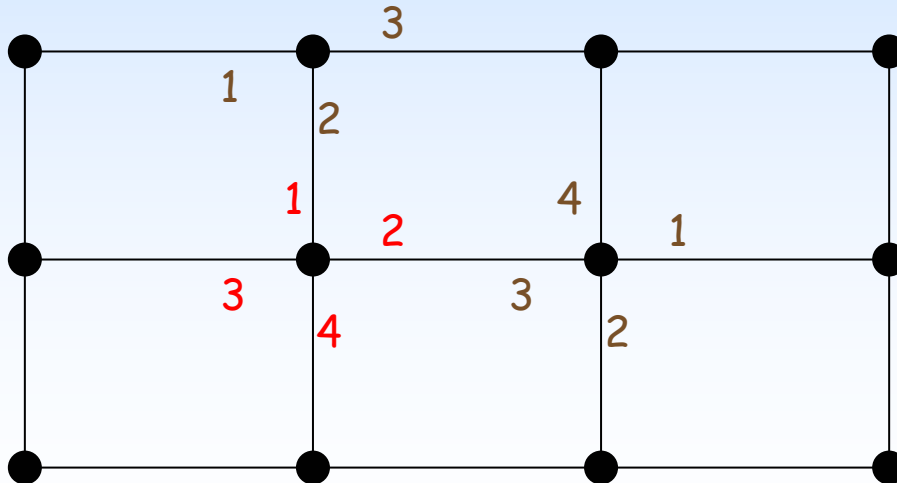
- Graph of unknown size and of unknown topology
- No node labeling

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Robot

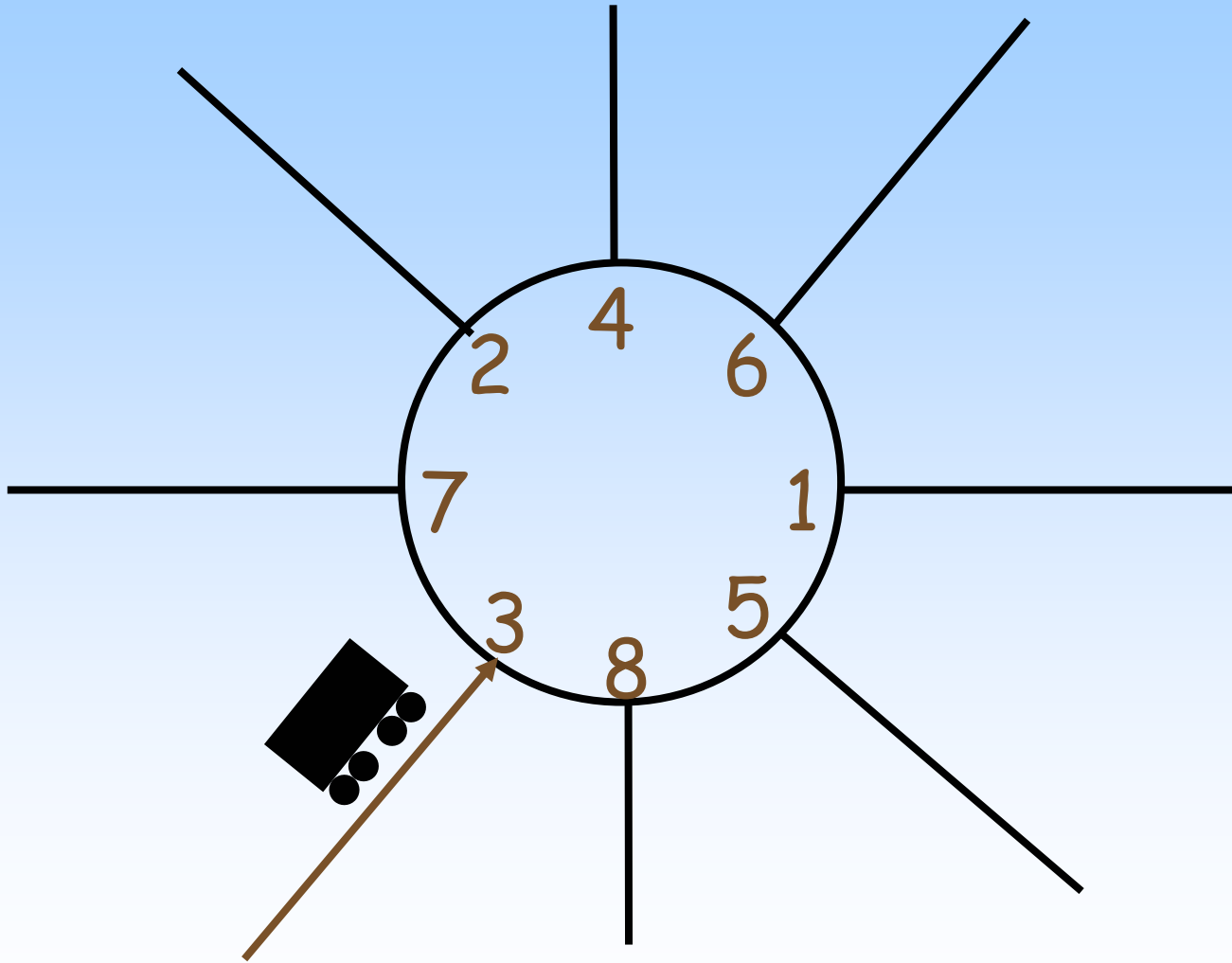
Robot

- Deterministic automaton with K states
→ robot with $\lceil \log K \rceil$ memory bits

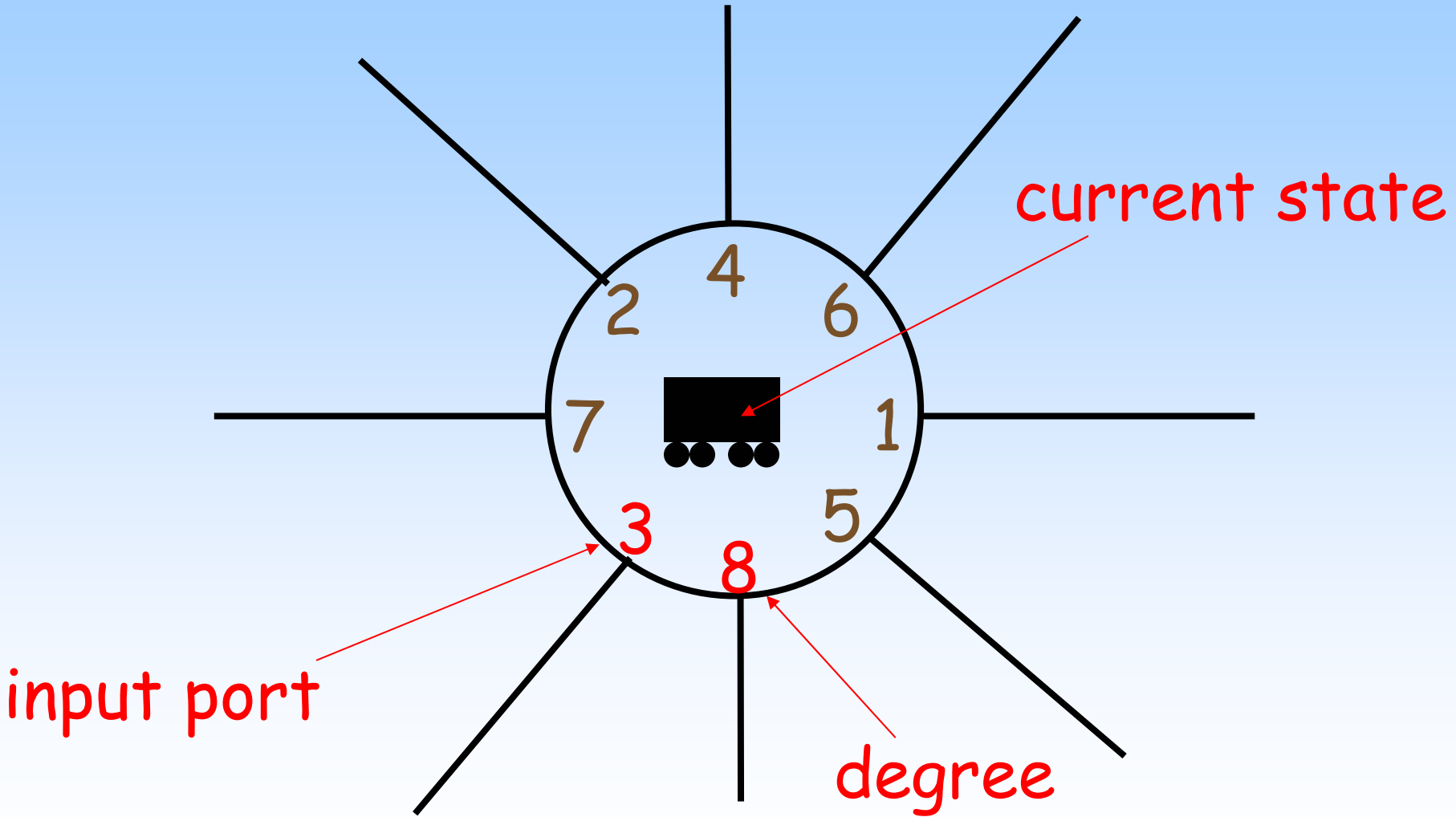
Robot

- Deterministic **automaton** with **K states**
→ robot with $\lceil \log K \rceil$ memory bits
- Moore automaton $\{I, P, S, \delta, \pi, s_0\}$
 - I : input set (degree, port number)
 - P : output set (port number)
 - S : set of states
 - δ : transition function ($I \times S \rightarrow S$)
 - π : output function ($S \rightarrow P$)
 - s_0 : initial state

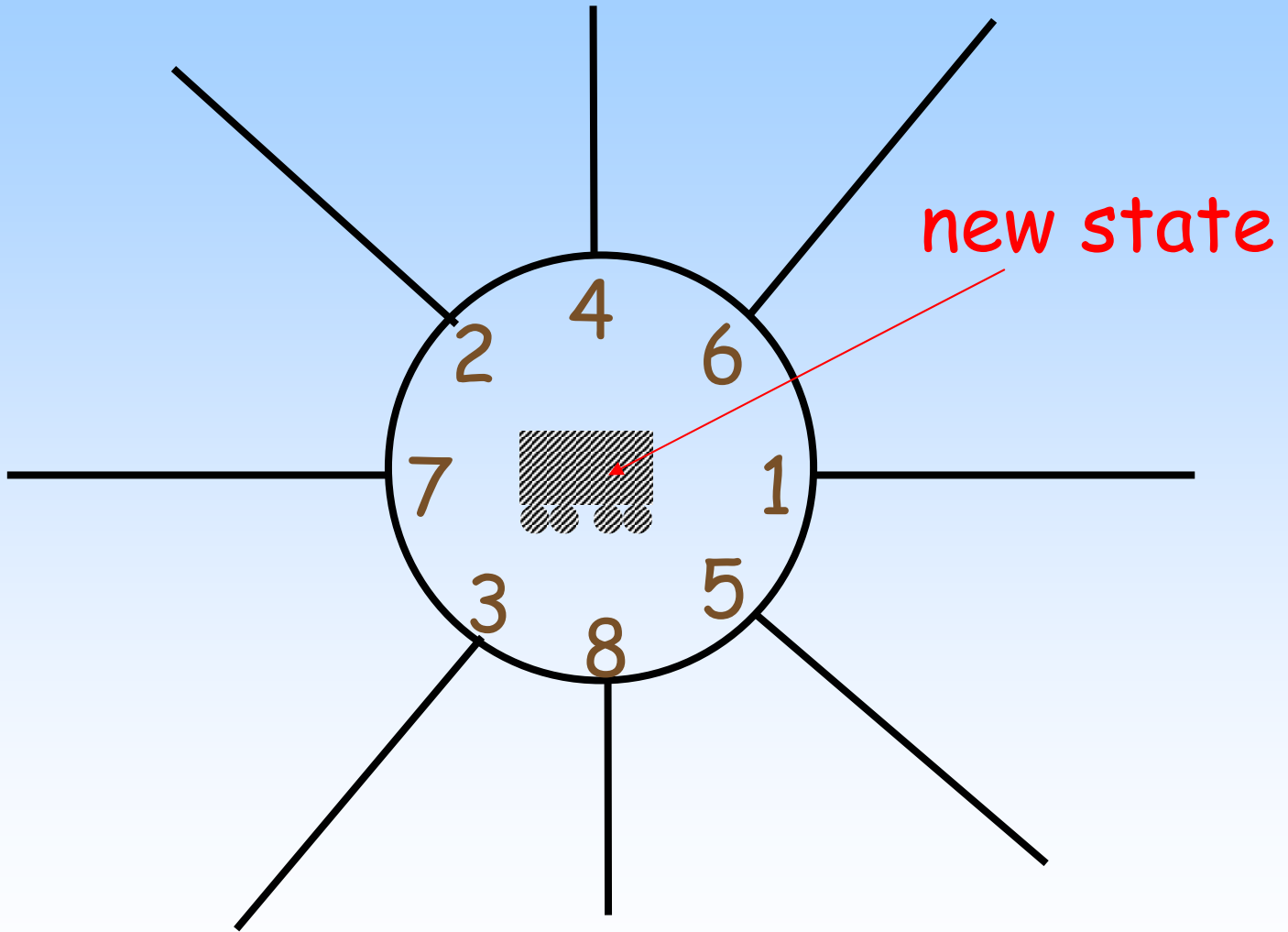
Robot's actions



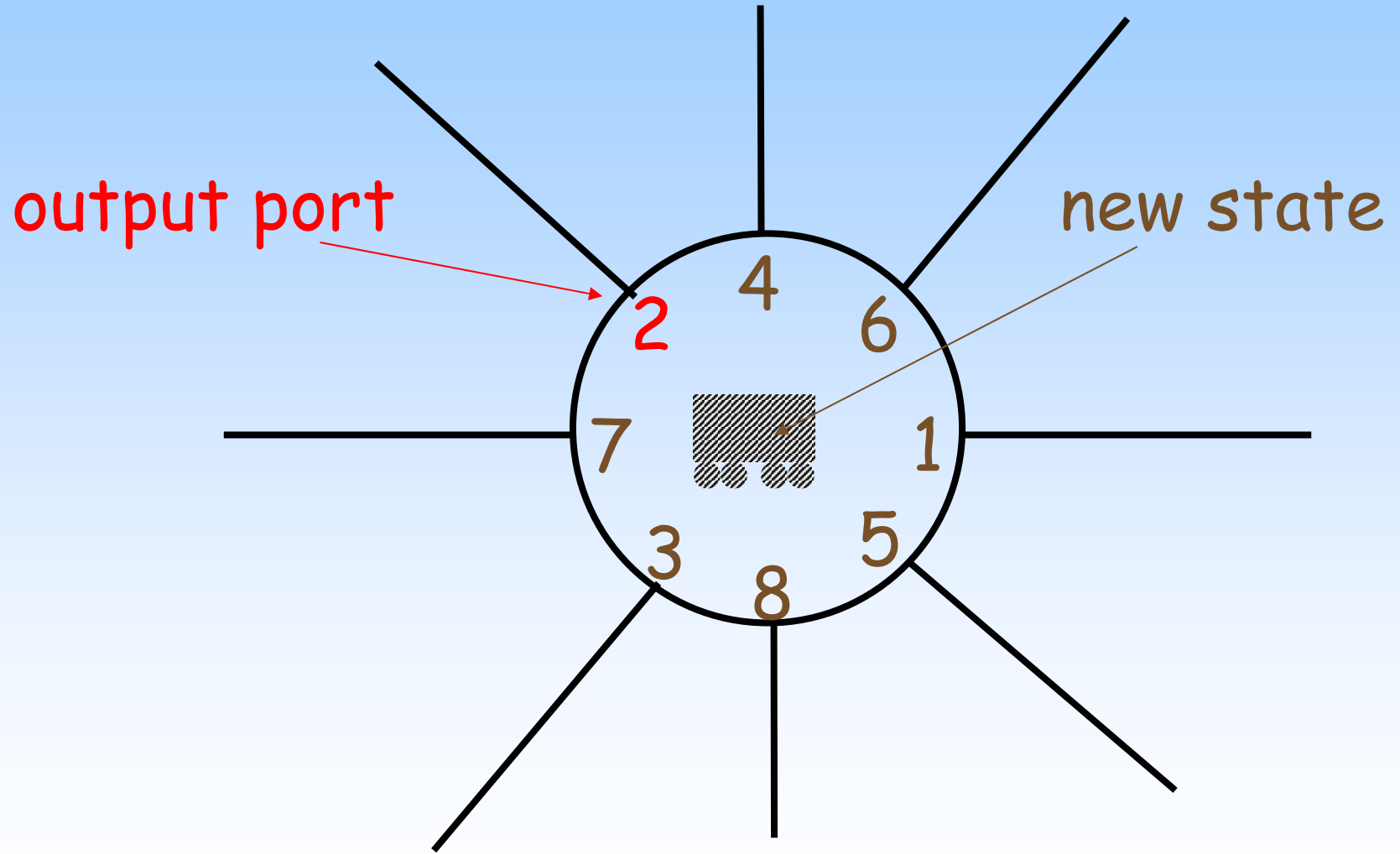
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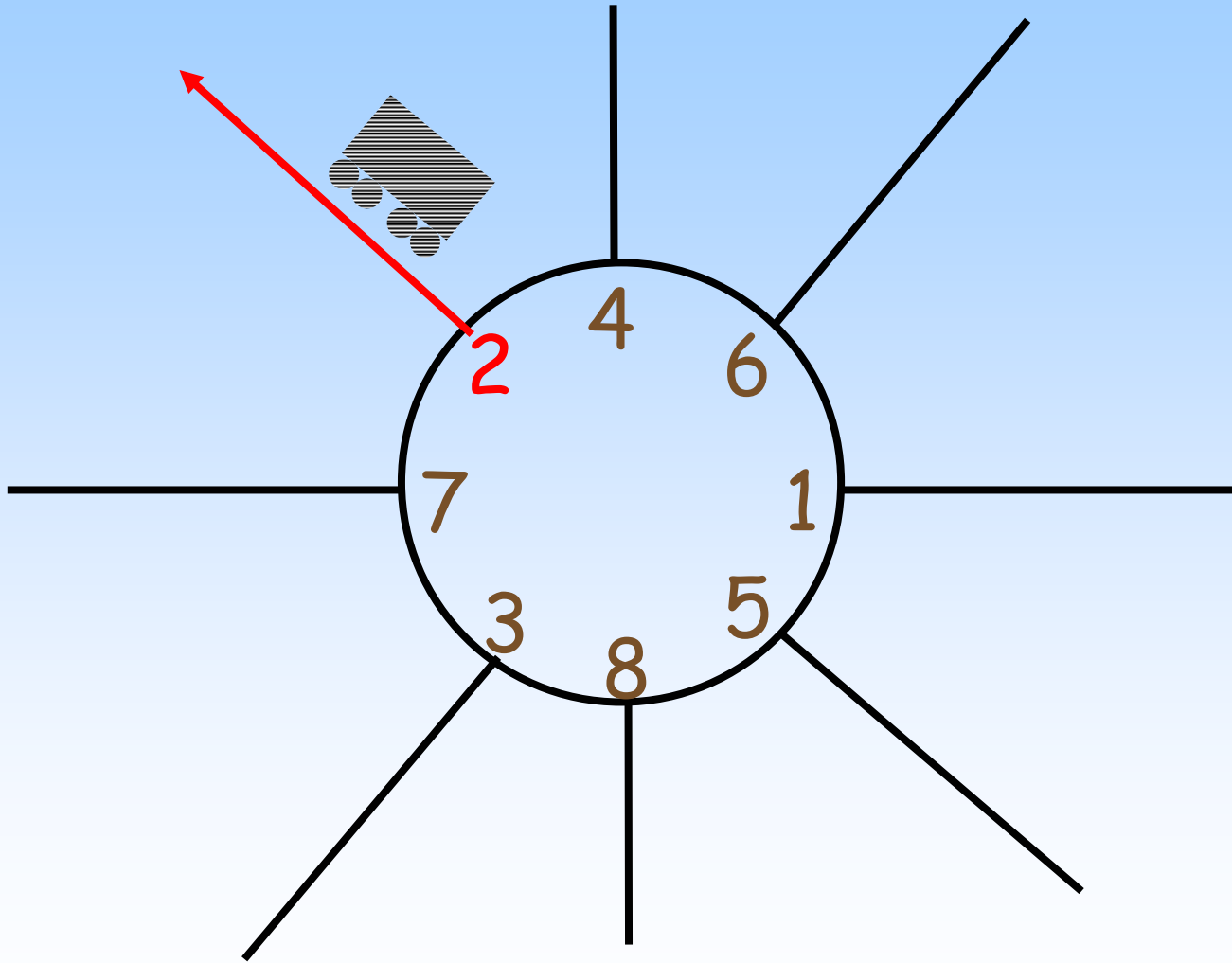
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Outline

- Related work
- Our results
- Trap construction (lower bound)
 - trap for the automaton A = graph that the automaton A cannot explore
- Optimal algorithm (upper bound)

Related work (1)

- First publication on labyrinths
 - Wiener, 1873
- The mouse of Shannon
 - Shannon, Presentation of a maze-solving machine, 1951
- Seminar talk
 - Rabin, Berkeley, 1966

No finite automaton with a finite number of pebbles can explore all graphs

Related work (2)

- Formal arguments
 - Müller, Endliche Automaten und Labyrinth, 1971
 - Coy, Automata in labyrinths, 1977
 - First trap (labyrinths)
 - Budach, Automata and Labyrinths, Math. Nachrichten, 1978
 - Trap for a team of automata
 - Rollik, Automaten in planaren Graphen, Acta informatica, 1980
- No finite set of finite automata can cooperatively perform exploration of all cubic planar graphs

Our results (1)

- Theorem 1

For every K -state robot and every $d \geq 3$, there exists a planar graph of maximum degree d , with at most $K+1$ nodes, that the robot cannot explore.

- Corollary

A robot that explores all n -node planar graphs requires at least $\lceil \log n \rceil$ memory bits.

Our results (2)

- Theorem 2

All graphs of diameter D and maximum degree d can be explored by a robot using $O(D \log d)$ memory bits.

- Theorem 3

A robot that explores all graphs of diameter D and maximum degree d requires at least $\Omega(D \log d)$ memory bits.

- Corollary

DFS-exploration is space optimal

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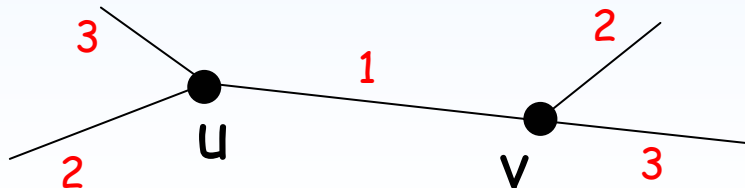
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 - Palindrome of even length : **2112**
 - pp-reduction : **3121123122331** \rightarrow 313

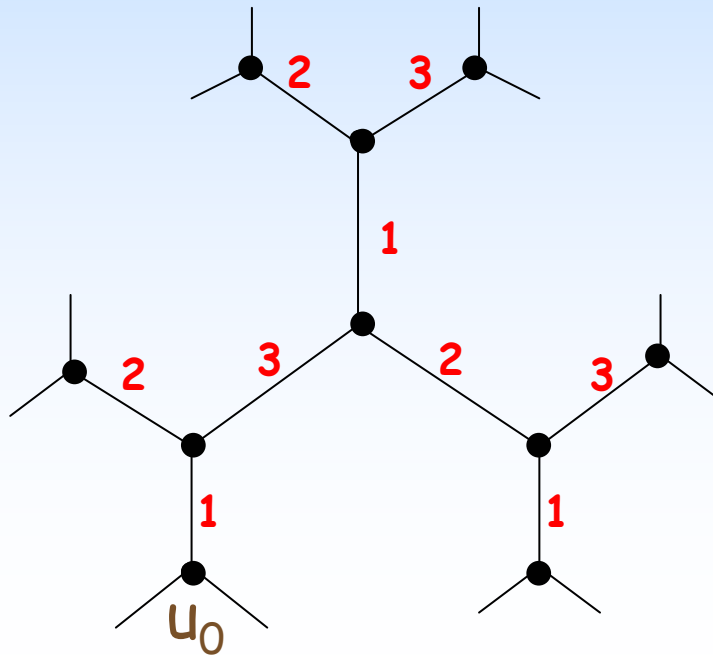
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- Edge-colored graph
 - same label at each extremity of the edge



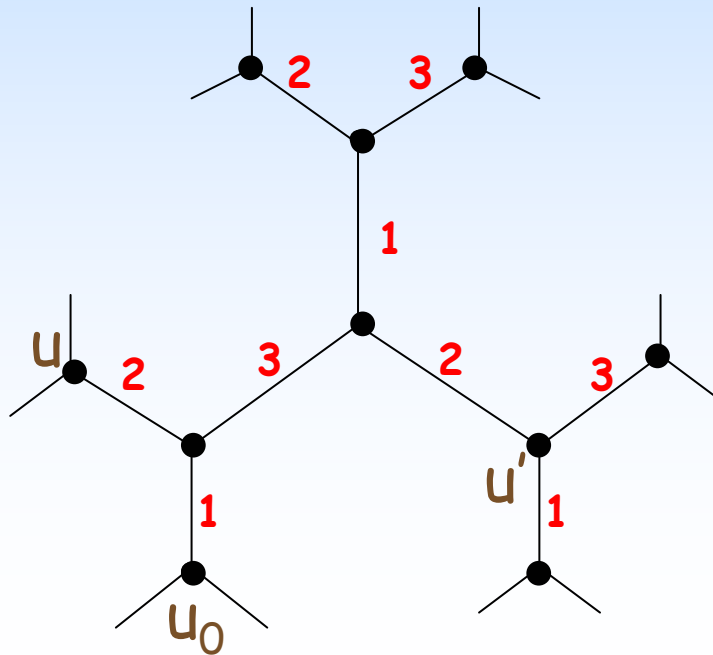
Skeleton of the trap (1)

- Trap based on a 3-regular edge-colored infinite tree
- Goal : trap the robot in a cycle



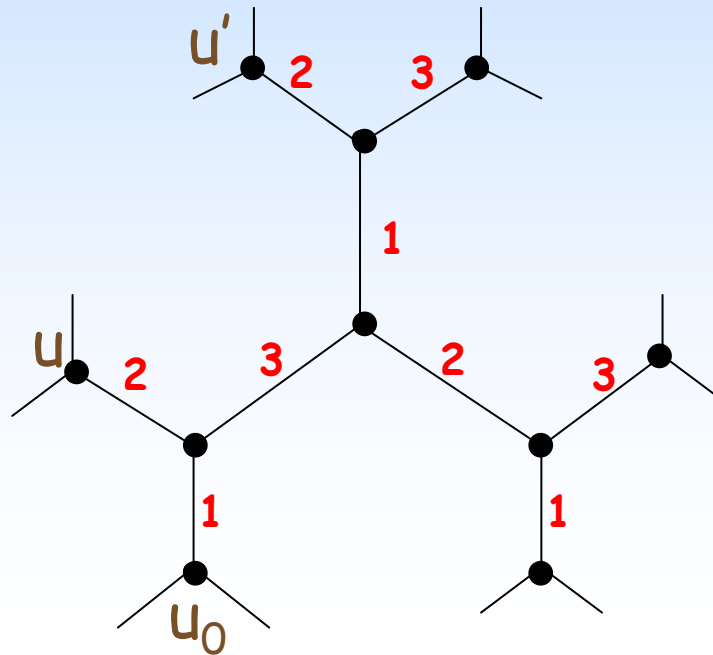
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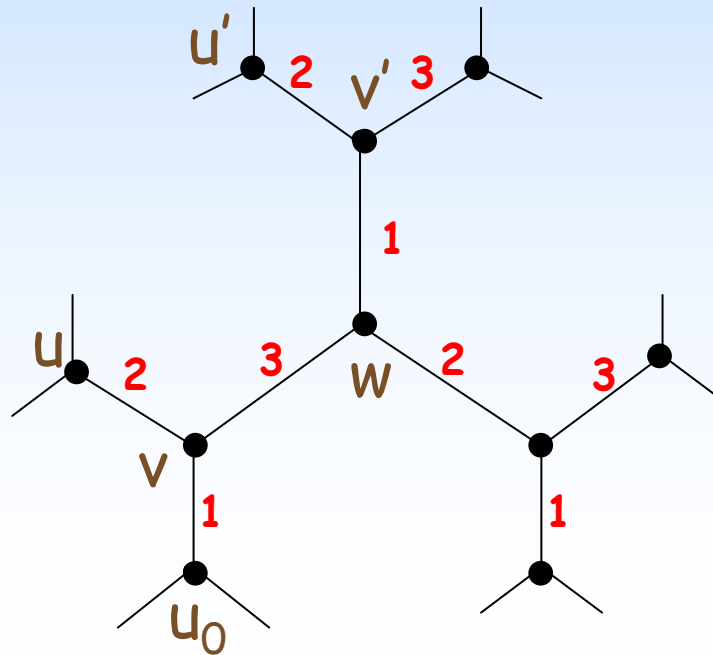
Skeleton of the trap (2)

- General case: the pp-reduction is not a palindrome
- Example : $1332-(23233212)^\infty \rightarrow 2312 \rightarrow 31$



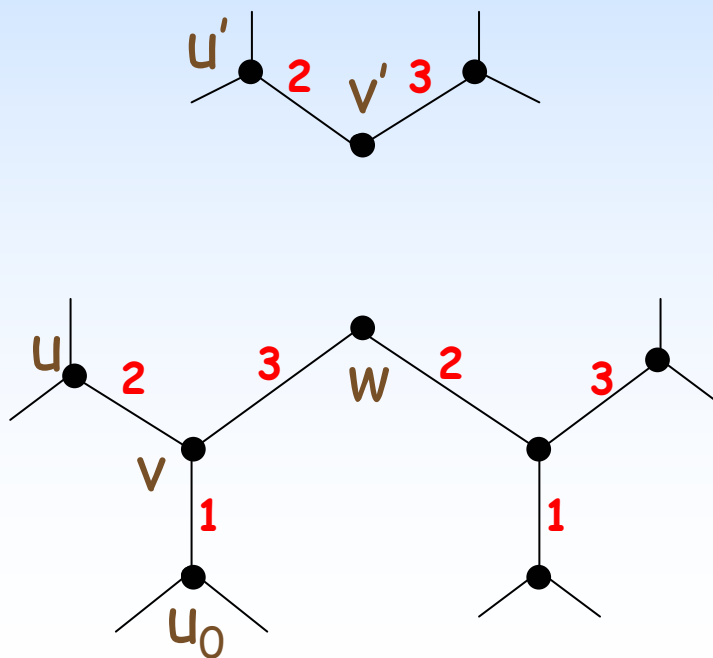
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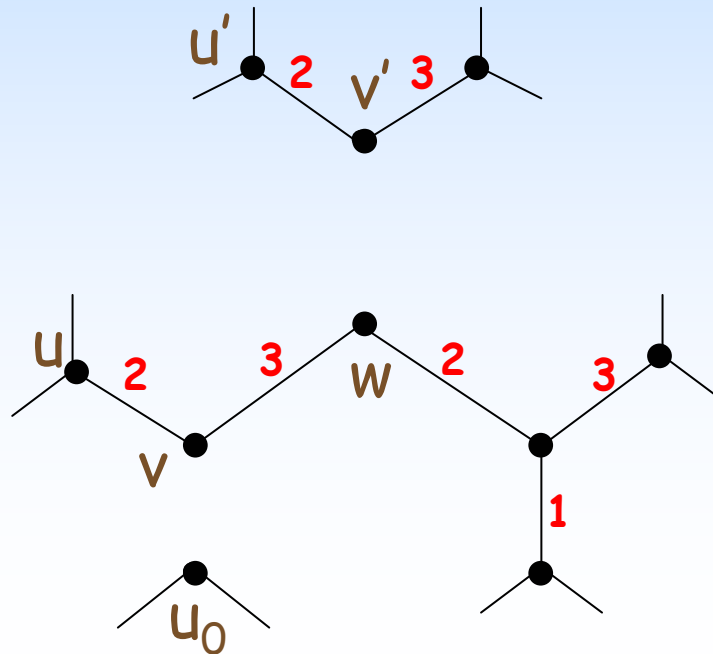
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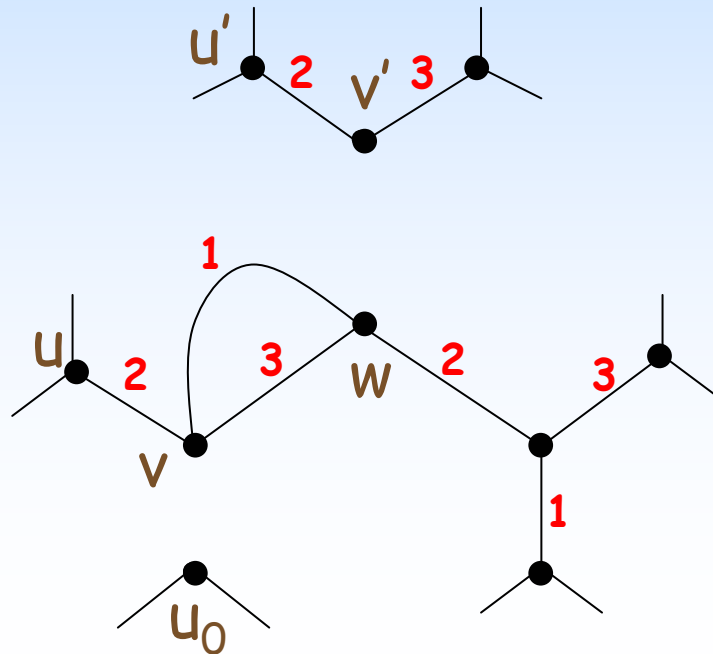
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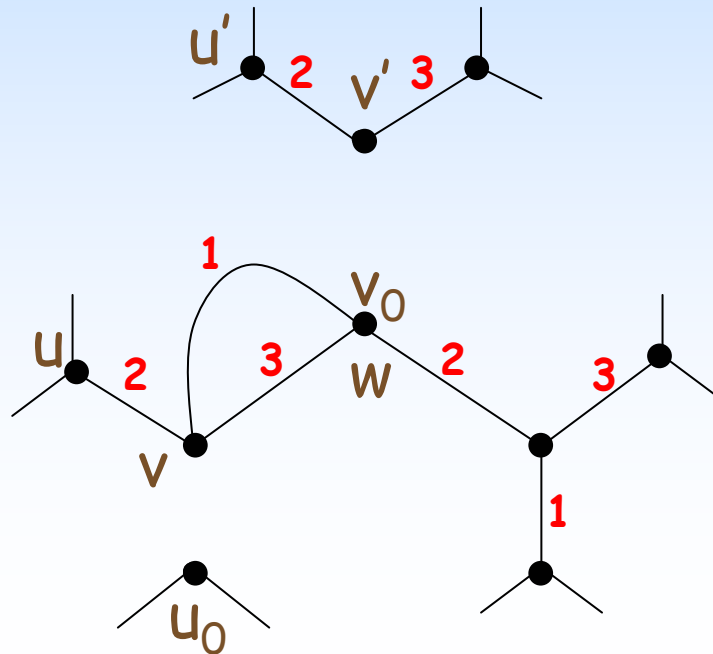
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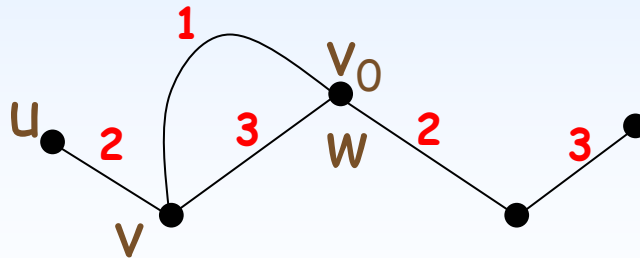


End of proof 1

- There exists a trap of at most $K+1$ vertices

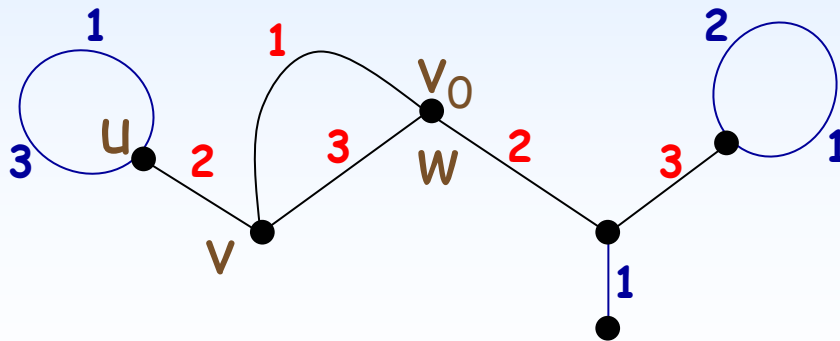
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Exploration algorithm

- Recall of Theorem 2

All graphs of **diameter D** and maximum **degree d** can be explored by a robot using **$O(D \log d)$ memory bits**.

- Proof 2

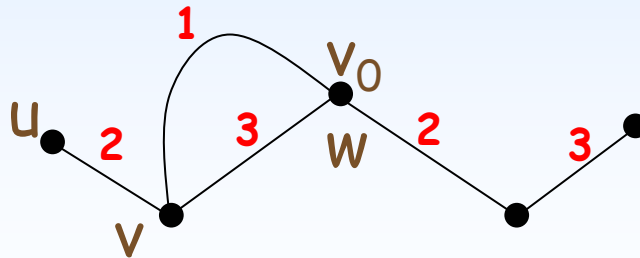
- Depth-First Search (**DFS**) of increasing depth
- Memory cost : **stack of port numbers** leading to the root $\rightarrow D \log(d)$ bits

More appropriate trap (proof 3)

- There exists a trap of diameter at most $4\lceil \log_{d-1}(K) \rceil + 2$

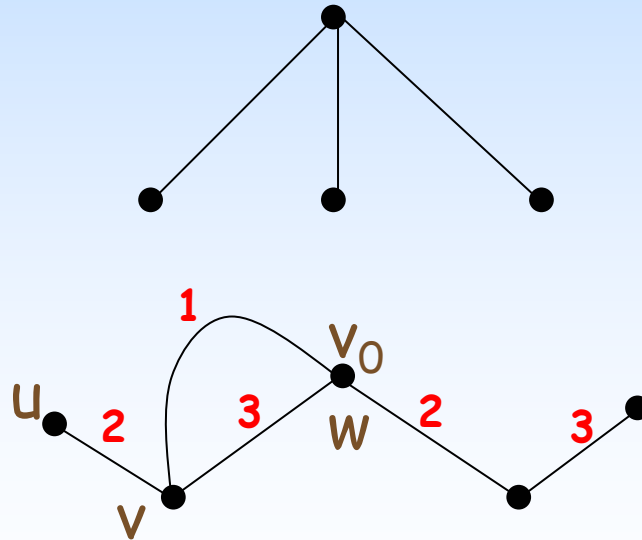
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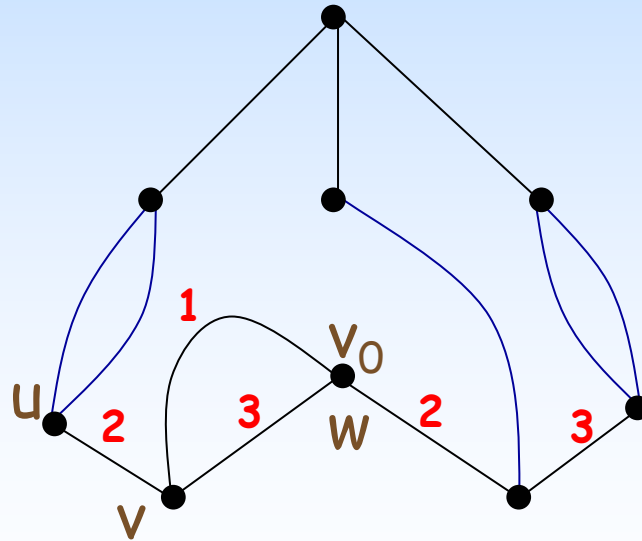
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End of proof 2

- Lemma

For every K -state automaton and for every degree $d \geq 3$, there exists a trap of diameter at most $4 \lceil \log_{d-1}(K) \rceil + 2$.

- Recall of Theorem 3

A robot that explores all graphs of diameter D and maximum degree d requires at least $\Omega(D \log d)$ memory bits.

Summary

- Our results
 - Trap of at most $K+1$ vertices for $d \geq 3$
 - Optimal bound for memory: $\Theta(D \log d)$
 - Holds even for planar graphs